

# Multiobjective Linear Programming Model for Scheduling Linear Repetitive Projects

Pandelis G. Ipsilandis<sup>1</sup>

**Abstract:** Linear repetitive construction projects require large amounts of resources which are used in a sequential manner and therefore effective resource management is very important both in terms of project cost and duration. Existing methodologies such as the critical path method and the repetitive scheduling method optimize the schedule with respect to a single factor, to achieve minimum duration or minimize resource work breaks, respectively. However real life scheduling decisions are more complicated and project managers must make decisions that address the various cost elements in a holistic way. To respond to this need, new methodologies that can be applied through the use of decision support systems should be developed. This paper introduces a multiobjective linear programming model for scheduling linear repetitive projects, which takes into consideration cost elements regarding the project's duration, the idle time of resources, and the delivery time of the project's units. The proposed model can be used to generate alternative schedules based on the relative magnitude and importance of the different cost elements. In this sense, it provides managers with the capability to consider alternative schedules besides those defined by minimum duration or maximizing work continuity of resources. The application of the model to a well known example in the literature demonstrates its use in providing explicatory analysis of the results.

**DOI:** 10.1061/(ASCE)0733-9364(2007)133:6(417)

**CE Database subject headings:** Scheduling; Project management; Computer programming.

## Introduction

Construction projects that are divided into sequential units involving the same repetitive activities usually require large amounts of resources which are used in a sequential manner and therefore effective resource management is very important with regard to optimizing the overall financial performance, as the latter is affected directly by the project's cost, and also indirectly by meeting final and intermediate delivery dates. Projects of this type are considered high risk, due to unforeseen natural causes, potential involvement in legal disputes, unpredicted weather conditions, etc., which can cause delays in overall project completion and cost overruns, thus making the management of resources and partial delivery times a very important issue.

The critical path method (CPM) is most commonly used for planning, scheduling, and control of such projects. Nonetheless, network analysis techniques are duration oriented and cannot sufficiently address resource management issues. Other techniques that focus on resource usage, such as the repetitive scheduling method (RSM) have been proposed as more suitable for scheduling and controlling repetitive projects. However, scheduling decisions for repetitive projects are more complex since several cost elements related to different aspects of the project (i.e., overall

duration, idleness of resources, timely delivery of project units, financing costs) must be considered and balanced by project managers in order to arrive at a cost efficient schedule. Decisions therefore cannot be based solely on optimizing a single criterion such as time or resource usage, but require a multiobjective approach. To address this need, new methodologies and approaches that can be applied through the use of decision support systems should be developed. In this paper we explore the multiobjective nature of decision making in repetitive construction projects and propose a parametric linear programming model formulation for supporting the multiobjective process of scheduling decisions.

The rest of the paper is organized as follows: the following section refers to definitions and clarifications, and presents a literature review with regard to classification of linear repetitive projects (LRPs) and associated scheduling methodologies. In the third section we introduce a linear programming formulation for modeling the scheduling of LRPs, the parametric design of which allows single or multiobjective optimization with respect to various time or cost related criteria. The next section illustrates the application of the proposed linear programming (LP) model to a well known literature example under different set of assumptions and parameters and provides demonstrative results. The same section also shows the utilization of the model to derive critical break points between the relative unit cost of project delays and that of resource work breaks that define distinct scheduling outcomes. Finally, the last section includes the conclusions and proposed directions of future research.

## Complex Nature of Linear-Repetitive Projects

In many instances construction projects consist of a set of activities that are repeated sequentially at different locations or units (construction sites). After an activity is completed at one site, it is

<sup>1</sup>Professor, Dept. of Project Management, School of Business, Technological Education Institute of Larissa, Perifereiaki Odos Larissis-Trikalon, 41110 Larissa, Greece. E-mail: ipsil@teilar.gr

Note. Discussion open until November 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on January 25, 2006; approved on December 28, 2006. This paper is part of the *Journal of Construction Engineering and Management*, Vol. 133, No. 6, June 1, 2007. ©ASCE, ISSN 0733-9364/2007/6-417-424/\$25.00.

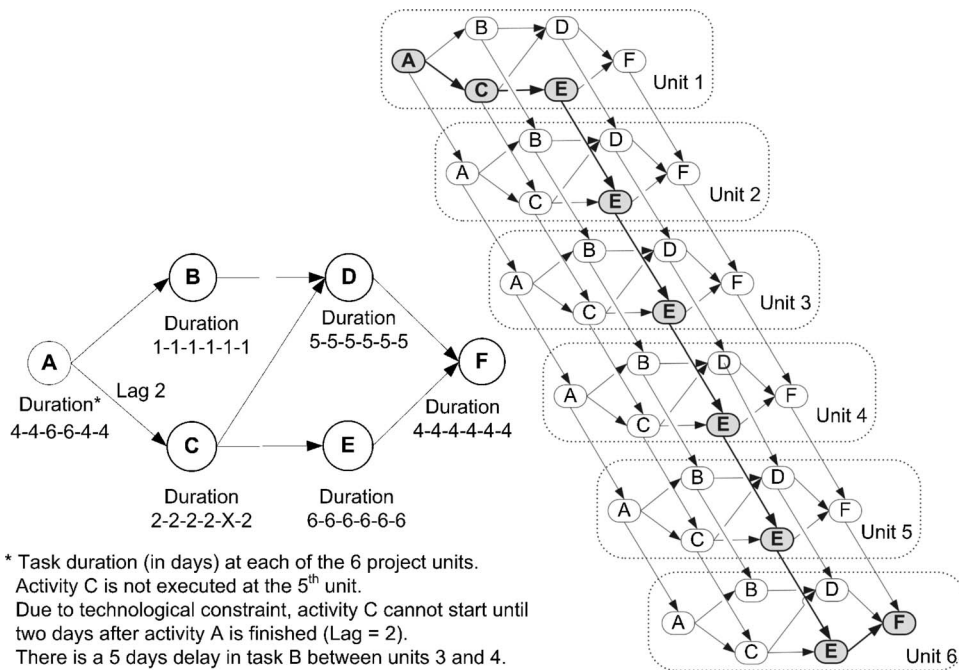


Fig. 1. Precedence network and PERT network for the LRP example

repeated in the next site (Mattila and Abraham 1998). The activities follow a logical and technological driven sequence and are subject to certain time or distance constraints imposed by internal (technological, managerial) or external causes that hold true for the entire life span of the project (Kallantzis and Lambropoulos 2004).

During the last decades various terms have been used in the literature to describe such projects. The most popular among them is LRPs, where the term linear initially referred mainly to construction projects, the activities of which are repeated continuously following a horizontal flow, such as in the case of highway segments, bridges, tunnels, railways, pipelines, sewers, etc., while the term repetitive described construction projects where activities are repeated in discrete repeated units in a vertical direction as in the case of high-rise and multistory buildings, multi-housing projects, etc. Different methods have been proposed for planning, scheduling, and controlling the construction process of these types of projects. The development of the RSM by Harris and Ioannou (1998) and Yang (2002a) which can be applied to both project categories, is the milestone for classifying these projects into a unique category as LRPs. Although the work breakdown structure of LRP could expand to hundreds of tasks, these can be usually grouped in few activity groups repeated in several units. El-Rayes and Moselhi (1998) examine a real life case of road construction consisting of five activity groups repeated in 15 units and El-Rayes et al. (2002) also use a case of housing construction project involving 13 activity groups repeated in ten units.

The most frequently used method for designing, planning, scheduling and control of construction projects is the network based PERT/CPM (Mattila and Abraham 1998). Nevertheless, since the 1970s various researchers have challenged its applicability in an attempt to prove its inadequacies, while at the same time their research led to the development of alternative approaches and methods more suitable for the construction process (Peer 1974; Carr and Mayer 1974; Dressler 1974; O'Brien 1975; Ashley 1980; Selinger 1980; Birrell 1980; Johnston 1981; Stradal

and Cacha 1982; Russell and Caselton 1988; Reda 1990; Harmelink and Rowings 1998; Harris and Ioannou 1998).

Network analysis has been characterized by its critics as insufficient to describe the repetitive nature of LRPs. The CPM treats the piecewise execution of an LRP task across the project units as a set of distinct activities connected only through precedence relationships. The size of the corresponding CPM network of the LRP is quickly exploding. A network representation of a repetitive project consisting of  $M$  tasks,  $P$  precedence relationships, that is repeated in  $N$  units will contain  $(M \cdot N)$  tasks and  $[P \cdot N + M \cdot (N - 1)]$  precedence relationships as it is illustrated in the example used in a later section (Fig. 1).

The objective of CPM is the minimization of the duration of the project through the definition of the critical path (CP) and the optimum time/cost tradeoff of the project by means of crashing the critical activities. Therefore, a CPM schedule cannot guarantee work continuity for the LRP tasks, and its CP may well consist of sections of different tasks, thus making CP-based project control impractical for construction managers who are interested in monitoring the location and the production rate of each task as it is progressing through the entire length of the project. Moreover CP crashing is not suitable for LRPs as it may change the production rate of a task in certain units introducing additional work breaks.

Utilization of resource leveling heuristic techniques to CPM schedules of LRPs still does not guarantee work continuity (Selinger 1980; Reda 1990; Russell and Wong 1993). Such techniques mainly address problems where the same resource is used in many tasks, making an initial assumption of unlimited availability of resources in the development of the project schedule, and through resource allocation require revision of the project schedule in order to comply with resource availability constraints. However, the main issue in LRPs is not the capacity of resources, which are distinctly identified with specific tasks, but their scheduling. The development of more efficient genetic algorithm based techniques, which address simultaneously resource allocation and leveling in projects and operations as a multiobjective problem

(Chan et al. 1996; Hegazy 1999), are mainly targeted to dynamic scheduling and allocation of resources to tasks, using self-improvement techniques, do not also provide a solution to the work continuity problem in LRPs.

Finally, in traditional scheduling techniques like PERT/CPM, the learning effect is not accounted for when time estimates are produced for separate tasks. In a recent survey for the Michigan State Transportation Department concerning 20 major roadway construction projects in the years 1999–2001, findings show that the contractors' time schedule usually overestimated the duration of the tasks (Mattila and Bowman 2004).

The alternative approaches that have been proposed for LRP scheduling are derived mainly from graphical representations on *X-Y* diagrams, where one axis indicates time and the other work progress. These methods can be organized, according to Mattila and Abraham (1998), in three basic categories: (1) those that are based on the line-of-balance (LOB) technique, applied mainly to discrete projects; (2) those that are based on the linear scheduling method (LSM), more appropriate for continuous projects; and (3) those that combine the previous techniques with other operations research techniques such as dynamic programming, stochastic programming, linear programming, simulation, etc.

While all of these techniques overcome the inadequacies of PERT/CPM, they failed to obtain wide acceptance and practical use. The main reasons for their limited acceptance are that: (1) they have been more complicated than they should have been and (2) there is not an acceptable algorithm (process) for identifying the project's critical path which determines the completion time (Harris and Ioannou 1998; Harmelink and Rowings 1998; Kallantzis and Lambropoulos 2003). The lack of a critical path identification algorithm can be attributed to the fact that in all these methods the underlying assumptions is that all activities must be considered as critical, for better control of a project (Peer 1974). Significant efforts however have been made the last few years toward the development of an acceptable and accurate algorithm to determine the critical path in LRPs (Harmelink and Rowings 1998; Harmelink 2001; Mattila and Park 2003; Kallantzis and Lambropoulos 2003, 2004).

The RSM was introduced by Harris and Ioannou (1998), and was further developed in the following years (Yang and Ioannou 2001, 2004; Yang 2002a,b; Ioannou and Harris 2003; Ioannou and Yang 2003, 2004). It is also based on a graphical representation of the project on an *X-Y* diagram and its objective is to integrate existing methods into a generalized one that ensures continuous resource utilization. RSM can be used for the scheduling of both discrete and continuous projects. For discrete projects, the repeated units (work progress) are usually drawn on the *Y*-axis and the elapsed project time on the *X*-axis, while for continuous projects the time is drawn on the *Y*-axis and the repeated units on the *X*-axis.

RSM follows the activity relationships concept of the CPM and adopts three activity types, which were first introduced by Vorster et al. (1992), as basic elements of graphical methods for scheduling linear projects. These are: (1) line activity which indicates the work progress from one unit to another as a function of time; (2) block activity which represents work that occupies a specific area over a certain period; and (3) bar activity which defines nonrepetitive work. Moreover, three relationships are defined for controlling the links between activities: the time-controlled, the distance-controlled, and the continuity. The RSM also introduces the terms controlling sequence and control points for the determination of the critical activities that belong to the controlling sequence not only in the case where a delay affects the

project completion time but also in the case where a delay introduces an interruption in the resource utilization (work break). RSM employs a pull-system approach, where the finish time of the predecessor activity is pulled forward to meet the start date of the successor in order to achieve work continuity and uninterrupted resource utilization, in contrast with the CPM push system, where the start of every activity is pushed in time to maintain the precedence relationships with its predecessors. The objective in RSM is not minimization of the project completion time but achieving work continuity which leads to minimizing the overall project cost. In construction projects, the minimization of the cost may be more desirable than the reduction of the project duration (Yang 2002b). However this assumption may not be true since prolonging the project duration may lead not only to delay penalties, but also to lost revenues. Particularly in projects which involve a high risk of long delays because of legal disputes or other causes delivering parts of the project and making them operational is vital for avoiding severe financial losses.

The repetitive project planner (RP2) (Yang 2002a) is the computer implementation of the RSM. The algorithm involves two stages: The first stage is similar to the forward pass computations of CPM and results in the computation of the minimum project duration. In the second stage each continuity relationship "pulls" the predecessors to eliminate the time gap with the successor to ensure work continuity, under the CPM duration constraint.

Although RSM optimizes for work continuity its analysis features are limited. It can only allow tradeoffs between time gaps and project duration on a trial and error basis. Cost considerations and other control variables are not taken directly into consideration in the computation, but only as back-end calculations.

## **Multiobjective LP Scheduling Model for LRPs (MOLPS-LRP)**

### ***Multiobjective Nature of Scheduling Decisions in LRPs***

Scheduling of LRPs is in practice more complicated and relevant decisions could involve more control variables than just minimizing duration or achieving resource continuity which is the case in CPM and RSM, respectively. A more comprehensive list of criteria important to construction managers in their decision making regarding the overall project performance may include the following:

1. **Duration:** the project duration is a key variable to any project;
2. **Resource idle time:** the execution of each of the project's tasks requires the use of certain resources. In a repetitive project, the same task is repeated sequentially in the different project units. Violating the continuity of the same task between successive project units introduces work gaps that increase the cost of the project because of idle resources;
3. **Unit completion time:** completion of the work on a project unit affects the project deliverables and it could have significant financial implications since the project's cash receipts depend on completing intermediate deliverables. In other cases, the completed units of the project can become operational before the completion of the entire project thus resulting in earlier cash inflows;
4. **Slack time:** reducing activity slack time may result in achieving a high level of work continuity but at the same time introduces higher risk, regarding completion time of the units and the overall project duration; and
5. **Number of units the project is divided into:** in nondiscrete

projects such as highways the division of the project is not always a result of physical constraints as it is the case in discrete projects (i.e., floors in a high rise building). In a highway construction project a project unit represents a segment of the highway of a certain length and not all units necessarily have the same length. A certain fixed cost of maintaining a construction site is associated with each project unit, which on one hand could increase the cost of the project but on the other hand result in smaller better managed deliverables which could improve the project's cash flow.

Furthermore scheduling decisions are rarely based only on any single variable. Alternative project schedules, comparisons, and cost tradeoffs are often needed to arrive at an acceptable or optimum project schedule. In this aspect the scheduling problem can be seen as a multiobjective problem that can be addressed efficiently by linear programming techniques. The following linear programming formulation for LRP scheduling is based on a parametric objective function which, according to the values of its parameters, could aim either at single target optimization (i.e., duration, unit completion time, work breaks), or a multiobjective optimization by combining different criteria into a single cost criterion.

### Formal Description of MOLPS-LRP

Any LRP can be defined by a set of  $M$  tasks and  $P$  project dependency relationships (SS, FS, SF, and FF, with or without time lag). The project is divided into  $N$  separate units in a linear way where without loss of generality the following general assumptions hold for the most part:

1. All tasks are performed in all units;
2. A task cannot be performed in any project unit before the same task is completed in the previous unit; and
3. The set of dependencies remain the same in all units. Yang (2002a) lists a set of practical concerns in scheduling repetitive projects that are exceptions to these general assumptions which, however, can be easily handled in the LP formulation that follows.

### Constraint Definitions

The following set of constraints describes the operation of activities in an LRP (see Notation):

Task duration constraints

$$f_{ij} = s_{ij} + d_{ij} \quad \forall i = 1, 2 \dots M, \quad j = 1, 2 \dots N \quad (1)$$

Project linearity constraints

$$s_{ij+1} \geq s_{ij} + l_i \quad \forall i = 1, 2 \dots M, \quad j = 1, 2 \dots N - 1 \quad (2)$$

Any task in unit  $j+1$  can start after the elapsed time from the start of the same task in unit  $j$ . When  $l_i = d_{ij}$ , the constraint takes the form of a finish to start relationship. Exceptions to this rule can be handled accordingly.

Technological dependencies

$$s_{ij} \geq f_{kj} \quad \forall i = 1, 2 \dots M, \quad j = 1, 2 \dots N, \quad \forall k \in P_i \quad (3)$$

The exact form of the constraint depends on the type of the dependency. Without loss of generality here we assume that all dependencies are of FS type. Exceptions can be handled accordingly.

Unit completion time

$$UC_j \geq f_{kj} \quad \forall k \in E, \quad j = 1, 2 \dots N \quad (4)$$

Completion time for unit  $j$ ,  $UC_N$  equals the project's duration.

Resource delay

$$WB_i = \sum_{j=1}^{N-1} (s_{ij+1} - f_{ij}), \quad \forall i = 1, 2 \dots M \quad WB = \sum_{i=1}^M WB_i \quad (5)$$

The sum of time gaps for task  $i$ , and the total resource time lost in work break delay.

### Global Objective Function

Depending on the values of the parameters  $c_j$  and  $f_i$ , the following general objective function

$$\text{Minimize} \sum_{j=1}^N c_j \cdot (UC_j - D_j) + \sum_{i=1}^M f_i \cdot WB_i \quad (6)$$

can be used accordingly as follows for optimizing:

Project duration

$$\text{Minimize } UC_N \quad c_N = 1, \quad \text{rest of } c_j \text{ and } f_i \text{ equal } 0 \quad (7)$$

Total work-break time

$$\text{Minimize } WB \quad \text{All } f_i \text{ equal } 1, \quad \text{all } c_j \text{ equal to } 0 \quad (8)$$

Unit completion time

$$\text{Minimize} \sum_{i=1}^M UC_i \quad \text{All } f_i \text{ equal } 0, \quad \text{all } c_j \text{ equal to } 1 \quad (9)$$

Total cost of work break

$$\text{Minimize} \sum_{i=1}^M f_i \cdot WB_i \quad \text{All } c_j \text{ equal to } 0 \quad (10)$$

Delay cost in unit completion

$$\text{Minimize} \sum_{i=1}^M c_j \cdot (UC_j - D_j) \quad \text{All } f_i \text{ equal } 0 \quad (11)$$

Tradeoffs between costs of project delays and resource delays (work breaks)

$$\text{Minimize} \sum_{j=1}^N c_j \cdot (UC_j - D_j) + \sum_{i=1}^M f_i \cdot WB_i \quad (12)$$

### MOLPS-LRP Application and Results

#### Case Study Example

In this section we demonstrate the type of answers and analysis that can be supported by the proposed model through the use of a specific example. The LP model that was described in the previous section is applied to the same example of a small linear discrete project that was initially used by the RSM authors (Harris and Ioannou 1998), which is repeated here with certain modifications regarding the task duration times. The project is divided into six repetitive units, with six discrete tasks each of which is performed by a specific crew, repeated at each unit. All task



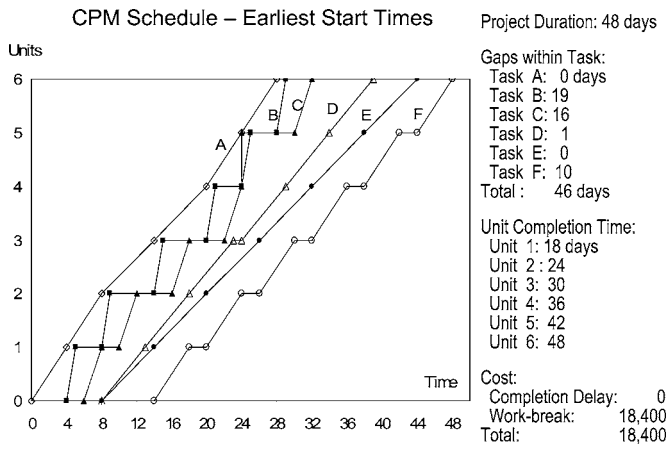


Fig. 2. Linear scheduling (CPM-ES)

dependencies are finish to start as shown in the precedence network diagram on the left of Fig. 1 along with the duration of each task at each of the six units, and other technological constraints. The multiplicative complexity of the resulting PERT diagram for all six units of the project, is shown on the right part of Fig. 1. The critical path of the entire project consists of Tasks A, and C, in unit 1, the sequence of Task E on all units and Task F in unit 6.

Furthermore let us assume that the cost of work break ( $f_i$ ) is 400/day, the same for all tasks, while a penalty of 1,000/day exists for delays in project delivery ( $c_6=1,000$ ), and an additional cost of 100/day is imposed for delays in the completion of units 1–5 ( $c_i=100, i=1, 2, \dots, 5$ ). Completion delays are calculated as deviations from the dates defined in the CPM early start (ES) time schedule.

### Minimizing Unit Completion Time

The CPM ES schedule minimizes the project duration and at the same time the completion time for all units. Completing and delivering project units as early as possible could be critical because of seasonal constraints (avoiding delays due to bad weather conditions) and could also affect the financial standing of the projects in cases where payments depend on the delivery of completed units. In linear projects like highway construction the delivery of

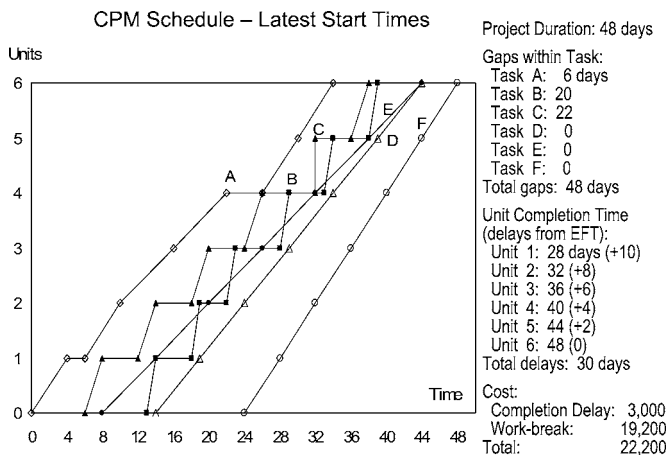


Fig. 3. Linear scheduling (CPM-LS)

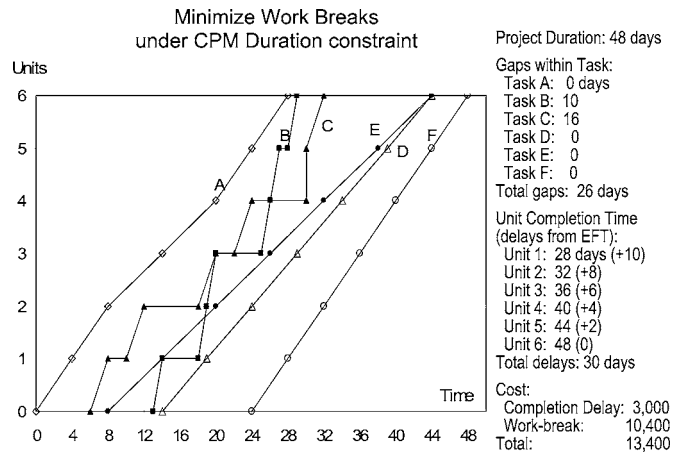


Fig. 4. Linear schedule—minimization of work breaks under CPM duration

a certain unit could signal income generation milestones, therefore delays in unit completion could change negatively the net present value of the project. In this case the objective is set to minimization of completion time of all or certain units, even if that means sacrifices in work continuity. Setting the objective function of the MOLP-LRP as in Eq. (9) the solution is identical to the CPM ES schedule which yields a project duration of 48 days, with unit completion times and resource delays set as shown in Fig. 2, resulting in an additional cost of 18,400 due to work breaks in the schedule. In the corresponding linear scheduling diagram, the progress of each task through the project units is represented by a piecewise straight line. The slope of the line corresponds to the production rate of the specific task at each unit. Horizontal segments on the progress line correspond to work breaks and therefore in resource idleness between the execution of a task in successive units. Vertical segments represent specific cases, where a task is not included in the corresponding unit.

As mentioned previously, CPM is insufficient in addressing work continuity objectives. Work breaks cannot be eliminated or even reduced by scheduling tasks according to the latest start (LS) time as it is demonstrated in Fig. 3. Pushing tasks to their LS time transfers work breaks from the last project activities to those in the beginning of the projects. In the specific example, the LS schedule creates even more work breaks, while at the same time produces delays in intermediate deliveries, raising the total cost to 22,200. Additionally, the LS schedule consumes all the slack, hence making the project more vulnerable to unexpected delays.

### Minimizing Work-Break Time

Introducing the 48 days CPM duration in constraint Eq. (4) ( $UC_6=48$ ), and setting the objective function as in Eq. (8), the LP model was run with an objective of total resource work-break time minimization. The resulting schedule is shown in Fig. 4. The minimum project duration of 48 days can be achieved with a minimum of 26 days of work breaks at Tasks B and C. Further reduction of task work-break time cannot be achieved without extending the project's duration beyond 48 days.

If the CPM duration constraint is relaxed work breaks can be further reduced to a minimum of 5 days, with a negative effect in project's duration which is extended to 62 days as shown

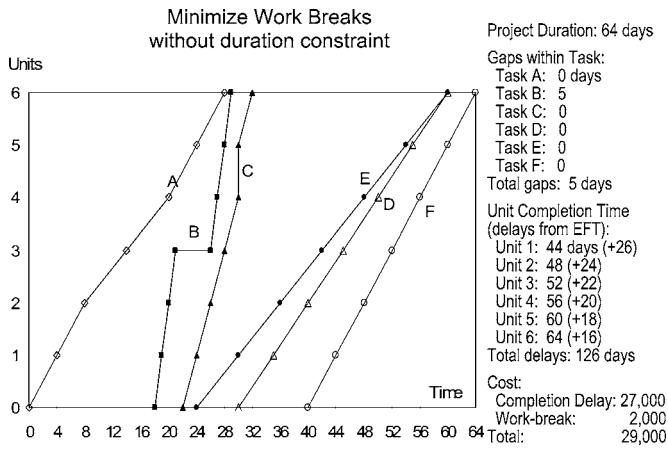


Fig. 5. Linear schedule—minimization of work breaks without duration constraints

in Fig. 5. The work break of Task C is eliminated, while that of Task B is reduced to 5 days as set by the task's technological constraints. The finishing time of all units is also pulled to 14 days later than in the previous schedule.

### Multicriteria Optimization

All the schedules derived under the previous conditions were based on a single criterion each time. By trying different evaluation criteria as they are defined in Eqs. (7)–(11), alternative schedules optimized with respect to the criterion selected could be produced. The objective function defined in Eq. (12) consolidates the criteria of duration, unit completion time, and work break into a single cost evaluation criterion. Using the cost factors for completion delays, duration, and work breaks, defined earlier, the MOPLS-LRP multiobjective optimum solution is displayed in Fig. 6.

### Cost Trade-Offs

Resource idleness cost varies among different activities according to the type and scarceness of the resources consumed. The same is true with the cost associated with delays in completion time of different units which can affect the overall cost of the project

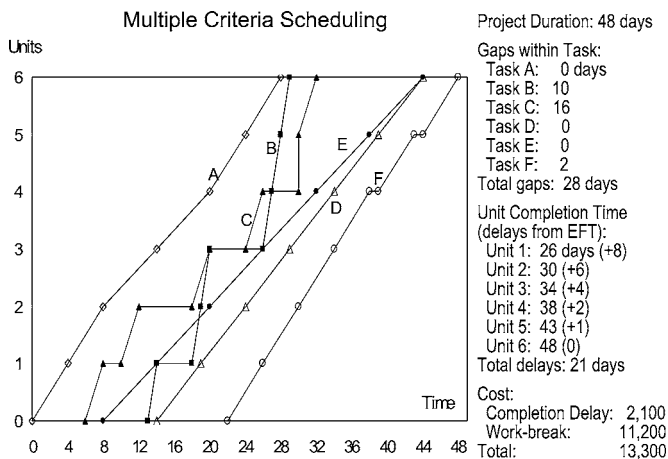


Fig. 6. Linear schedule—Multicriteria cost minimization

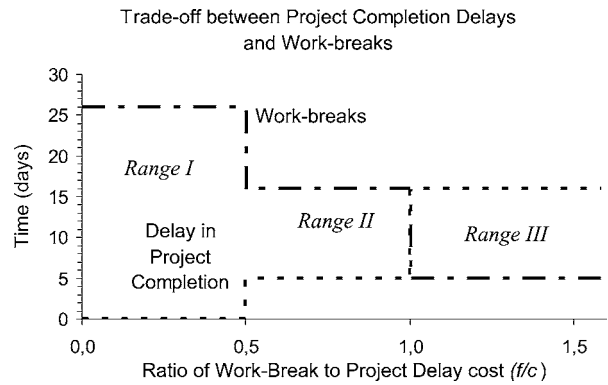


Fig. 7. Tradeoff between project completion delays and work breaks

either directly (i.e., delay penalties) or indirectly (i.e., financial cost due to late cash receipts, or delays in revenue generation). The cost objective functions in Eqs. (10)–(12) can be used either for minimizing a total cost function as shown before, or for performing a tradeoff analysis between the different types of costs. The MOLPS-LRP model can be used to establish optimum schedules at different levels of cost relations, in the case where no exact cost data exist, by using the standard tools of LP range and sensitivity analysis. The following two examples demonstrate this type of analysis.

### Trade-Off between Project Duration Delay Cost and Work-Break Cost

The first case refers to a tradeoff analysis between the cost associated with delays in project completion and that of task work breaks. Delays are measured as deviations from the earliest finish date of the project as it is set by the CPM or from any predefined delivery date. It is also assumed at the moment that intermediate delays in completing individual project units do not impose any additional cost to the project, and that the cost of work breaks is the same for all tasks.

In this case the objective function [Eq. (12)] of the LP model is equivalent to

$$\text{Minimize } c(UC_N) + f(WB) \text{ or Minimize } c[UC_N + (f/c)WB] \quad (13)$$

where  $c$  and  $f$  denote the daily cost of project delay and work break, respectively.

The results of the sensitivity analysis on the values of the coefficient  $f/b$  of the objective function in Eq. (13) can be used to set optimality ranges, associated with alternative optimum solutions (schedules) as shown in Fig. 7. For the specific example three optimality ranges exist related to three optimum solutions: When the work-break unit cost ranges between 0 and up to 50% of the lateness cost (Range I), the optimum scheduling results in project duration of 48 days (minimum possible) with a maximum work break time of 26 days. When the work-break unit cost ranges between 50 and 100% of the lateness cost (Range II), it is more economical to let the project duration slip by 5 days in order to reduce work breaks to 16 days. And finally when the work-break cost exceeds the lateness cost (Range III) the optimum schedule is the one that reduces work breaks to the minimum of 5 days, which results in extending the project duration by 16 days.

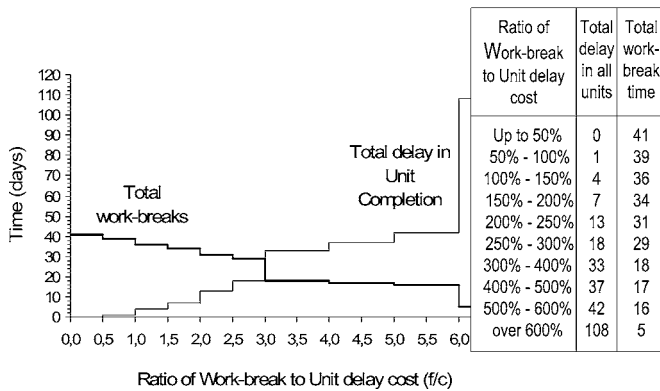


Fig. 8. Tradeoff between unit completion delays and work breaks

### Trade-Off between Unit Completion Delay Cost and Work-Break Cost

In the second example, the scenario where a penalty cost is associated with delays in delivery time of individual project units is examined. Delays could be measured as deviations from the earliest finish dates of the project units (Fig. 2) or from a promised time. The choice does not affect at all the range analysis that follows, since the cost coefficients of the objective function remain unchanged. For simplicity purposes we assume here that the same penalty applies to delays in any unit. Also as in the previous case, the cost of work breaks is assumed to be the same for all tasks.

In this case the objective function [Eq. (12)] of the LP model can be written as

$$\begin{aligned} \text{Minimize } & \sum_{j=1}^N c \cdot (UC_j - D_j) + \sum_{i=1}^M f \cdot WB_i \\ & = c \left[ \sum_{j=1}^N UC_j + (f/c) \sum_{i=1}^M WB_i \right] - c \sum_{j=1}^N D_j \end{aligned} \quad (14)$$

Since the second part of Eq. (14) is constant, the analysis to define ranges of optimality is based on the level of the  $f/c$  ratio. The results shown in Fig. 8 indicate ten optimality ranges corresponding to ten different optimum solutions according to the relation of the work break to unit completion cost. A schedule which minimizes the work-break time (5 days) is optimum only when the associated work-break cost is six times higher or more than the cost paid for unit delays, and it is achieved by introducing a total of 108 days of delay in the completion of all units. As the relative size of the work break to unit delay cost drops, alternative solutions that allow for work breaks may be more cost efficient. A significant break point in optimality conditions occurs when work break cost is three times higher than the cost of delays. Under this level total delays in the units are kept below 18 days in total (average 3 days per unit) while above this level, they range from 33 to 108 days (about 5.5 to 18 days per unit). Fig. 8 gives a graphical representation of the results. In general, any distinct segment of a cost coefficient ratio defined by the sensitivity analysis, corresponds to an optimal schedule associated with optimum level of project duration, delays in unit completion, and work breaks. The number of alternative optimum solutions and the tradeoff brake points and quantities depend on the constraints of the specific problem that define the set of all feasible schedules.

## Conclusions

Scheduling of linear repetitive construction projects is not a single dimension decision process. A scheduling decision must take into consideration more than a single factor and most of the times tradeoffs are required between unit completion times, project duration, and work breaks. CPM and RSM while efficient in optimizing with respect to duration and work continuity, respectively, do not provide the facilities to optimize a project schedule in a holistic way that takes into account various cost considerations. The MOLPS-LRP model can address these issues efficiently and has the capacity to provide optimum schedules reflecting not only single but multiple complex objectives and assist project managers in producing and selecting among alternative schedules based on the relative magnitude of different cost elements. In this sense it provides managers with the capability to consider alternative schedules besides those defined by minimum duration (CPM) or minimum resource work breaks (RSM). A fully integrated software implementation of this approach will enhance the applicability of the methodology to real world projects and provide compatibility in data interexchange with other common PM software tools. Although the MOLPS-LRP model, as it stands, can handle issues related to problem formulation such as production learning curves, the introduction of distance constraints, different types of time constraints, etc., there are other issues that need further research. One main issue to be further investigated is the risk level associated with the alternative scheduling decisions, as it is indicated by the slack time of the tasks and the probability of meeting the objectives set (delivery times, work breaks, etc.) since unexpected events in one task or unit may affect not only the duration of the project and delivery times of the units but also the work continuity in other resources. Similarly the use of simulation techniques could provide further insight into the stability of the different alternative optimum solutions defined by the tradeoff approach. Another issue that needs further consideration is whether the MOLPS-LRP model can be used to determine the optimum number of units in a linear project when these are not defined as physical units (i.e., floors, apartments, etc.), given that an increase in the number of units could affect positively the duration of the project but could also increase the cost of work breaks and the total employment of the resources.

## Notation

The following symbols are used in this paper:

- $c_j$  = cost per time unit (penalty or financial) for delays in finishing unit  $j$ ;
- $D_j$  = promised delivery time of unit  $j$ ;
- $d_{ij}$  = duration of task  $i$  in unit  $j$ ;
- $d_{ij} = w_{ij}/p_{ij}$  = duration at each unit;
- $E$  = set of all activities without successors;
- $f_i$  = cost per time unit of work break (idle resources) in task  $i$ ;
- $i = 1, 2, \dots, M$  = project tasks;
- $j = 1, 2, \dots, N$  = project units;
- $l_i$  = minimum time for starting task  $i$  between successive units;
- $P_i$  = set of predecessor activities to task  $i$ ;
- $p_i^o$  = beginning production rate at unit 1;
- $p_i^o + p_i^d$  = corresponding ceiling on the production rate;

$p_{ij}$  = production rate at unit  $j$  = function of a learning factor  $p_{ij} = p_i^o + p_i^d(1 - (1 - r_i)^{j-1})$ ;  
 $r_j$  = rate of improvement from unit to unit;  
 $s_{ij}, f_{ij}$  = start and finish time, respectively, of task  $i$  in unit  $j$ ;  
 $UC_j$  = completion time of project unit  $j$ ;  
 $w_{ij}$  = amount of the corresponding work; and  
 $WB_i$  = total time of work breaks for task  $i$  because of discontinuities in successive units.

## References

- Ashley, D. B. (1980). "Simulation of repetitive-unit construction." *J. Constr. Div.*, 106(2), 185–194.
- Birrell, G. S. (1980). "Construction planning—Beyond the critical path." *J. Constr. Div.*, 106(3), 389–407.
- Carr, R. I., and Meyer, W. L. (1974). "Planning construction of repetitive building units." *J. Constr. Div.*, 100(3), 403–412.
- Chan, W. T., Chua, D. K. H., and Kannan, C. G. (1996). "Construction resource scheduling with genetic algorithms." *J. Constr. Eng. Manage.*, 122(2), 125–132.
- Dressler, J. (1974). "Construction scheduling of linear construction sites." *J. Constr. Div.*, 100(4), 571–587.
- El-Rayes, K., and Moselhi, O. (1998). "Resource-driven scheduling of repetitive activities." *Constr. Manage. Econom.*, 16, 433–446.
- El-Rayes, K., Ramanathan, R., and Moselhi, O. (2002). "An object-oriented model for planning and control of housing construction." *Constr. Manage. Econom.*, 20, 201–210.
- Harmelink, D. J. (2001). "Linear scheduling model: Float characteristics." *J. Constr. Eng. Manage.*, 127(4), 255–260.
- Harmelink, D. J., and Rowings, J. E. (1998). "Linear scheduling model: Development of controlling activity path." *J. Constr. Eng. Manage.*, 124(4), 263–268.
- Harris, R. B., and Ioannou, P. G. (1998). "Scheduling project with repeating activities." *J. Constr. Eng. Manage.*, 124(4), 269–278.
- Hegazy, T. (1999). "Optimization of resource allocation and leveling using genetic algorithms." *J. Constr. Eng. Manage.*, 125(3), 167–175.
- Ioannou, P. G., and Harris, R. B. (2003). "Discussion of algorithm for determining controlling path considering resource continuity." *J. Comput. Civ. Eng.*, 17(1), 68–70.
- Ioannou, P. G., and Yang, I. T. (2003). "Discussion of algorithm for determining controlling path considering resource continuity." *J. Comput. Civ. Eng.*, 17(1), 70–72.
- Ioannou, P. G., and Yang, I. T. (2004). "Discussion of comparison of linear scheduling model and repetitive scheduling method." *J. Compos. Constr.*, 8(3), 461–463.
- Johnston, D. W. (1981). "Linear scheduling method for highway construction." *J. Constr. Div.*, 107(2), 247–261.
- Kallantzis, A., and Lambropoulos, S. (2003). "Correspondence of activity relationships and critical path between time-location diagrams and critical path method." *Proc., 16th National Conf., Hellenic Operational Research Society (HELORS)*, Larissa, Greece, 67–76.
- Kallantzis, A., and Lambropoulos, S. (2004). "Critical path determination by incorporation minimum and maximum time and distance constraints into linear scheduling." *Eng., Constr., Archit. Manage.*, 11(3), 211–222.
- Mattila, K. G., and Abraham, D. M. (1998). "Linear scheduling: Past efforts and future directions." *Eng., Constr., Archit. Manage.*, 5(3), 294–303.
- Mattila, K. G., and Bowman, M. R. (2004). "Accuracy of highway contractor's schedules." *J. Constr. Eng. Manage.*, 130(5), 647–655.
- Mattila, K. G., and Park, A. (2003). "Comparison of linear scheduling model and repetitive scheduling method." *J. Constr. Eng. Manage.*, 129(1), 56–64.
- O'Brien, J. J. (1975). "VPM scheduling for high-rise buildings." *J. Constr. Div.*, 101(4), 895–905.
- Peer, S. (1974). "Network analysis and construction planning." *J. Constr. Div.*, 100(3), 203–210.
- Reda, R. B. (1990). "RPM: Repetitive project modeling." *J. Constr. Eng. Manage.*, 116(2), 316–330.
- Russell, A. D., and Ceselton, W. F. (1988). "Extensions to linear schedule optimization." *J. Constr. Eng. Manage.*, 114(4), 36–52.
- Russell, A. D., and Wong, W. C. M. (1993). "New generation of planning structures." *J. Constr. Eng. Manage.*, 119(2), 196–214.
- Selinger, S. (1980). "Construction planning for linear projects." *J. Constr. Div.*, 106(2), 195–205.
- Stradal, O., and Cacha, J. (1982). "Time space scheduling method." *J. Constr. Div.*, 108(3), 445–457.
- Vorster, M. C., Beliveau, Y. J., and Bafna, T. (1992). "Linear scheduling and visualization." *Transp. Res. Rec.*, 1351, 32–39.
- Yang, I. T. (2002a). "Repetitive project planner: Resource-driven scheduling for repetitive construction projects." Ph.D. dissertation, Univ. of Michigan, Ann Arbor, Mich.
- Yang, I. T. (2002b). "Stochastic analysis on project duration under the requirement of continuous resource utilization." *Proc., 10th Int. Conf. Group for Lean Construction*, Gramado, Brazil, 527–540.
- Yang, I. T., and Ioannou, P. G. (2001). "Resource-driven scheduling for repetitive projects: A pull-system approach." *Proc., 9th Int. Conf. Group for Lean Construction*, Singapore, 365–377.
- Yang, I. T., and Ioannou, P. G. (2004). "Scheduling with focus on practical concerns in repetitive projects." *Constr. Manage. Econom.*, 22, 619–630.



Copyright of *Journal of Construction Engineering & Management* is the property of American Society of Civil Engineers and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.