Characterization of Model Uncertainty in Immediate Settlement Calculations for Spread Footings on Clays

Caractérisation de l'incertitude des calculs du modèle de tassement immédiat des semelles de répartition réposant sur les sols argileux

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ABSTRACT: Immediate settlement calculations for spread footings supported by clay soils are generally based on displacement influence factors derived from elastic stress fields and soil stiffness estimated from triaxial compression strength tests or correlations to various measureable characteristics such as plasticity, strength, or stress history. As a consequence of the linear elastic design models, curvature in load-displacement behaviour cannot be characterized unless the stiffness degradation of the subgrade is explicitly incorporated. This paper uses a load test database of spread footings on clay to evaluate the accuracy of an elasticity-based immediate settlement estimation method, which was shown to significantly reduce in accuracy with increasing magnitudes of displacement and exhibit significant variability. A method to predict immediate settlements using a non-linear constitutive model set within an elastic stress field is presented, and is shown to capture the general non-linear shape of footing load tests and maintain its accuracy over a broad range in displacements with similar uncertainty to that of the elasticity-based method. Recommendations are made to estimate an appropriate initial stiffness for use with non-linear and linear elastic models based on back-calculated undrained soil modulus.

RÉSUMÉ: Les modèles acceptés pour prévoir le tassement immédiat des semelles de répartition reposant sur les sols argileux sont généralement basés sur des facteurs d'influence de déplacement calculé à partir des champs de contraintes élastiques en conjonction avec la rigidité du sol estimée par des essais triaxiaux, ou les corrélations aux paramètres mesurables variés comme la plasticité, la résistance au cisaillement non drainé, et la contrainte de préconsolidation. À cause de l'utilisation du modèle élastique linéaire, la non-linéarité de la relation charge-déplacement ne peut pas être caractérisée sauf la dégradation de la rigidité du sol de fondation est incorporée explicitement. Cet article utilise une base de données des semelles de répartition essayées sur les sols argileux pour évaluer la précision d'une méthode d'estimation du tassement immédiat à cause d'élasticité. Cette méthode se dégrade sérieusement en termes de précision avec l'augmentation de déplacement, et, elle est aussi caractérisée par une grande variabilité. Dans cet article, une méthode est présentée qui prévoit les tassements immédiats à l'aide d'une modèle constitutif non-linéaire dans un champ de contrainte élastique et il est démontré qu'elle affiche une forme générale non-linéaire des essais de charge des semelles de répartition et qu'elle garde une bonne précision pour un grand nombre de déplacements ayant une incertitude similaire à celle de la méthode basée sur l'élasticité. Des recommandations ont été faites pour estimer une rigidité initiale appropriée pour l'utilisation avec les modèles d'élasticité non-linéaires et élastiques linéaires qui sont fonction des modules de sols non-drainés calculés rétrospectivement..

KEYWORDS: elastic settlement, settlement, shallow foundations, clay soils, undrained loading.

1 INTRODUCTION

Spread footings are used throughout the world as viable foundation support systems for structures. They are typically constructed of reinforced concrete, can assume any shape, and generally meet the following criteria (Vesic, 1973; Das, 2011):

1. The depth of footing embedment, D_f, lies between the ground surface and up to four times the footing width, B, below the adjacent grade, and
2. Additional support, such as driven piles or drilled shafts, are not located beneath the footing.

Designers must evaluate two conditions to ensure that the foundation will perform adequately (Perloff and Baron, 1976): safety against overall bearing failure in the supporting soil, and displacements leading to unsatistfactory structural performance must not occur. The first condition is often considered the most critical limit state; however, immediate settlement can lead to a serviceability limit state and must be included in design.

Generally accepted methods for estimating immediate settlement of spread footings require the use of linear elastic models to simulate soil behavior; this approach does not capture the true non-linear behavior of soil. This study presents a statistical evaluation of a commonly used elasticity-based method and soil stiffness correlation using a load test database. Then, a simple non-linear model capturing observed load-displacement curvature in footing load tests is presented and its accuracy is characterized. The undrained initial elastic modulus is back-calculated using the load test database, and is found to vary as a function of overconsolidation ratio.

2 IMMEDIATE SETTLEMENT OF SHALLOW FOUNDATIONS ON CLAY

2.1 *Elasticity-based design methodology*

Carrier and Christian (1978) found that stress distributions developed from finite element analyses (FEA) used in conjunction with embedment factors proposed by Burland (1970) produced the most reasonable values of displacement assuming an undrained Poisson's ratio, ν_s, equal to 0.5 for clay. Mayne and Poulos (1999) modified Burland's work and developed an improved distortion settlement estimation approach for circular foundations that accounts for variations in Poisson's ratio, soil modulus, foundation rigidity, and embedment effects. The resulting expression for immediate settlement can be constructed as (Mayne and Poulos 1999):

$$\delta_i = \frac{q_{app} B_{eq} I_E I_F I_G}{E_s}\left(1-\nu_s^2\right) \tag{1}$$

where q_{app} = applied bearing stress, B_{eq} = equivalent diameter of the footing, I_E, I_F, I_G are displacement influence factors that control the magnitude of displacement (described below), and E_s is the Young's modulus of the soil.

The stresses below a spread footing, and therefore the immediate settlement, are affected by the amount of footing embedment. Burland's embedment influence factor, I_E, is used to modify the stress distribution for embedment effects.

Regression analyses on Burland's charts yielded a simple representation of I_E (Strahler 2012):

$$I_E = 1 - \frac{1}{3.5e^{(1.22\nu_s - 0.4)}\left(\frac{B_{eq}}{D_f}\right)} \tag{2}$$

In addition to embedment effects, stresses below a spread footing are also affected by the rigidity of the foundation. The rigidity correction factor, I_F, is used to modify the stress distribution to account for foundation rigidity and is given by (Mayne and Poulos 1999):

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10K_f} \tag{3}$$

where K_f = the foundation flexibility factor (Brown, 1969) and is a function of the modulus of the soil as well as the modulus, thickness and radius of the foundation.

Soil profiles that exhibit a linear increase in modulus with depth, termed a Gibson profile (e.g., Mayne & Poulos, 1999), may be modeled using the Gibson displacement influence factor, I_G, given by:

$$I_G = \frac{1}{\left[1.27 + 0.75\left(\frac{E_o}{k_E B_{eq}}\right)^{-0.8}\right]} \tag{4}$$

where E_o is Young's modulus of the soil directly beneath foundation, k_E is the rate of increase of modulus with depth.

The use of Eqn. (1) requires an estimate of soil stiffness; for undrained loading of footings on clay the appropriate stiffness for linear elastic models is the undrained Young's modulus, E_u. Although many correlations to E_u exist (e.g., Kulhawy and Mayne 1990), Duncan and Buchignani (1987) suggested that E_u was linearly proportional to undrained shear strength, s_u, and proposed the following commonly used expression:

$$E_u = Ks_u \tag{5}$$

where K = the constant of proportionality and is a function of stress history and soil plasticity. Duncan and Buchignani (1987) proposed Figure 1 to indicate the sensitivity of K to plasticity index (PI) and overconsolidation ratio (OCR).

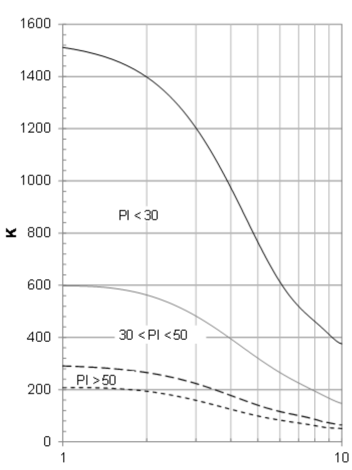

Figure 1. Variation in the K-factor based on OCR and PI (adapted from Duncan and Buchignani 1987).

2.2 *Non-linear distortion displacement models*

Several researchers have pointed to the limitations of linear elastic-perfectly plastic model behavior and developed non-linear distortion displacement models that attempt to more accurately estimate displacements (Osman and Bolton, 2004; Elhakim and Mayne, 2006; Foye, et al., 2008). These methods are either computationally intensive, require significant or potentially expensive subsurface information, or rely on FEAs that assume homogeneous or isotropic soil conditions and are limited to specific stress conditions. As a result they may not be applicable to many realistic design scenarios and are limited in their appropriate uses.

3 LOAD TEST DATABASE AND STATISTICAL APPROACH FOR IMMEDIATE SETTLEMENT MODEL EVALUATION

3.1 *Development of load test database*

To evaluate the uncertainty in the linear elastic distortion settlement calculation and provide the basis for a new model, a database of case histories was developed. The database was initially populated with 24 case histories and was subsequently reduced to 12 with 30 individual footing load tests based on the quality of soil and load test information. The stress histories represented in the database largely consist of lightly to heavily overconsolidated soil profiles, with just one true normally consolidated soil profile. The database included 13 square foundations and 17 circular footings. Twenty-eight of the footings were embedded below the ground surface. Further details on the load test database are given in Strahler (2012), and are not described here for brevity.

3.2 *Statistical approach*

The accuracy of the immediate settlement models evaluated herein was characterized using the mean bias, λ, defined as the ratio of an observed and calculated displacement, and its distribution. Distributions of the sample bias values were assessed using goodness of fit metrics, and appropriate second moment statistics were determined. The coefficient of variation (COV) of the bias, defined as the standard deviation in bias divided by its mean, is used herein as a convenient representation of dispersion. Details regarding distribution fitting are given by Strahler (2012).

4 EVALUATION OF THE ELASTICITY-BASED APPROACH

Equation (1) was rearranged to compute the elasticity-based bearing pressure, q^e_{app}, for each displacement, δ_i, for a given load-displacement curve:

$$q^e_{app} = \frac{E_u \delta_i}{B_{eq} I_G I_F I_E \left(1 - \nu_s^2\right)} \tag{6}$$

To evaluate the performance of Eqn (6) using the footing load test database, the undrained shear strength was averaged over B_{eq} and the constant of proportionality, K, was linearly interpolated from Figure 1 for data pairs of PI and OCR. The upper dark line was assumed to correspond to a PI = 0, whereas the lower dashed line was assumed to correspond to a PI = 100. For case histories with soil layers characterized with OCRs greater than 10, K was assumed to be equal to the value at OCR = 10 (Figure 1).

Following the computation of bearing pressures, the sample biases were calculated and their statistical distribution determined. The mean bias for a displacement of 10 mm was 0.85, indicating that the undrained Young's modulus estimated using Figure 1 and Eqn. (6) is moderately un-conservative (i.e.

the calculated bearing pressure for 10 mm of displacement is greater than that measured). However, the model exhibited significant variability, with COV = 85 percent. The accuracy in the selected approach decreases significantly with increases in magnitude of displacement. For example, at displacements of 25 and 50 mm, Eqn. (6) and Figure 1 produced mean biases and COVs of 0.46 and 88 percent, and 0.17 and 54 percent, respectively. The COV at 50 mm is somewhat smaller due to the reduction in the number of bearing pressure-displacement data pairs at larger displacements available in the database.

5 DEVELOPMENT OF PROPOSED MODEL

The evaluation of the elasticity-based approach presented above indicated a need for more accurate immediate settlement calculations. An accurate model should account for the non-linear response of footings loaded rapidly on clays. An approach that incorporates common triaxial strength test data within an elastic stress field is described below.

5.1 *Selected constitutive response*

The Duncan-Chang hyperbolic model (Duncan and Chang, 1970; Duncan et al. 1980) is a non-linear soil constitutive model that expresses the development of the principal stress difference as a function of axial strain, initial Young's modulus, and effective confining pressure. The stress path that develops below the center of a footing is similar to an undrained triaxial compression stress path (Stuedlein and Holtz, 2010). The failure criterion can be defined as the point at which half of the principal stress difference exceeds the available shear strength:

$$(\sigma'_1 - \sigma'_3)_{ult} = 2s_u \tag{8}$$

where $(\sigma'_1 - \sigma'_3)_{ult}$ is the principal stress difference at failure. The original hyperbolic model developed by Kondner (1963) is given as:

$$\sigma'_1 - \sigma'_3 = \frac{\varepsilon}{\dfrac{1}{E_{in}} + \dfrac{\varepsilon}{(\sigma'_1 - \sigma'_3)_{ult}}} \tag{9}$$

where σ'_1 and σ'_3 can represent the vertical and horizontal stresses below the center of a footing, respectively, ε is the axial strain and E_{in} is the initial undrained Young's modulus, which remains constant during undrained loading (Duncan, et al., 1980). Note that E_{in} represents the initial tangent Young's modulus and is typically measured at small strains; the range in strain associated with E_u as reported by Duncan and Buchignani (1987) is not known.

5.2 *Calculation of footing displacements*

The distribution of vertical, horizontal, and shear stress beneath the center of the footing was generated for each footing in the load test database using elasticity theory assuming undrained conditions (v_s = 0.5). During loading, the change in vertical and horizontal stresses, $\Delta\sigma_1$ and $\Delta\sigma_3$, can be modeled as the change in vertical and radial stresses, $\Delta\sigma_v$ and $\Delta\sigma_r$, respectively, by assuming that square footings can be treated as equivalent circles (Davis and Poulos, 1972).

Substitution of Equation (8) into Equation (9) and rearranging for axial strain produces an expression for displacement based on the integration of strains over the assumed depth of influence. This study considered an effective depth of $2B_{eq}$ for the integration of strains. The displacement resulting from an applied load, δ_i, can be calculated using:

$$\delta_i = \int_0^{2B_{eq}} \frac{\Delta\sigma_{vr_{\Delta Z_j}}}{E_{in_{\Delta Z_j}} + \dfrac{\Delta\sigma_{vr_{\Delta Z_j}}}{2s_{u_{\Delta Z_j}}}} d\Delta Z_j \tag{10}$$

where $\Delta\sigma_{vr}$ is the principal stress difference and ΔZ_j = an increment of depth. Pertinent soil parameters (s_u, OCR, and PI) were averaged over a depth of B_{eq} below the footing where the majority of the large strains develop.

Due to the asymptotic nature of the constitutive model adopted, unreasonable displacements are computed when the applied shear stress approaches s_u within a given ΔZ_j. To mitigate this effect, the shear stresses were limited to 99 percent of the available s_u (i.e., $\Delta\sigma_{vr}/2 < 0.99s_u$). Although, excessive displacements result at higher loads, the calibrated hyperbolic model may be used to estimate the non-linear pre-failure displacements without performing a time-consuming numerical study.

5.3 *Displacement prediction using the non-linear model*

Bearing pressure-displacement curves were calculated using the Duncan-Chang model and elastic stress fields. The E_u was estimated using Figure 1 and Equation (5). The observed and predicted q-δ curves were compared statistically with the bias. Bearing pressure-displacement points corresponding to $\Delta\sigma_{vr}/2 \geq 0.99s_u$ were omitted.

On average, the non-linear approach produced a slight under-prediction of displacements for a given bearing pressure, with a mean λ = 1.13 for each bearing pressure-displacement curve in the database, but exhibited significant variability (COV = 105 percent). The relatively large COV is the result of the inherent variability in soil strength, transformation model error in the calculation of undrained modulus, and model error. The tendency for the selected non-linear constitutive model and elastic stress-field to under-predict the displacement at a given bearing pressure resulted from excessive strains calculated as the mobilized shear stresses approached the undrained shear strength.

6 BACK-CALCULATION OF INITIAL MODULUS

Another application of a non-linear constitutive model within an elastic stress field is the estimation of the initial Young's modulus of the soil. Equation (10) can be rearranged for initial Young's modulus and its value back-calculated using least squares regression on the observed bearing pressure-displacement curve:

$$E_{in} = \int_0^{2B_{eq}} \frac{\Delta\sigma_{vr_{\Delta Z_j}}}{\delta_{\Delta Z_j}\left(1 - \dfrac{\Delta\sigma_{vr\Delta Z_j}}{2s_{u_{\Delta Z_j}}}\right)} d\Delta Z_j \tag{11}$$

where E_{in} is the initial undrained Young's modulus averaged over a depth B_{eq}. Again, data-pairs corresponding to shear stresses approaching the ultimate stress difference were omitted.

The back-calculated initial Young's modulus depends on the shape of the predicted bearing pressure-displacement curve. In some cases the predicted curvature of the bearing pressure-displacement curve was not in agreement with the observed curvature and in these instances the fitting procedure was modified to estimate the initial portion of the bearing pressure displacement curve. This was done to focus on the initial stiffness characteristics (Strahler 2012).

6.1 *Young's modulus comparison*

The calculated undrained Young's modulus and back-calculated initial Young's modulus, E_{in}, were compared using the bias and its distribution. In general, the E_u calculated using the Duncan and Buchignani (1987) correlation under-predicts the back-calculated initial modulus (mean λ = 3.05) and exhibits a significant amount of variability (COV = 99%). This level of under-prediction is not surprising, given that the Duncan-Chang model uses an initial undrained Young's modulus that is typically based on the first 0.1 to 0.25% of axial strain or less. The type or strain level of the Young's modulus referenced by Duncan and Buchignani (1987) is not specified, but the relationship was developed from in-situ testing and could potentially represent a tangent or secant modulus at 50% of peak strength. Thus, the strain levels for the estimated E_u and E_{in} may not be similar, and could explain the inaccuracy and uncertainty shown in Figure 2.

A new K was calculated using the back-calculated E_{in} and the results are presented in Figure 2. The relationship proposed by Duncan and Buchignani (1987) has been overlaid on the data for comparison and appears to be independent of *PI*.

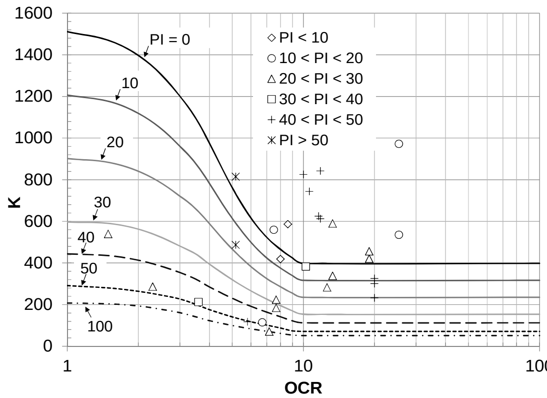

Figure 2. Back-calculated K-factor using non-linear model compared to Duncan & Buchignani (1987).

6.2 *Correlation to initial undrained Young's modulus*

When plotted against OCR, the back-calculated initial Young's modulus normalized by the atmospheric pressure, p_{atm}, exhibits a linear trend line. The stiffness appears to increase with *OCR*. A single footing used a 21 cm diameter tendon extended to bedrock beneath the center of the footing in order to develop displacements (Bauer 1976). The tendon likely interfered with the failure mechanism of the soil beneath the footing and produced a higher initial Young's modulus. It was included in the database because it was not considered a support mechanism (drilled shaft, driven pile, etc.); however, it was omitted in Figure 3 due to its clear departure from the trend.

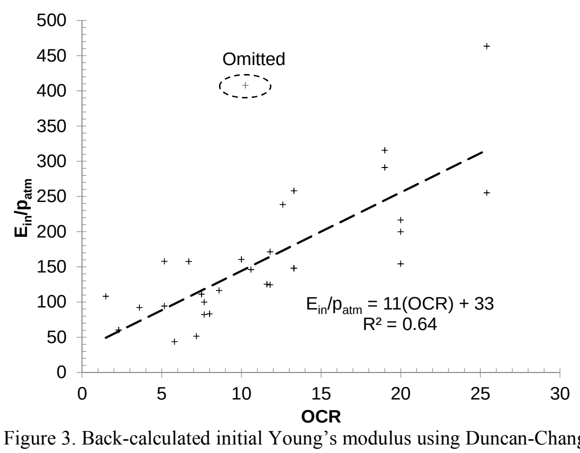

Figure 3. Back-calculated initial Young's modulus using Duncan-Chang model, based on Duncan & Buchignani (1987).

7 SUMMARY AND CONCLUSIONS

The use of a single undrained Young's modulus to predict the highly non-linear response of footings supported on cohesive soil has been shown to be slightly conservative at low displacements but increases in error with increasing displacement. A method to estimate displacements based on the non-linear Duncan-Chang model was shown to be slightly conservative and more accurately captures the overall load-displacement curve. The proposed method also allowed the estimation of an initial undrained Young's modulus, which appears to be correlated with *OCR*. This trend can be used to estimate the initial Young's modulus for use in the non-linear model or additionally modified to be used in elasticity based methods.

Despite the improvement in modeling footing response reported herein, signficant uncertainty in the response remains without the adequate characterization of inherent soil variability, transformation error associated with correlations, and model error. Improved site characterization presents the best approach to reducing the uncertainty of footing load-displacement response.

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