Chapter 8

Beam-Columns and Frame Behavior

KFC Yum! Center, Louisville, KY
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8.1 INTRODUCTION

Beam-columns are members subjected to axial forces and bending moments simultaneously; thus their behavior falls somewhere between that of an axially loaded column and that of a beam under pure bending. It is thus possible to consider the beam or axially loaded member as special cases of the beam-column. Practical applications of the beam-column are numerous. They occur as chord members in trusses, as elements of rigidly connected frameworks, and as members of pin-connected structures with transverse or eccentric loads. It is not always possible to look at a member and determine whether it is a beam-column or not; some knowledge of the actual forces being carried by the member is required to categorize it as a beam-column. However, many structural members are subjected to these combined forces, and the beam-column is a very common element in building structures.

The manner in which the combined loads are transferred to a particular beam-column significantly impacts the ability of the member to resist those loads. Starting with the axially loaded column, bending moments can occur from various sources. Lateral load can be applied directly to the member, as is the case for a truss top chord or a column supporting the lateral load from a wall. Alternatively, the axial force can be applied at some eccentricity from the centroid of the column as a result of the specific connections. In addition, the member can receive end moments from its connection to other members of the structure, such as in a rigid frame. In all cases, the relation of the beam-column to the other elements of the structure is important in determining both the applied forces and the strength of the member.

To understand the behavior of beam-columns, it is common practice to look at the response predicted by an interaction equation. The response of a beam-column to an axial load P, major axis moment M_x, and minor axis moment M_y is presented on the three-dimensional diagram shown in Figure 8.1. Each axis in this diagram represents the capacity of the member when it is subjected to loading of one type only, whereas the curves represent the combination of two types of loading. The surface formed by connecting the three curves represents the interaction of axial load and biaxial bending. This interaction surface is of interest to the designer.

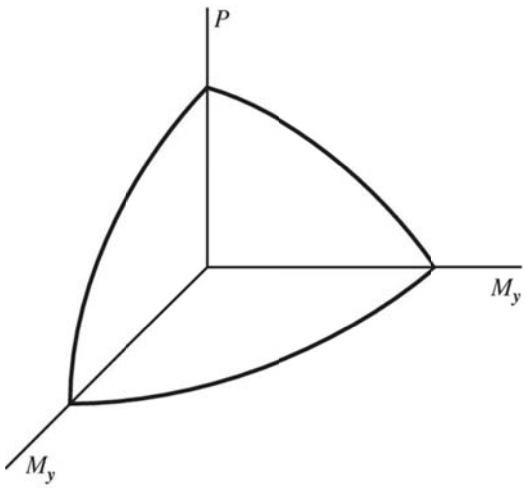


Figure 8.1 Ultimate Interaction Surface for a Stocky Beam-Column.

The end points of the curves shown in Figure 8.1 depend on the strength of the beam-columns as described for compression members (Chapter 5) and bending members (Chapter 6). The shape of the curves between these end points depends on the properties of the particular member as well as the properties of other members of the structure.

Table 8.1 lists the sections of the *Specification* and parts of the *Manual* discussed in this chapter.

8.2 SECOND-ORDER EFFECTS

The single most complicating factor in the analysis and design of a beam-column is what are known as *second-order effects*. Second-order effects are the changes in member forces and moments as the direct result of structural deformations. Because the commonly used elastic methods of structural analysis assume that all deformations are small, and because the equations of equilibrium are written using the undeformed configuration of the structure, these methods are not able to capture the additional second-order effects that occur in real structures without adjustment. The results of that type of analysis are called *first-order effects*—that is, first-order forces, first-order moments, and first-order displacements. To account for the influence of the deformations, an additional analysis must be performed. The results of this additional analysis are referred to as the second-order effects.

Several approaches are available for including second-order effects in an analysis. A complete second-order inelastic analysis would take into account the actual deformation of the

Table 8.1 Sections of *Specification* and Parts of *Manual* Covered in This Chapter

	Specification
B3	Design Basis
C	Design for Stability
H	Design of Members for Combined Forces and Torsion
Appendix 6	Stability Bracing for Columns and Beams
Appendix 7	Alternative Methods of Design for Stability
Appendix 8	Approximate Second-Order Analysis
	Manual
Part 1	Dimensions and Properties
Part 3	Design of Flexural Members
Part 4	Design of Compression Members
Part 6	Design of Members Subject to Combined Loading

structure and the resulting forces, as well as the sequence of loading and the behavior of the structure after any of its components are stressed beyond the elastic limit. This approach to analysis is generally more complex than is necessary for normal design. A similar approach that includes the actual deformations but that does not include inelastic behavior is usually sufficient.

An approach that is consistent with normal design office practice and with how beam-columns have been handled for many years uses a first-order elastic analysis and amplification factors to approximate the second-order effects. This approach applies these amplification factors as multipliers to the results of the first-order analysis to obtain the second-order effects.

Two different deflection components that could occur in a beam-column influence the moments in that beam-column. The first, illustrated in Figure 8.2a, is the deflection along the length of the member that results from the moment along the member. In this case, the member ends must remain in their original position relative to each other; thus, no sway is considered. The moment created by the load, P, acting at an eccentricity δ_2 from the deformed member, is superimposed on the moment resulting from the applied end moments. Because the magnitude of this additional moment depends on the properties of the column itself, this is called the *member effect*.

When the beam-column is part of a structure that is permitted to sway, the displacements of the overall structure also influence the moments in the member. For a beam-column that is permitted to sway an amount Δ_2, as shown in Figure 8.2b, the additional moment is given by $P\Delta_2$. Because the lateral displacement of a given member is a function of the properties of all of the members in a given story, this is called the *structure effect*.

To understand the magnitude of the potential increase in moments on a column due to second-order effects, two simple calculations will be carried out. The first is for a 20 ft long column similar to that shown in Figure 8.2a. A W12×96 member is used to carry an axial load P_u = 400 kips and equal end moments of M_u = 200 ft-kips bending the member in single curvature. A first-order analysis yields an axial force in the column of 400 kips and a bending moment at every point along the column length of 200 ft-kips. The maximum deflection of the member at mid height due to the moment is

$$\delta = \frac{M_u L^2}{8EI} = \frac{200(20)^2(1728)}{8(29{,}000)(833)} = 0.715 \text{ in.}$$

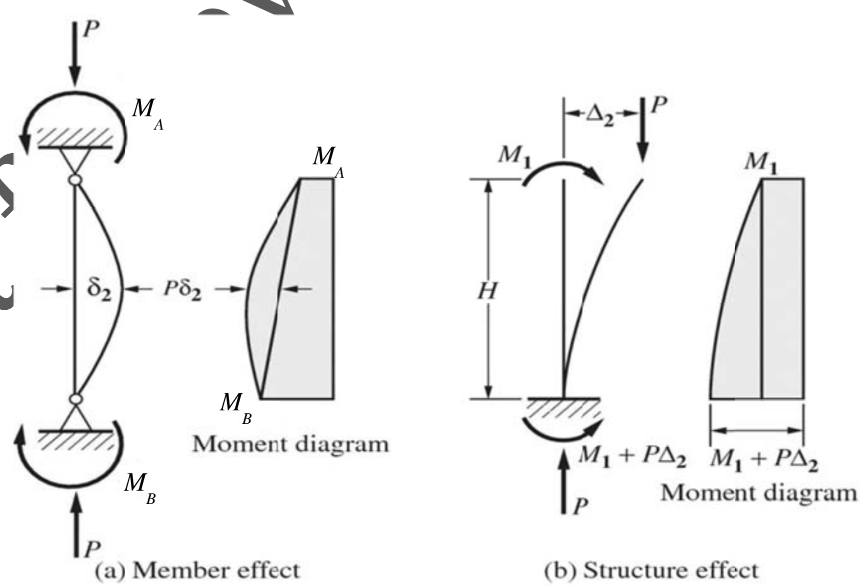


Figure 8.2 Column Displacements for Second-Order Effects.

With the occurrence of this deflection, the applied load of 400 kips is now at an eccentricity from the member in its displaced position. Thus, an additional moment is induced into the member equal to

$$M_{\text{additional}} = \frac{400(0.715)}{12} = 23.8 \text{ ft-kips}$$

The addition of this additional moment to the original internal moment of 200 ft-kips yields the second-order moment,

$$M_{2nd} = 200 + 23.8 = 224 \text{ ft-kips}$$

Thus, there is an amplification of the moment by 224/200 = 1.12. If this were the final case, second-order analysis would be fairly simple. Unfortunately, the additional moment just determined also causes additional deflection, which, in turn, causes additional moment. This process continues until equilibrium is reached. The process is an iterative one, and is nonlinear.

A second example is a column similar to that shown in Figure 8.2b. The same W12×96 member is used, and the axial force is again P_u = 400 kips. In this case, the column is a cantilever with a moment of M_u = 200 ft-kips applied at the top. This moment will cause a horizontal deflection at the top of the column of

$$\Delta = \frac{M_u L^2}{2EI} = \frac{200(20)^2(1728)}{2(29{,}000)(833)} = 2.86 \text{ in.}$$

In this displaced position, the 400 kip load is now at an eccentricity from the fixed support, which induces an additional moment

$$M_{\text{additional}} = \frac{400(2.86)}{12} = 95.3 \text{ ft-kips}$$

The addition of this additional moment to the original support moment of 200 ft-kips yields the second-order moment

$$M_{2nd} = 200 + 95.3 = 295 \text{ ft-kips}$$

which is an increase of 1.48 times the first-order moment. Again, this is not the end of the required calculations; this additional moment causes additional deflections and additional moments.

Both of these second-order effects are significant in real structures and must be accounted for in the design of beam-columns according to Section C1 of the *Specification*. Procedures for incorporating these effects will be addressed once an overall approach to beam-column design is established.

8.3 INTERACTION PRINCIPLES

The interaction of axial load and bending within the elastic response range of a beam-column can be investigated through the straightforward techniques of *superposition*. This is the approach normally considered in elementary strength of materials in which the normal stress due to an axial force is added to the normal stress due to a bending moment.

Although the superposition of individual stress effects is both simple and correct for elastic stresses, there are significant limitations when applying this approach to the limit states of real structures. These include:

1. Superposition of stress is correct only for behavior within the elastic range, and only for similar stress types.
2. Superposition of strain can be extended only into the inelastic range when deformations are small.
3. Superposition cannot account for member deformations or stability effects such as local buckling.
4. Superposition cannot account for structural deflections and system stability.

With these limitations in mind, it is desirable to develop interaction equations that will reflect the true limit states behavior of beam-columns. Any limit state interaction equation must reflect the following characteristics:

Axial Load

1. Maximum column strength
2. Individual column slenderness

Bending Moment

1. Lateral support conditions
2. Sidesway conditions
3. Member second-order effects
4. Structure second-order effects
5. Moment variation along the member

The resulting equations must also provide a close correlation with test results and theoretical analyses for beam-columns, including the two limiting cases of pure bending and pure compression.

Application of the resulting interaction equations can be regarded as a process of determining available axial strength in the presence of a given bending moment or determining the available moment strength in the presence of a given axial load. An applied bending moment consumes a portion of the column strength, leaving a reduced axial load strength. When the two actions are added together, the resulting total load must not exceed the total column strength. Conversely, the axial load can be regarded as consuming a fraction of the moment strength. This fraction, plus the applied moments, must not exceed the maximum beam strength.

8.4 INTERACTION EQUATIONS

A simple form of the three-dimensional interaction equation is

$$\frac{P_r}{P_c}+\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\leq 1.0 \tag{8.1}$$

where the terms with the subscript *r* represent the required strength and those with the subscript *c* represent the available strength.

This interaction equation is plotted in Figure 8.3. The figure shows that this results in a straight line representation of the interaction between any two of the load components. The

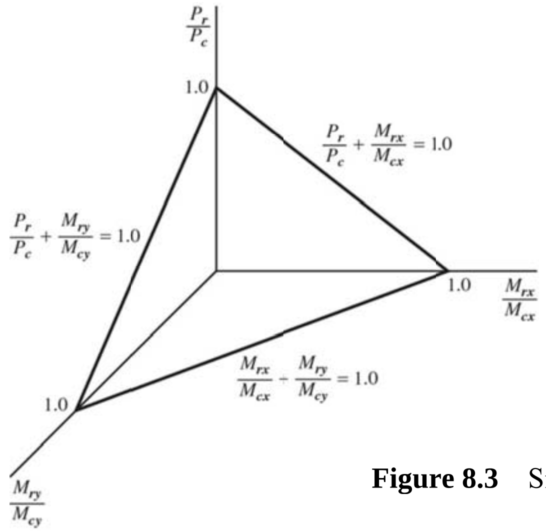


Figure 8.3 Simplified Interaction Surface.

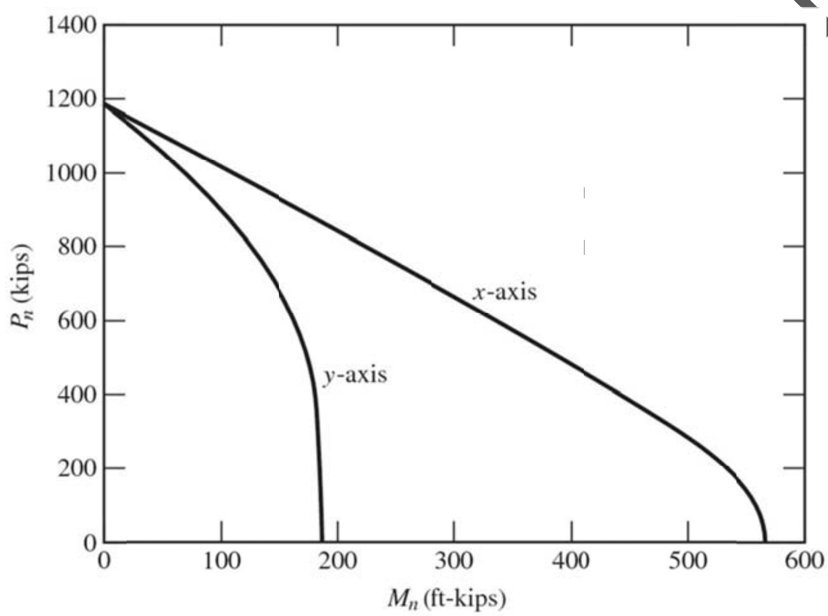


Figure 8.4a Interaction Diagram for Stub W14×82 Column.

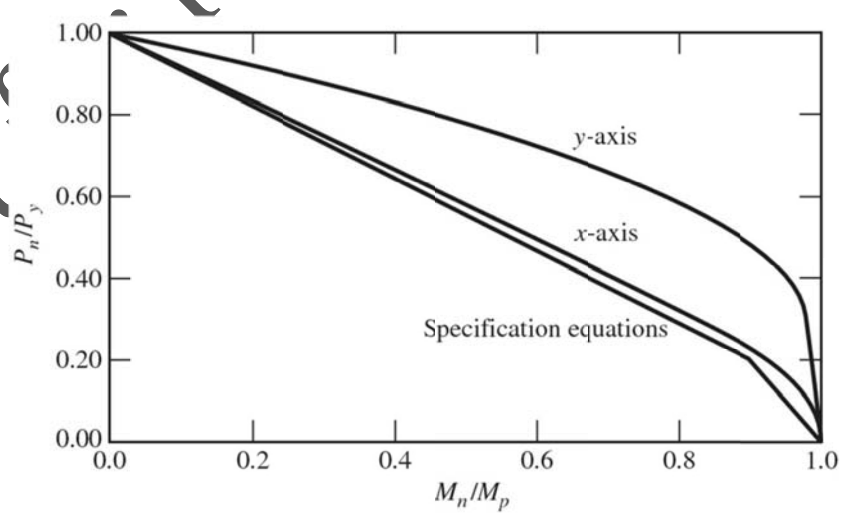


Figure 8.4b Normalized Interaction Diagram for Stub W14×82 Column.

horizontal plane of Figure 8.3 represents the interaction of moments in the two principal axis directions, called biaxial bending, whereas the vertical planes represent the interaction of axial compression plus either major or minor axis bending. It should also be apparent that the three-dimensional aspect is represented by a plane with intercepts given by the straight lines on the three coordinate planes.

The interaction equations in Chapter H of the *Specification* result from fitting interaction equations that are similar to the form of Equation 8.1 to a set of data developed from an analysis of forces and moments for various plastic stress distributions on a stub column. Figure 8.4a shows the actual analysis results for a W14×82 stub column. Figure 8.4b shows the same data plotted as functions of the normalized axial strength P_y and flexural strength M_p. In both cases, the influence of length on the axial or flexural strength is not included. Using curves of this type, developed for a wide variety of steel beam-column shapes, two equations were developed that are conservative and accurate for x-axis bending. When applied to y-axis bending, they are significantly more conservative; however, simplicity of design and the infrequent use of weak axis bending justify this extra level of conservatism.

An additional modification to these equations is required to account for length effects. Rather than normalizing the curves on the yield load and the plastic moment as was done in Figure 8.4b, the equations were developed around the nominal strength of the column and the nominal strength of the beam. The resulting equations are Equations H1-1a and H1-1b in the *Specification* and are plotted in Figure 8.5.

The equations shown here consider bending about both principal axes, whereas the plot in Figure 8.5 is for single-axis bending.

For $\dfrac{P_r}{P_c} \geq 0.2$,

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC H1-1a)}$$

For $\dfrac{P_r}{P_c} < 0.2$,

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC H1-1b)}$$

where

P_r = required compressive strength, kips
P_c = available compressive strength, kips
M_r = required flexural strength, ft-kips
M_c = available flexural strength, ft-kips
x = subscript relating symbol to strong axis bending
y = subscript relating symbol to weak axis bending

It is important to note that

1. The available column strength, P_c, is based on the axis of the column with the largest slenderness ratio. This is not necessarily the axis about which bending takes place.
2. The available bending strength, M_c, is based on the bending strength of the beam without axial load, including the influence of all the beam limit states.

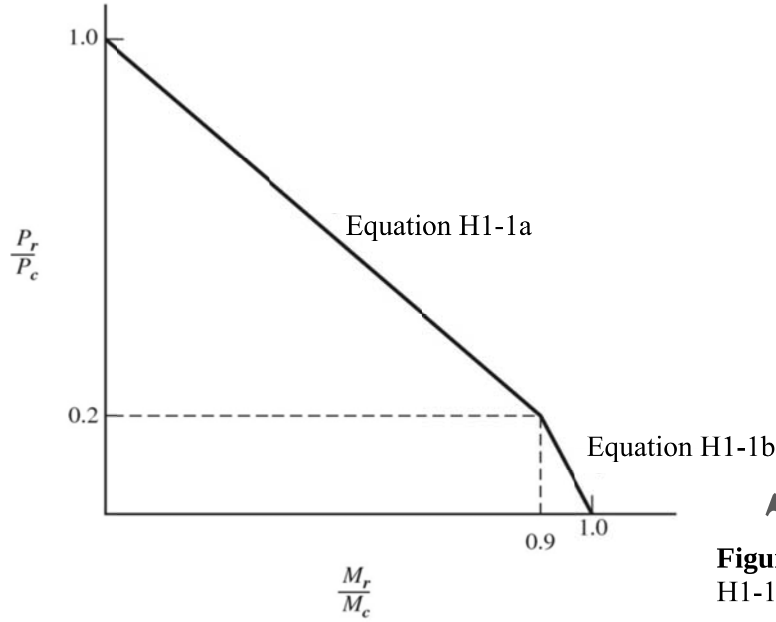


Figure 8.5 Interaction Equations H1-1a and H1-1b.

3. The required compressive strength, P_r, is the force in the member, including second-order effects.

4. The required flexural strength, M_r, is the bending moment in the member, including second-order effects.

Second-order forces and moments can be determined through a second-order analysis or by a modification of the results of a first-order analysis using amplification factors as mentioned earlier. These amplification factors will be discussed as they relate to braced frames (Section 8.5) and moment frames (Section 8.6).

Additional provisions are available for cases where the axial strength limit state is out-of-plane buckling and the flexural strength limit state is lateral-torsional buckling for bending in plane. Equations H1-1a and b are conservative for this situation, but an additional approach is available. *Specification* Section H1.3 provides that (1) for the limit state of in-plane instability, Equations H1-1a and H1-1b should be used where the compressive strength is determined for buckling in the plane of bending and $M_{cx} = M_p$, and (2) for the limit states of out-of-plane buckling and lateral-torsional buckling

$$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \le 1.0 \qquad \text{(AISC H1-3)}$$

where

P_{cy} = available compressive strength out of the plane of bending
M_{cx} = available lateral-torsional buckling strength for strong axis bending with C_b = 1.0
C_b = lateral-torsional buckling factor discussed in Chapter 6

If there is significant biaxial bending, meaning that the required-moment-to-available-moment ratio for y-axis bending is greater than or equal to 0.05, then this option is not available. Although this optional approach can provide a more economical solution in some cases, it is not used in the examples or problems in this book.

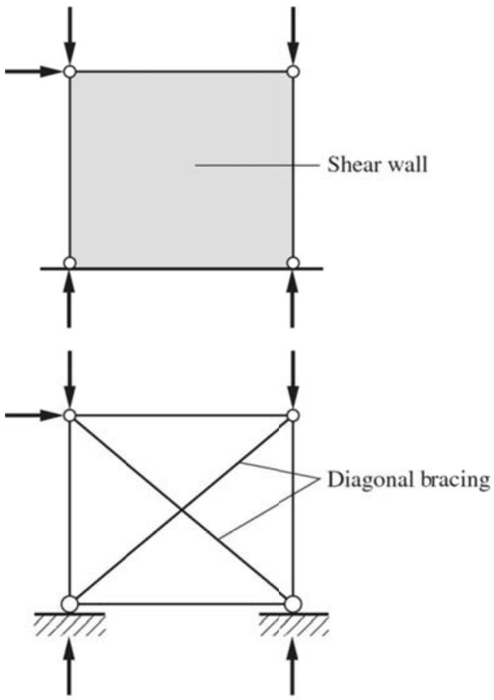


Figure 8.6 Braced Frame.

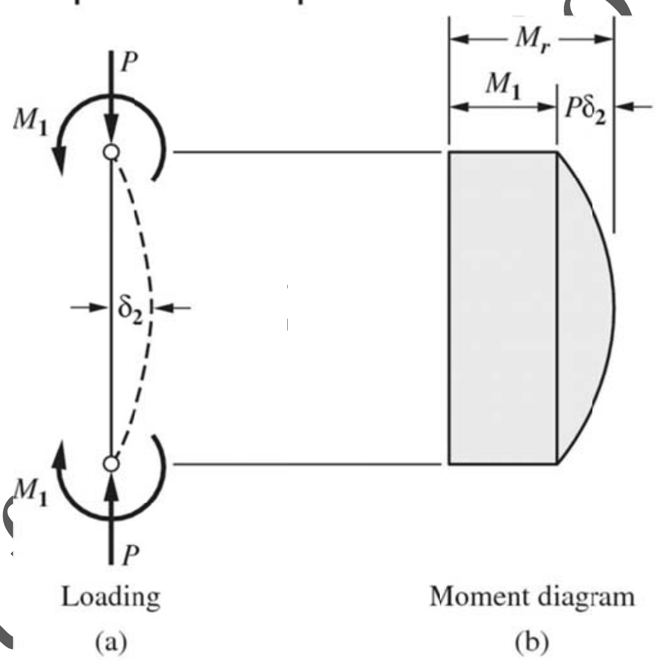


Figure 8.7 An Axially Loaded Column with Equal and Opposite End Moments.

8.5 BRACED FRAMES

A frame is considered to be braced if a positive system—that is, an actual system such as a shear wall (masonry, concrete, steel, or other material) or diagonal steel member—as illustrated in Figure 8.6, serves to resist the lateral loads, stabilize the frame under gravity loads, and resist lateral displacements. In these cases, columns are considered braced against lateral translation and the in-plane K-factor can be taken as 1.0, according to Appendix Section 7.2.3(a), unless a rational analysis indicates that a lower value is appropriate. This is the type of column that was discussed in Chapter 6. Later in this chapter the requirements for bracing to ensure that a structure can be considered a braced frame, as found in Appendix 6, are discussed.

If the column in a braced frame is rigidly connected to a girder, bending moments result from the application of the gravity loads to the girder. These moments can be determined through a first-order elastic analysis. The additional second-order moments resulting from the displacement along the column length can be determined through the application of an amplification factor.

The full derivation of the amplification factor has been presented by various authors.[1,2] Although this derivation is quite complex, a somewhat simplified derivation is presented here to help establish the background. An axially loaded column with equal and opposite end moments is shown in Figure 8.7a. This is the same column that was discussed in Section 8.2. The resulting moment diagram is shown in Figure 8.7b where the moments from both the end moments and the secondary effects are given.

The maximum moment occurring at the mid-height of the column, M_r, is shown to be

$$M_r = M_1 + P\delta_2$$

The amplification factor is defined as

$$AF = \frac{M_r}{M_1} = \frac{M_1 + P\delta_2}{M_1}$$

Rearranging terms yields

$$AF = \frac{1}{1 - \dfrac{P\delta_2}{M_1 + P\delta_2}}$$

Two simplifying assumptions will be made. The first is based on the assumption that δ is sufficiently small that

$$\frac{\delta_2}{M_1 + P\delta_2} \approx \frac{\delta_1}{M_1}$$

and the second, using the beam deflection, $\delta_1 = M_1 L^2/8EI$, assumes that

$$\frac{M_1}{\delta_1} = \frac{8EI}{L^2} \approx \frac{\pi^2 EI}{L^2} = P_e$$

Because these simplifying assumptions are in error in opposite directions, they tend to be offsetting. This results in a fairly accurate prediction of the amplification. Thus,

$$AF = \frac{1}{1 - P/P_e} \tag{8.2}$$

A comparison between the actual amplification and that given by Equation 8.2 is shown in Figure 8.8.

The discussion so far has assumed that the moments at each end of the column are equal and opposite, and that the resulting moment diagram is uniform. This is the most severe loading case for a beam-column braced against translation. If the moment is not uniformly distributed, the

[1]Galambos, T. V., *Structural Members and Frames*. Englewood Cliffs, NJ: Prentice Hall, Inc., 1968.
[2]Johnson, B. G., Ed., *Guide to Stability Design Criteria for Metal Structures*, 3rd ed., SSRC, New York: Wiley, 1976.

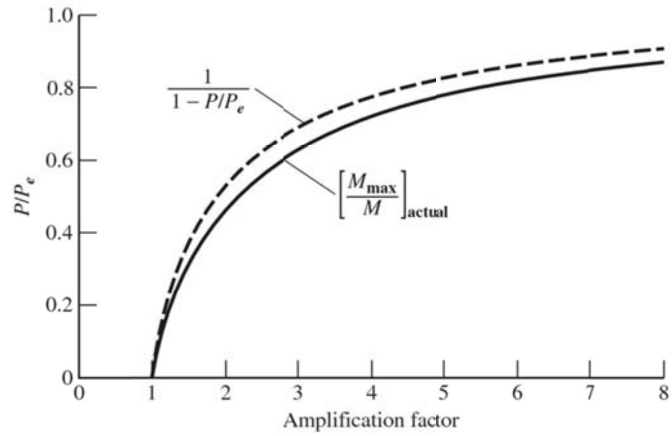


Figure 8.8 Amplified Moment: Exact and Approximate.

displacement along the member is less than previously considered and the resulting amplified moment is less than indicated. It has been customary in design practice to use the case of uniform moment as a base and to provide for other moment distributions by converting them to an equivalent uniform moment through the use of an additional factor, C_m.

Numerous studies have shown that a reasonably accurate correction results for beam-columns braced against translation and not subject to transverse loading between their supports, if the moment is reduced through its multiplication by C_m, where

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) \qquad \text{(AISC A-8-4)}$$

M_1/M_2 is the ratio of the smaller to larger moments at the ends of the member unbraced length in the plane of bending. M_1/M_2 is positive when the member is bent in reverse curvature and negative when bent in single curvature.

For beam-columns in braced frames where the member is subjected to transverse loading between supports, C_m may be taken from Commentary Table C-A-8.1, or conservatively taken as 1.0.

The combination of the amplification factor, AF, and the equivalent moment factor, C_m, accounts for the total member secondary effects. This combined factor is given as B_1 in Appendix 8 of the *Specification* as

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1.0 \qquad \text{(AISC A-8-3)}$$

where

α = 1.6 for ASD and 1.0 for LRFD to account for the nonlinear behavior of the structure at its ultimate strength
P_r = required strength, which may be taken as the first-order required strength, $P_{nt} + P_{lt}$, when used in moment frames
P_{e1} = Euler buckling load for the column in the plane of bending with an effective length factor, $K = 1.0$

Thus, the value of M_r in Equations H1-1a and H1-1b is taken as

$$M_r = B_1 M_{nt}$$

where M_{nt} is the maximum moment on the beam-column. The subscript *nt* indicates that for this case, the column does not undergo any lateral translation of its ends. It is possible for C_m to be less than 1.0 and for Equation A-8-3 to give an amplification factor less than 1.0. This indicates

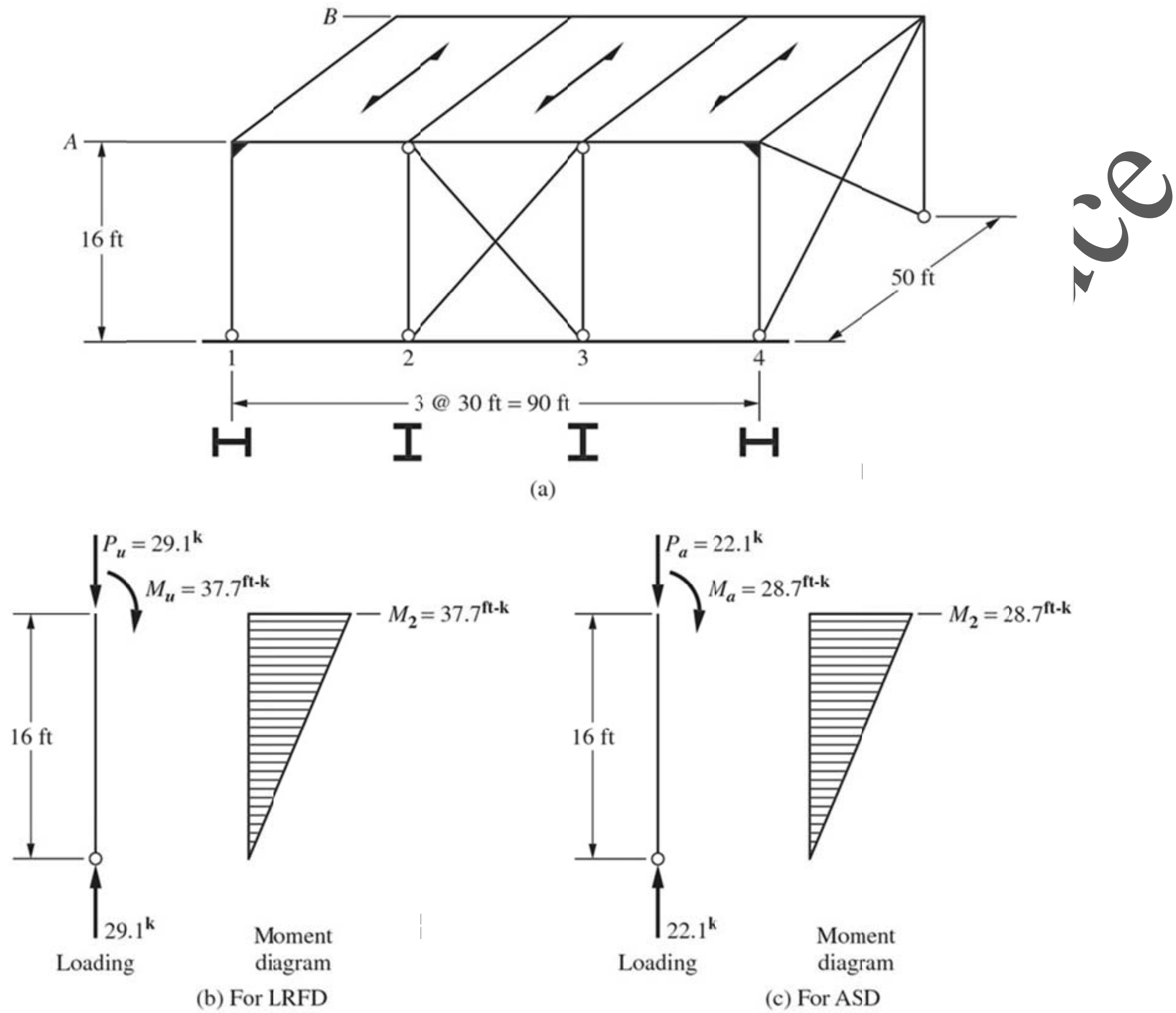


Figure 8.9 Three-Dimensional Braced Frame for a Single-Story Structure.

that the combination of the $P\delta$ effects and the nonuniform moment results in a moment less than the maximum moment on the beam-column from a first-order analysis. In this case, the amplification factor $B_1 = 1.0$.

EXAMPLE 8.1a
Braced Frame Column Design for Combined Axial and Bending by LRFD

Goal: Design column A1 in Figure 8.9 for the given loads using the LRFD provisions and the second-order amplification factor provided in Appendix 8 of the *Specification*.

Given: The three-dimensional braced frame for a single-story structure is given in Figure 8.9. Rigid connections are provided at the roof level for columns A1, B1, A4, and B4. All other column connections are pinned. Dead Load = 50 psf, Snow Load = 20 psf, Roof Live Load = 10 psf, and Wind Load = 20 psf horizontal. Use A992 steel. Assume that the X-bracing is sufficiently stiffer than the rigid frames to resist all lateral load.

SOLUTION

Step 1: Determine the appropriate load combinations. From ASCE 7, Section 2.3, the following two combinations are considered.

ASCE 7 load combination 3

$$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (0.5L \text{ or } 0.5W)$$

ASCE 7 load combination 4

$$1.2D + 1.0W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R)$$

Step 2: Determine the factored roof gravity loads for each load combination. For load combination 3

$$1.2(50) + 1.6(20) = 92 \text{ psf}$$

and for load combination 4

$$1.2(50) + 0.5(20) = 70 \text{ psf}$$

Because column A1 does not participate in the lateral load resistance, the worst case loading will use the uniformly distributed roof load of 92 psf.

Step 3: Carry out a preliminary first-order analysis. Because the structure is indeterminate, a number of approaches can be taken. If an arbitrary 6:1 ratio of moment of inertia for beams to columns is assumed, a moment distribution analysis yields the moment and force given in Figure 8.9b. Thus, the column will be designed to carry
P_u = 29.1 kips and M_u = 37.7 ft-kips

Step 4: Select a trial size for column A1 and determine its compressive strength and bending strength.

Try W10×33. (Section 8.8 addresses trial section selection.)

From *Manual* Table 1-1

$$A = 9.71 \text{ in.}^2,\ r_x = 4.19 \text{ in.},\ r_y = 1.94 \text{ in.},\ I_x = 171 \text{ in.}^4,\ r_x/r_y = 2.16$$

The column is oriented so that bending is about the *x*-axis of the column. It is braced against sidesway by the diagonal braces in panel A2–A3 and is pinned at the bottom and rigidly connected at the top in the plane of bending. The column is also braced out of the plane of bending by the brace in panel A1–B1. Because this column is part of a braced frame, $K = 1.0$ can be used. Although the *Specification* permits the use of a lower K-factor if justified by analysis, this is not recommended because it would likely require significantly more stiffness in the braced panel.

From *Manual* Table 4-1a, for *y*-axis buckling

$$\phi P_n = 214 \text{ kips for } L_c = 16.0 \text{ ft}$$

From *Manual* Table 3-10

$$\phi M_n = 113 \text{ ft-kips for } L_b = 16.0 \text{ ft}$$

Step 5: Check the W10×33 for combined axial load and bending in-plane.

For an unbraced length of 16 ft, the Euler load is

$$P_{e1} = \frac{\pi^2 EI}{L_{c1}^2} = \frac{\pi^2(29{,}000)(171)}{(16.0(12))^2} = 1330 \text{ kips}$$

The column is bent in single curvature between bracing points, the end points, and the moment at the base is zero, so $M_1/M_2 = 0.0$. Thus

$$C_m = 0.6 - 0.4(0.0) = 0.6$$

Therefore, the amplification factor, with $\alpha = 1.0$, becomes

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{1.0(29.1)}{1330}} = 0.613 \leq 1.0$$

The *Specification* requires that B_1 not be less than 1.0. Therefore, taking $B_1 = 1.0$,

$$M_{rx} = B_1(M_x) = 1.0(37.7) = 37.7 \text{ ft-kips}$$

To determine which equation to use, calculate

$$\frac{P_u}{\phi P_n} = \frac{29.1}{214} = 0.136 < 0.2$$

Therefore, use Equation H1-1b

$$\frac{P_u}{2\phi P_n} + \frac{M_u}{\phi M_n} \leq 1.0$$

$$0.5(0.136) + \frac{37.7}{113} = 0.402 < 1.0$$

Thus, the W10×33 will easily carry the given loads.

The solution to Equation H1-1b indicates that there is a fairly wide extra margin of safety. It would be appropriate to consider a smaller column for a more economical design.

EXAMPLE 8.1b
Braced Frame Column Design for Combined Axial and Bending by ASD

Goal: Design column A1 in Figure 8.9 for the given loads using the ASD provisions and the second-order amplification factor provided in Appendix 8 of the *Specification*.

Given: The three-dimensional braced frame for a single-story structure is given in Figure 8.9. Rigid connections are provided at the roof level for columns A1, B1, A4, and B4. All other column connections are pinned. Dead Load = 50 psf, Snow Load = 20 psf, Roof Live Load = 10 psf, and Wind Load = 20 psf horizontal. Use A992 steel. Assume that the X-bracing is sufficiently stiffer than the rigid frames to resist all lateral load.

SOLUTION

Step 1: Determine the appropriate load combinations. From ASCE 7, Section 2.3, the following two combinations are considered.

ASCE 7 load combination 3

$$D + (L_r \text{ or } S \text{ or } R)$$

ASCE 7 load combination 6

$$D + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$$

Step 2: Determine the factored roof gravity loads for each load combination. For load combination 3

$$50 + 20 = 70 \text{ psf}$$

and for load combination 6

$$50 + 0.75(20) = 65 \text{ psf}$$

Because column A1 does not participate in the lateral load resistance, the worst case loading will use the uniformly distributed roof load of 70 psf.

Step 3: Carry out a preliminary first-order analysis. Because the structure is indeterminate, a number of approaches can be taken. If an arbitrary 6:1 ratio of moment of inertia for beams to columns is assumed, a moment distribution analysis yields the moment and force given in Figure 8.9c. Thus, the column will be designed to carry

$$P_a = 22.1 \text{ kips and } M_a = 28.7 \text{ ft-kips}$$

Step 4: Select a trial size for column A1 and determine its compressive strength and bending strength.

Try W10×33. (Section 8.8 addresses trial section selection.)

From *Manual* Table 1-1

$$A = 9.71 \text{ in.}^2,\ r_x = 4.19 \text{ in.},\ r_y = 1.94 \text{ in.},\ I_x = 171 \text{ in.}^4,\ r_x/r_y = 2.16$$

The column is oriented so that bending is about the x-axis of the column. It is braced against sidesway by the diagonal braces in panel A2–A3 and is pinned at the bottom and rigidly connected at the top in the plane of bending. The column is also braced out of the plane of bending by the brace in panel A1–B1. Because this column is part of a braced frame, $K = 1.0$ can be used. Although the *Specification* permits the use of a lower K-factor if justified by analysis, this is not recommended because it would likely require significantly more stiffness in the braced panel.

From *Manual* Table 4-1a for y-axis buckling

$$P_n/\Omega = 142 \text{ kips for } L_c = 16.0 \text{ ft}$$

From *Manual* Table 3-10

$$M_n/\Omega = 74.9 \text{ ft-kips for } L_b = 16.0 \text{ ft}$$

Step 5: Check the W10×33 for combined axial load and bending in-plane.

For an unbraced length of 16 ft, the Euler load is

$$P_{e1} = \frac{\pi^2 EI}{L_{c1}^2} = \frac{\pi^2 (29{,}000)(171)}{(16.0(12))^2} = 1330 \text{ kips}$$

The column is bent in single curvature between bracing points, the end points, and the moment at the base is zero, so $M_1/M_2 = 0.0$. Thus

$$C_m = 0.6 - 0.4(0.0) = 0.6$$

Therefore, the amplification factor, with α = 1.6. becomes

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{1.6(22.1)}{1330}} = 0.616 \le 1.0$$

The *Specification* requires that B_1 not be less than 1.0. Therefore, taking B_1 = 1.0,

$$M_{rx} = B_1(M_x) = 1.0(28.7) = 28.7 \text{ ft-kips}$$

To determine which equation to use, calculate

$$\frac{P_u}{P_n/\Omega} = \frac{22.1}{142} = 0.156 < 0.2$$

Therefore, use Equation H1-1b

$$\frac{P_a}{2P_n/\Omega} + \frac{M_a}{M_n/\Omega} \le 1.0$$

$$0.5(0.156) + \frac{28.7}{74.9} = 0.461 < 1.0$$

Thus, the W10×33 will easily carry the given loads.

The solution to Equation H1-1b indicates that there is a fairly wide extra margin of safety. It would be appropriate to consider a smaller column for a more economical design.

8.6 Moment Frames

A moment frame depends on the stiffness of the beams and columns that make up the frame for stability under gravity loads and under combined gravity and lateral loads. Unlike braced frames, there is no external structure to lean against for stability. Columns in moment frames are subjected to both axial load and moment and experience lateral translation.

The same interaction equations, Equations H1-1a and H1-1b, are used to design beam-columns in moment frames as were previously used for braced frames. However, in addition to the member second-order effects discussed in Section 8.5, there is the additional second-order effect that results from the sway or lateral displacement of the frame.

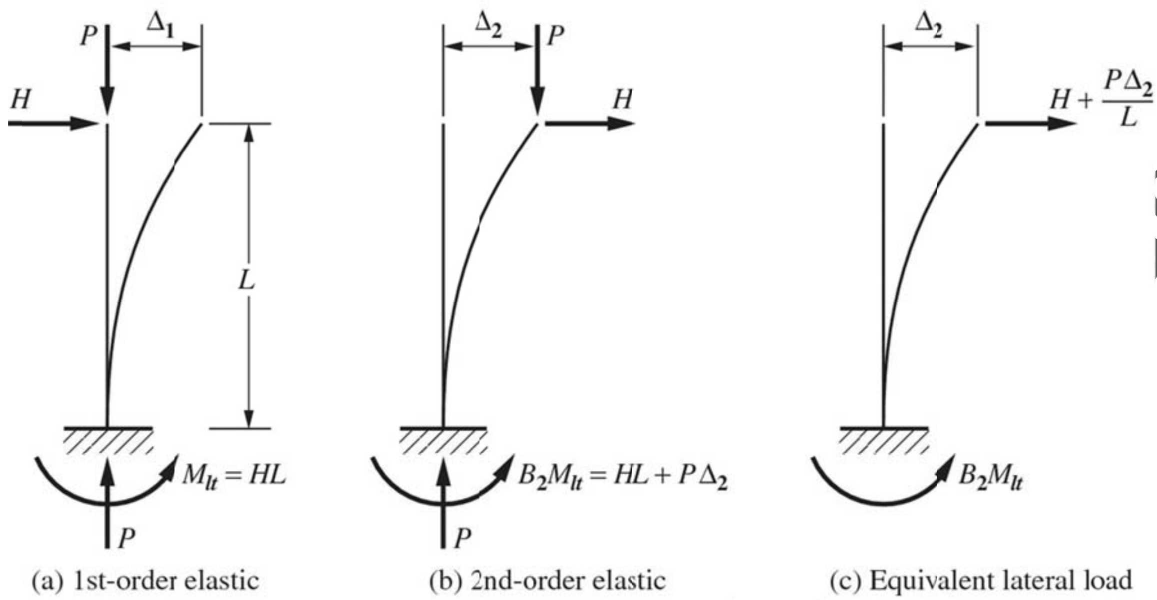


Figure 8.10 Structure Second-Order Effect: Sway.

Figure 8.10 shows a cantilever or flag pole column under the action of an axial load and a lateral load. Figure 8.10a is the column as viewed for a first-order elastic analysis where equilibrium requires a moment at the bottom, $M_{lt} = HL$. The deflection that results at the top of the column, Δ_1, is the elastic deflection of a cantilever, so

$$\Delta_1 = \frac{HL^3}{3EI} \tag{8.3}$$

A second-order analysis yields the forces and displacements as shown in Figure 8.10b. The displacement, Δ_2, is the total displacement, including second-order effects, and the moment, including second-order effects, is

$$B_2 M_{lt} = HL + P\Delta_2 \tag{8.4}$$

An equivalent lateral load can be determined that results in the same moment at the bottom of the column as in the second-order analysis. This load is $H + P\Delta_2/L$ and is shown in Figure 8.10c.

It may be assumed, with only slight error, that the displacements at the top of the column for the cases in Figures 8.10b and c are the same. Thus, using the equivalent lateral load

$$\Delta_2 = \frac{(H + P\Delta_2/L)L^3}{3EI} = \frac{HL^3}{3EI}\left(1 + \frac{P\Delta_2}{HL}\right) = \Delta_1\left(1 + \frac{P\Delta_2}{HL}\right) \tag{8.5}$$

Equation 8.5 can now be solved for Δ_2, where

$$\Delta_2 = \frac{\Delta_1}{1 - \dfrac{P\Delta_1}{HL}} \tag{8.6}$$

and the result substituted into Equation 8.4. Solving the resulting equation for the amplification factor, B_2, and simplifying yields

$$B_2 = \frac{\Delta_2}{\Delta_1} = \frac{1}{1 - \dfrac{P\Delta_1}{HL}} \tag{8.7}$$

Considering that the typical beam-column will be part of some larger structure, this equation must be modified to include the effect of the multistory and multibay characteristics of the actual structure. This is easily accomplished by summing the total gravity load on the columns in the story and the total lateral load in the story. Thus, Equation 8.7 becomes

$$B_2 = \frac{1}{1 - \dfrac{\Sigma P\Delta_1}{\Sigma HL}} \tag{8.8}$$

This amplification factor is essentially that given in Appendix 8 of the *Specification* as Equation A-8-6, when combined with Equation A-8-7

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \dfrac{\alpha P_{story}\Delta_H}{R_M HL}} \geq 1.0 \qquad \text{(AISC A-8-6)}$$

where

P_{story} = total gravity load on the story

$P_{e\,story}$ = measure of lateral strength of the structure = $R_M \dfrac{HL}{\Delta_H}$ (AISC A-8-7)

Δ_H = story drift from a first-order analysis due to the lateral load, H

α = 1.0 for LRFD and 1.6 for ASD to account for the nonlinear behavior of the structure at its ultimate strength

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right) \qquad \text{(AISC A-8-8)}$$

P_{mf} = the total vertical load in columns that are part of the lateral load resisting system

The variable R_M accounts for the influence of the member effect on the sidesway displacement that could not be accounted for in the simplified derivation above. If all the columns are moment frame columns, P_{mf}/P_{story} = 1.0 and R_M = 0.85. For braced frames, P_{mf} = 0 and R_M = 1.0. For frames with a combination of columns resisting lateral load through bending and gravity only or *leaning columns*, the value of R_M will be between these limits.

It is often desirable to limit the lateral displacement, or drift, of a structure during the design phase. ASCE 7 Appendix C Commentary provides some general guidance. This limit can be defined using a drift index, which is the story drift divided by the story height, Δ_H/L. The design then proceeds by selecting members so that the final structure performs as desired. This is similar to beam design, where deflection is the serviceability criterion. Because a limit on the drift index can be established without knowing member sizes, it can be used in Equation A-8-6; thus an analysis with assumed member sizes is unnecessary.

With this amplification for sidesway, the moment, M_r, to be used in Equations AISC H1-1a and AISC H1-1b, can be evaluated. M_r must include both the member and structure second-order effects. Thus, a first-order analysis without sidesway is carried out, yielding moments, M_{nt}, that is without translation, to be amplified by B_1. Next, a first-order analysis including lateral loads and permitting translation is carried out. This yields moments, M_{lt}, with translation, to be amplified by B_2. The resulting second-order moment is

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(AISC A8-1)}$$

where

B_1 is given by Equation A-8-3

B_2 is given by Equation A-8-6

M_{nt} = first-order moments when the structure is not permitted to translate laterally

M_{lt} = first-order moments that result from just the lateral translation

M_{lt} could include moments that result from unsymmetrical frame properties or loading as well as from lateral loads. In most real structures, however, moments resulting from this lack of symmetry are usually small and are thus often ignored.

The second-order force is

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(AISC A-8-2)}$$

The sum of P_{nt} and P_{lt} for the entire structure will equal the total gravity load on the structure, since the sum of P_{lt} will be zero. For the individual column, however, it is important to amplify the portion of the individual column force that comes from the lateral load.

For situations where there is no lateral load on the structure, it may be necessary to incorporate a minimum lateral load in order to capture the second-order effects of the gravity loads. This is covered in Section 8.7 where the three methods provided in the *Specification* for treating stability analysis and design are discussed.

EXAMPLE 8.2a
Moment Frame Strength Check for Combined Compression and Bending by LRFD

Goal: Using the LRFD provisions, determine whether the W14×90, A992 column shown in Figure 8.11 is adequate to carry the imposed loading.

Given: An exterior column from an intermediate level of a multi-story moment frame is shown in Figure 8.11. The column is part of a braced frame out of the plane of the figure. Figure 8.11a shows the elevation of the frame with the member to be checked labeled AB. The same column section will be used for the level above and below the column AB. A first-order analysis of the frame for gravity loads plus the minimum lateral load (the minimum lateral load will be discussed in Section 8.7) results in the forces shown in Figure 8.11b, whereas the results for gravity plus wind are shown in Figure 8.11c. Assume that the frame drift under service loads is limited to height/300 for a story shear, H = 148 kips.

SOLUTION

Step 1: Determine the column effective length factor in the plane of bending.

Using the effective length alignment chart introduced in Chapter 5 and given in Commentary Figure C-A-7.2, determine the effective length for buckling in the plane of the moment frame. At each joint there are two columns and one beam framing in. Thus,

$$G_A = G_B = \frac{\Sigma(I/L)_c}{\Sigma(I/L)_g} = \frac{2\left(\frac{999}{12.5}\right)}{\left(\frac{2100}{30.0}\right)} = 2.28$$

Thus, from Figure 5.20, K = 1.66.

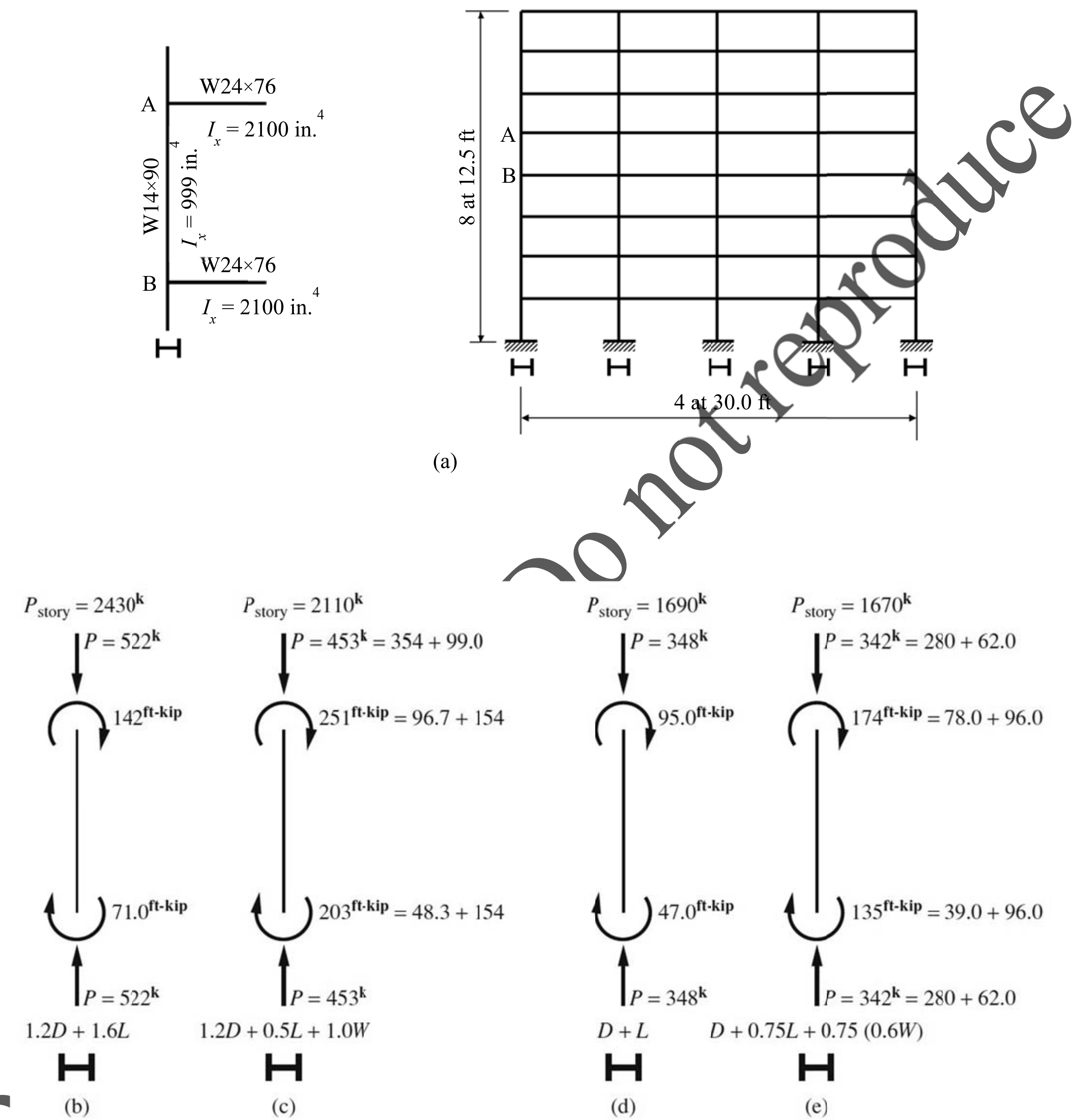


Figure 8.11 Exterior Column From an Intermediate Level of a Multistory Rigid Frame (Example 8.2).

Step 2: Determine the controlling effective length.

With $r_x/r_y = 1.66$ for the W14 × 90,

$$\left(L_{cx}\right)_{eff} = \left(KL\right)_{eff} = \frac{\left(KL\right)_x}{r_x/r_y} = \frac{1.66\left(12.5\right)}{1.66} = 12.5 \text{ ft}$$

$$L_{cy} = KL_y = 1.0(12.5) = 12.5 \text{ ft}$$

Step 3: Since the effective length about each axis is 12.5 ft, determine the column design axial strength using $L_c = 12.5$.

From the column tables, *Manual* Table 4-1a, for $L_c = 12.5$ ft,
$\phi P_n = 1060$ kips

Step 4: Determine the first-order moments and forces for the loading combination that includes wind, 1.2D +0.5L + 1.0W.

The column end moments given in Figure 8.11c are a combination of moments resulting from a nonsway gravity load analysis and a wind analysis:

Moment for end *A*:

$M_{nt} = 96.7$ ft-kips
$M_{lt} = 154$ ft-kips

Moment for end *B*:

$M_{nt} = 48.3$ ft-kips
$M_{lt} = 154$ ft-kips

Compression:

$P_{nt} = 354$ kips
$P_{lt} = 99.0$ kips

Step 5: Determine the second-order moments by amplifying the first-order moments.

No-translation amplification: The no-translation moments must be amplified by B_1. From Figure 8.11c it is seen that the end moments bend the column in reverse curvature:

$$\frac{M_1}{M_2} = \frac{48.3}{96.7} = 0.50$$

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4(0.50) = 0.4$$

$$P_{e1} = \frac{\pi^2 E}{L_{c1}^2} = \frac{\pi^2(29{,}000)(999)}{(1.0(12.5)(12))^2} = 12{,}700 \text{ kips}$$

Thus, with $\alpha = 1.0$ for LRFD and $P_r = 354 + 99 = 453$ kips, Equation A-8-3 yields

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.4}{1 - \dfrac{(1.0)(453)}{12{,}700}} = 0.415 < 1.0$$

Therefore, $B_1 = 1.0$.

Translation amplification: The translation forces and moments must be amplified by B_2. The design drift limit of height/300 and Equation A-8-6 are used to determine B_2.

The total service lateral load on this story is given as

$$H = 148 \text{ kips}$$

Additional given information is that the total gravity load for this load combination in Figure 8.11c is

$$P_{story} = 2110 \text{ kips}$$

The drift limit under the service lateral load of 148 kips is

$$\Delta_H = L/300 = 12.5(12)/300 = 0.50 \text{ in.}$$

Remember that in the calculation of B_2, H can be taken as any convenient magnitude, as long as Δ_H is the corresponding displacement. This is because it is the ratio of H to Δ_H that is used in the determination of $P_{e\,story}$.

Thus, with $\alpha = 1.0$ for LRFD and $R_M = 0.85$ assuming all columns are moment frame columns, Equation A-8-7 gives

$$P_{e\,story} = \frac{R_M HL}{\Delta_H} = \frac{0.85(148)(12.5)(12)}{0.50} = 37{,}700 \text{ kips}$$

and Equation A-8-6 gives

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \left(\dfrac{(1.0)2110}{37{,}700}\right)} = 1.06 > 1.0$$

Thus, the second-order compressive force and moment are

$$P_r = P_{nt} + B_2(P_{lt}) = 354 + 1.06(99) = 459 \text{ kips}$$
$$M_r = B_1(M_{nt}) + B_2(M_{lt}) = 1.0(96.7) + 1.06(154) = 260 \text{ ft-kips}$$

These represent the required strength for this load combination.

Step 6: Determine whether the W14×90 will provide the required strength based on the appropriate interaction equation.

The unbraced length of the compression flange for pure bending is 12.5 ft, which is less than L_p = 15.1 ft for this section, taking into account that its flange is noncompact. Thus, from *Manual* Table 3-2, the design moment strength of the section is

$$\phi M_n = 574 \text{ ft-kips}$$

Determine the appropriate interaction equation. From Step 3, ϕP_n = 1060 kips;

$$\frac{P_u}{\phi P_n} = \frac{459}{1060} = 0.433 > 0.2$$

so use Equation H1-1a, which yields

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_u}{\phi M_n}\right) \leq 1.0$$

$$0.433 + \frac{8}{9}\left(\frac{260}{574}\right) = 0.836 < 1.0$$

Thus,

the W14×90 is adequate for this load combination.

Step 7: Check the section for the gravity-only load combination, 1.2D + 1.6L.

Because this is a gravity-only load combination, *Specification* Appendix Section 7.2.2, by reference to Section C2.2b, requires that the analysis include a minimum lateral load of 0.002 times the gravity load. This will be further discussed in Section 8.7. For this load combination, the total story gravity load must also be known and is given in Figure 8.11b as P_{story} = 2430 kips. Thus, for this frame the minimum lateral load is $0.002P_{story}$ = 0.002(2430) = 4.86 kips at this level.

The forces and moments given in Figure 8.11b include the effects of this minimum lateral load. The magnitude of the lateral translation effect is small in this case. Since both the moment due to the minimum lateral load and the amplification factor, B_2, are expected to be small, the forces and moments used for this check will be assumed to come from a no-translation case, with little error. If the minimum lateral load would produce large moments or the amplification factor, B_2, calculated in Step 5, were large, this would not be a good assumption. Therefore, at end A, M_{nt} = 142 ft-kips, at end B M_{nt} = 71.0 ft-kips, and P_{nt} = 522 kips.

A quick review of the determination of B_1 from the first part of this solution shows that the only change is in the magnitude of the axial force and the member end moments; thus

$$\frac{M_1}{M_2} = \frac{71.0}{142} = 0.50$$

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4(0.50) = 0.4$$

$$P_{e1} = \frac{\pi^2 EI}{L_{c1}^2} = \frac{\pi^2(29{,}000)(999)}{(1.0(12.5)(12))^2} = 12{,}700 \text{ kips}$$

Thus, with $\alpha = 1.0$ for LRFD,

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.4}{1 - \dfrac{(1.0)(522)}{12{,}700}} = 0.417 < 1.0$$

Note that B_1 is again 1.0.

With the assumption that there is no lateral translation, $M_{lt} = 0.0$ and B_2 is unnecessary,thus

$$P_r = 522 \text{ kips}, \quad M_r = 1.0(142) = 142 \text{ ft-kips}$$

Again using Equation H1-1a,

$$\frac{P_u}{\phi P_n} + \frac{8}{8}\left(\frac{M_u}{\phi M_n}\right) \le 1.0$$

$$\frac{522}{1060} + \frac{8}{9}\left(\frac{142}{574}\right) = 0.712 < 1.0$$

Thus,

the W14×90 is adequate for both load combinations.

EXAMPLE 8.2b
Moment Frame Strength Check for Combined Compression and Bending by ASD

Goal: Using the ASD provisions, determine whether the W14×90, A992 column shown in Figure 8.11 is adequate to carry the imposed loading.

Given: An exterior column from an intermediate level of a multi-story moment frame is shown in Figure 8.11. The column is part of a braced frame out of the plane of the figure. Figure 8.11a shows the elevation of the frame with the member to be checked labeled AB. The same column section will be used for the level above and below the column AB. A first-order analysis of the frame for gravity loads plus the minimum lateral load (the minimum lateral load will be discussed in Section 8.7) results in the forces shown in Figure 8.11d, whereas the results for gravity plus wind are shown in Figure 8.11e. Assume that the frame drift under service loads is limited to height/300 for a story shear, $H = 148$ kips.

SOLUTION

Step 1: Determine the column effective length factor in the plane of bending.

Using the effective length alignment chart introduced in Chapter 5 and given in Commentary Figure C-A-7.2, determine the effective length for buckling in the plane of the moment frame. At each joint there are two columns and one beam framing in. Thus,

$$G_A = G_B = \frac{\Sigma(I/L)_c}{\Sigma(I/L)_g} = \frac{2\left(\frac{999}{12.5}\right)}{\left(\frac{2100}{30.0}\right)} = 2.28$$

Thus, from Figure 5.20, $K = 1.66$.

Step 2: Determine the controlling effective length.

With $r_x/r_y = 1.66$ for the W14 × 90,

$$(L_{cx})_{eff} = (KL)_{eff} = \frac{(KL)_x}{r_x/r_y} = \frac{1.66(12.5)}{1.66} = 12.5 \text{ ft}$$

$$L_{cy} = KL_y = 1.0(12.5) = 12.5 \text{ ft}$$

Step 3: Since the effective length about each axis is 12.5 ft, determine the column allowable axial strength using $L_c = 12.5$.

From the column tables, *Manual* Table 4-1a, for $L_c = 12.5$ ft,

$$P_n/\Omega = 703 \text{ kips}$$

Step 4: Determine the first-order moments and forces for the loading combination that includes wind, D +0.75L + 0.75(0.6W).

The column end moments given in Figure 8.11e are a combination of moments resulting from a nonsway gravity load analysis and a wind analysis:

Moment for end *A*:

$M_{nt} = 78.0$ ft-kips
$M_{lt} = 96.0$ ft-kips

Moment for end *B*:

$M_{nt} = 39.0$ ft-kips
$M_{lt} = 96.0$ ft-kips

Compression:

$P_{nt} = 280$ kips
$P_{lt} = 62.0$ kips

Step 5: Determine the second-order moments by amplifying the first-order moments.

No-translation amplification: The no-translation moments must be amplified by B_1. From Figure 8.11e it is seen that the end moments bend the column in reverse curvature:

$$\frac{M_1}{M_2} = \frac{39.0}{78.0} = 0.50$$

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4(0.50) = 0.4$$

$$P_{e1} = \frac{\pi^2 E}{L_{c1}^2} = \frac{\pi^2(29{,}000)(999)}{(1.0(12.5)(12))^2} = 12{,}700 \text{ kips}$$

Thus, with $\alpha = 1.6$ for ASD and $P_r = 280 + 62 = 342$ kips, Equation A-8-3 yields

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} = \frac{0.4}{1 - \frac{(1.6)(342)}{12{,}700}} = 0.418 < 1.0$$

Therefore, $B_1 = 1.0$.

Translation amplification: The translation forces and moments must be amplified by B_2. The design drift limit of height/300 and Equation A-8-6 are used to determine B_2.

The total service lateral load on this story is given as

$$H = 148 \text{ kips}$$

Additional given information is that the total gravity load for this load combination in Figure 8.11e is

$$P_{story} = 1670 \text{ kips}$$

The drift limit under the service lateral load of 148 kips is

$$\Delta_H = L/300 = 12.5(12)/300 = 0.50 \text{ in.}$$

Remember that in the calculation of B_2, H can be taken as any convenient magnitude, as long as Δ_H is the corresponding displacement. This is because it is the ratio of H to Δ_H that is used in the determination of $P_{e\,story}$.

Thus, with $\alpha = 1.6$ for ASD and $R_M = 0.85$ assuming all columns are moment frame columns, Equation A-8-7 gives

$$P_{e\,story} = \frac{R_M HL}{\Delta_H} = \frac{0.85(148)(12.5)(12)}{0.50} = 37{,}700 \text{ kips}$$

and Equation A-8-6 gives

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} = \frac{1}{1 - \left(\frac{(1.6)1670}{37{,}700}\right)} = 1.08 > 1.0$$

Thus, the second-order compressive force and moment are

$$P_r = P_{nt} + B_2(P_{lt}) = 280 + 1.08(62.0) = 347 \text{ kips}$$
$$M_r = B_1(M_{nt}) + B_2(M_{lt}) = 1.0(78.0) + 1.08(96.0) = 182 \text{ ft-kips}$$

These represent the required strength for this load combination.

Step 6: Determine whether this shape will provide the required strength based on the appropriate interaction equation.

The unbraced length of the compression flange for pure bending is 12.5 ft, which is less than L_p = 15.1 ft for this section, taking into account that its flange is noncompact. Thus, from *Manual* Table 3-2, the allowable moment strength of the section is

$$M_n/\Omega = 382 \text{ ft-kips}$$

Determine the appropriate interaction equation. From Step 3, ϕP_n = 1060 kips;

$$\frac{P_a}{P_n/\Omega} = \frac{347}{703} = 0.494 > 0.2$$

so use Equation H1-1a, which yields

$$\frac{P_a}{P_n/\Omega} + \frac{8}{9}\left(\frac{M_a}{M_n/\Omega}\right) \le 1.0$$

$$0.494 + \frac{8}{9}\left(\frac{182}{382}\right) = 0.918 < 1.0$$

Thus,

the W14×90 is adequate for this load combination.

Step 7: Check the section for the gravity-only load combination, $D + L$.

Because this is a gravity-only load combination, *Specification* Appendix Section 7.2.2, by reference to Section C2.2b, requires that the analysis include a minimum lateral load of 0.002 times the gravity load. This will be further discussed in Section 8.7. For this load combination, the total story gravity load must also be known and is given in Figure 8.11d as P_{story} = 1690 kips. Thus, for this frame the minimum lateral load is $0.002P_{story}$ = 0.002(1690) = 3.38 kips at this level.

The forces and moments given in Figure 8.11d include the effects of this minimum lateral load. The magnitude of the lateral translation effect is small in this case. Since both the moment due to the minimum lateral load and the amplification factor, B_2, are expected to be small, the forces and moments used for this check will be assumed to come from a no-translation case, with little error. If the minimum lateral load would produce large moments or the amplification factor, B_2, calculated in Step 5, were large, this would not be a good assumption. Therefore, at end A, M_{nt} = 95.0 ft-kips, at end B M_{nt} = 47.0 ft-kips, and P_{nt} = 348 kips.

A quick review of the determination of B_1 from the first part of this solution shows that the only change is in the magnitude of the axial force and the member end moments; thus

$$\frac{M_1}{M_2} = \frac{47.0}{95.0} = 0.50$$

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4(0.50) = 0.4$$

$$P_{e1} = \frac{\pi^2 EI}{L_{c1}^2} = \frac{\pi^2(29{,}000)(999)}{(1.0(12.5)(12))^2} = 12{,}700 \text{ kips}$$

Thus, with $\alpha = 1.6$ for ASD,

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.4}{1 - \dfrac{(1.6)(348)}{12{,}700}} = 0.418 < 1.0$$

Note that B_1 is again 1.0.

With the assumption that there is no lateral translation,

$M_{lt} = 0.0$ and B_2 is unnecessary,

thus

$$P_r = 348 \text{ kips}, \quad M_r = 1.0(95.0) = 95.0 \text{ ft-kips}$$

Again using Equation H1-1a,

$$\frac{P_a}{P_n/\Omega} + \frac{8}{8}\left(\frac{M_a}{M_n/\Omega}\right) \le 1.0$$

$$\frac{348}{703} + \frac{8}{9}\left(\frac{95.0}{382}\right) = 0.716 < 1.0$$

Thus,

the W14×90 is adequate for both load combinations.

The moments in the beams and the beam-column connections must also be amplified for the critical case to account for the second-order effects. This is done by considering equilibrium of the beam-column joint. The amplified moments in the column above and below the joint are added together and this sum distributed to the beams which frame into the joint according to their stiffnesses. These moments then establish the connection design moments.

8.7 SPECIFICATION PROVISIONS FOR STABILITY ANALYSIS AND DESIGN

Up to this point, the discussion of the interaction of compression and bending has concentrated on the development of the interaction equations and one approach to incorporate second-order effects. The *Specification* actually provides three approaches to deal with these two closely linked issues. The most direct approach is to use a general second-order analysis in conjunction with the Direct Analysis Method described in Chapter C.

A general second-order analysis yields forces and moments that can be used directly in the interaction equations of Chapter H without the need to resort to amplification factors as just described. The disadvantage to this approach is that, since the extremely useful principal of superposition cannot be used (since the structural response is nonlinear), a complete nonlinear analysis must be carried out for each load combination. A discussion of general, or rigorous, methods of second-order analysis is beyond the scope of this book. Thus, in the remainder of this book, if second-order effects have not already been included in the analysis results given, the amplified first-order analysis approach will be used to obtain the required second-order forces and moments.

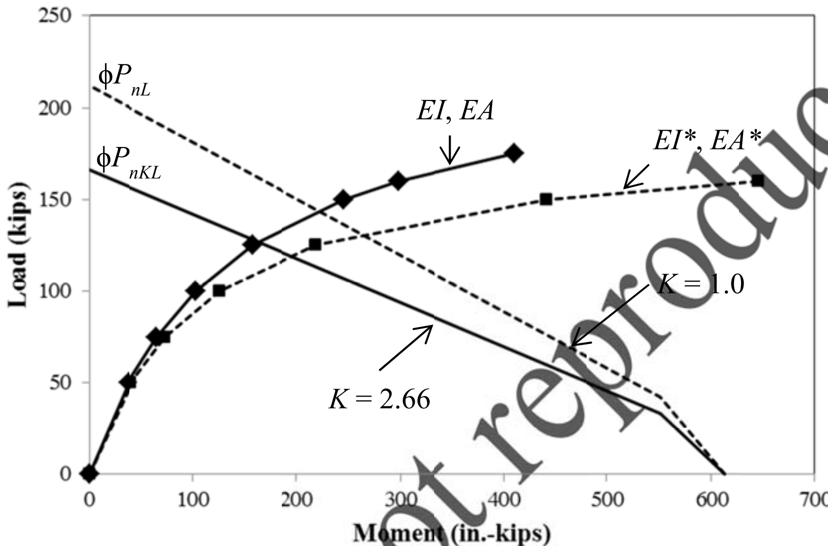


Figure 8.12 Comparison of the Effective Length Method and the Direct Analysis Method

8.7.1 Direct Analysis Method

The advantage of the direct analysis method of *Specification* Chapter C is that for the design of compression members, the effective length factor is taken as 1.0. Since for braced frames, K can always be taken as 1.0 based on Section 7.2.3(a), the direct analysis method is particularly useful for moment frames. The *Specification* requires that the stiffness of all elements contributing to the lateral load resistance of the structure be reduced. Thus, rather than using EA for the axial stiffness of the members, the modified stiffness, $EA^* = 0.8EA$, is used. Similarly for flexural stiffness, the modified stiffness $EI^* = 0.8\tau_b EI$, where τ_b accounts for the influence of residual stresses on second-order effects, is used. It should be remembered that the influence of residual stresses on the strength of compression and bending members was already discussed in Chapters 5 and 6. The use of τ_b in this instance is to capture the influence of those same residual stresses on displacements and thus on second-order effects. This is the same τ_b used with the alignment charts in the determination of the effective length in Chapter 5.

Figure 8.12 shows a comparison between the effective length method already presented and the direct analysis method for a simple structure. Equations H1-1a and H1-1b are plotted for the effective length method and labeled with $K = 2.66$. This indicates that the compressive strength of the member has been determined using $K = 2.66$. The nonlinear load-moment curve is identified with EI and EA to indicate that the nominal stiffnesses are used to determine this behavior. The intersection of these two curves indicates that this load and moment combination satisfy the interaction equation. Equations H1-1a and H1-1b are plotted for the direct analysis method and identified with $K = 1.0$. This indicates that the compressive strength of the member has been determined with $K = 1.0$. Note that regardless of which approach is selected to determine the compressive strength, the flexural strength is the same for both methods. The nonlinear load-moment curve is identified with EI^* and EA^* to indicate that the reduced stiffnesses were used to determine this behavior. The intersection of these two curves indicates that this load and moment combination satisfy the interaction equation. Next, note that the load magnitude for both of these intersections is nearly the same. Thus, the load that satisfies the interaction equation is the same regardless of which method is used. Since the direct analysis method did not require the determination of K, it is a significantly simpler method than the effective length method.

Another consideration that has only briefly been mentioned to this point is the requirement in Section C1 that the influence of geometric imperfections be considered. As with residual stresses, the influence of geometric imperfection on the strength of compression members has already been addressed through the *Specification* column strength equations. The requirement here is to consider the influence of out-of-plumbness on the stability of the structure. This may be accomplished by modeling the structure in its out-of-plumb condition or through the use of notional loads to simulate the out-of-plumbness. These notional loads will be discussed later in this section. It should be noted that this is not a requirement of the direct analysis method alone but a general requirement for determining required strength.

In addition to the direct analysis method, two other design methods are given in the *Specification*. They are found in Appendix 7. The limitations on the application of these methods are based on the direct analysis method.

8.7.2 Effective Length Method

Appendix 7.2 provides the requirements for the effective length method. This is the approach already described earlier in this chapter for braced and moment frames. It is valid so long as the ratio of second-order deflection to first-order deflection, Δ_2/Δ_1, is equal to or less than 1.5. Another way to state this requirement is to remember that $\Delta_2/\Delta_1 = B_2$, so the effective length method is valid as long as $B_2 \le 1.5$. Although this check was not made in Example 8.2, it can now be seen that it was acceptable to use the effective length method in that example, since for both LRFD and ASD, $B_2 \le 1.5$. A special case occurs when $B_2 \le 1.1$. In this case, columns in moment frames can be designed using $K = 1.0$. The effective length method is essentially the same method used in past practice with the addition of the requirement of a minimum lateral load to be applied in gravity-only load combinations. This is the notional load discussed above to account for initial out-of-plumbness. It is the same as the minimum lateral load used in Example 8.2 and will be discussed later in this section.

8.7.3 First-Order Analysis Method

A third method is given in the Appendix 7.3, the first-order analysis method. This approach permits design without direct consideration of second-order effects except through the application of additional notional lateral loads that account for structure out-of-plumbness and second-order effects. This is possible because of the limits placed on the implementation of this method. As with the effective length method, the structure must support gravity loads primarily through vertical columns, walls or frames and the ratio of the second-order drift to first-order drift must be less than or equal to 1.5. Additionally, compression members that participate in lateral load resistance must behave elastically according to

$$\alpha P_r \le 0.5 P_{ns} \qquad \text{(AISC A-7-1)}$$

With the foregoing limitations and the application of the notional load given by

$$N_i = 2.1\alpha\left(\Delta/L\right)Y_i \ge 0.0042Y_i \qquad \text{(AISC A-7-2)}$$

compression members may be designed using $K = 1.0$.

8.7.4 Geometric Imperfections

Now, consider in more detail the requirement to consider geometric imperfections. *Specification* Section C1 requires that geometric imperfections be considered in the analysis and design of structures. There are two types of geometric imperfections that must be considered: initial out-of-straightness and initial out-of-plumbness. The strength equations of *Specification*

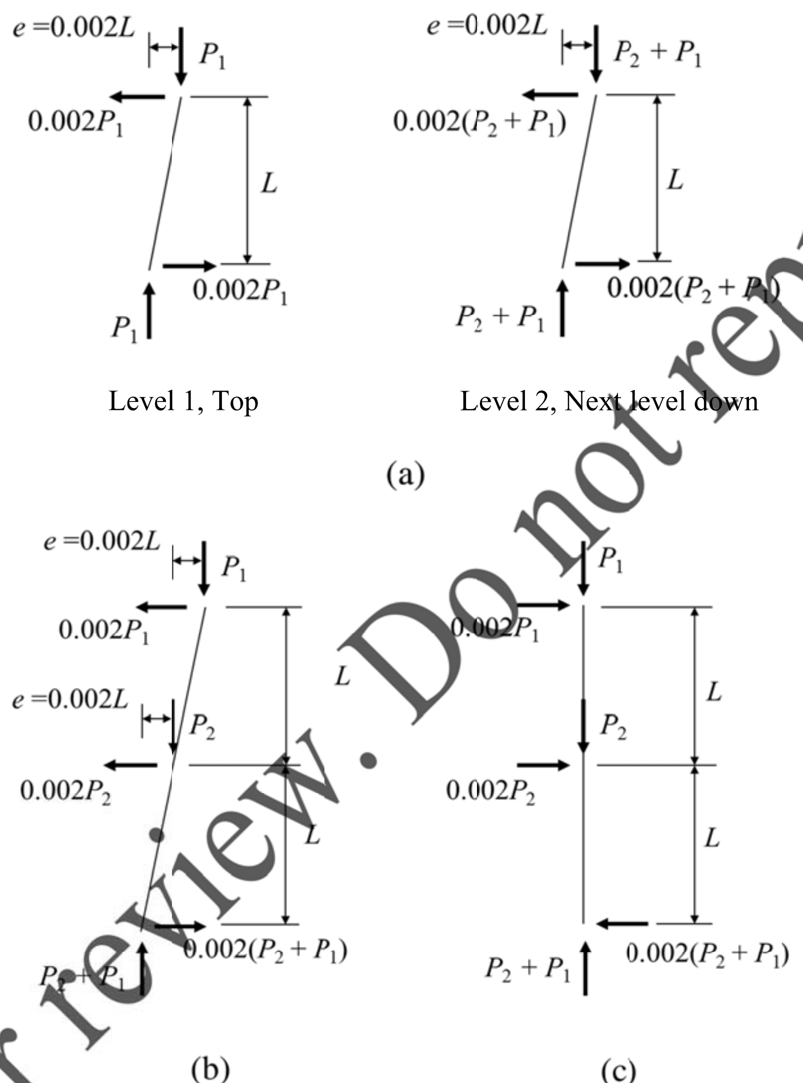


Figure 8.13 Notional Load Model for Geometric Imperfections.

Chapter E already account for initial out-of-straightness. To consider out-of-plumbness, the structure may be modeled in the out-of-plumb position or notional loads, as given in *Specification* Section C2.2b, may be used. The AISC *Code of Standard Practice* permits columns to be built with an out-of-plumbness tolerance of height/500.

Figure 8.13a shows the upper two stories of a column that is out-of-plumb, $L/500 = 0.002L$. For level 1, with a load P_1 applied, a horizontal force of $0.002P_1$ is required for equilibrium as shown. The next story down, level 2, is also out-of-plumb by the same amount and the load from above is added to the load introduced at that level so that the column must carry P_1 $_+$ P_2. Since this load is also applied at the eccentricity of $0.002L$, equilibrium requires that the column be restrained by a force of $0002(P_1 + P_2)$ as shown for level 2. When these two columns

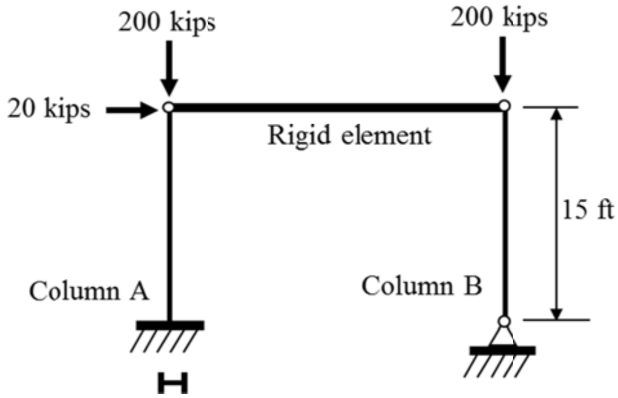


Figure 8.14 One-bay Unbraced Frame for Comparison of Analysis Methods

are put together, it can be seen in Figure 8.13b that the horizontal force at the intersection becomes 0.002 P_2. Thus, Figure 8.13b shows for this two story out-of-plumb column, the horizontal forces that are required to keep it in equilibrium. The same effect could be accomplished if the column was modeled plumb and the restraining forces from Figure 8.13b were applied as loads as shown in Figure 8.13c.

Thus, the out-of-plumbness can be modeled with a lateral load equal to 0.2 percent of the gravity load introduced at each level of the structure. This lateral load is called a notional load and is taken as $N_i = 0.002\alpha Y_i$ where Y_i is the total gravity load on story i and α is 1.0 for LRFD and 1.6 for ASD, as discussed earlier. The *Specification* includes α as described here, but when the amplified first-order analysis method is used to obtain second-order effects, this is not necessary, since α is included in the B_1 and B_2 calculations. This notional load is the same as the "minimum lateral load" used in Example 8.2.

8.7.5 Comparison of Methods

The three methods of frame stability analysis just described will be compared using a simple determinate structure. Figure 8.14 shows a one-bay unbraced frame with an LRFD gravity plus lateral load combination. Column A is a flag pole column and provides all of the lateral load resistance while column B is a gravity only column. Gravity only columns will be discussed in more depth in Section 8.10. Column A is a W14×90 bending about its strong axis, column B can be any size sufficient to support the gravity load since it does not contribute to the lateral load resistance, and the beam is assumed to be a rigid element.

***Effective Length Method*:** First the effective length method of Appendix Section 7.2 will be used along with the $B_1 - B_2$ amplification for second-order effects. If the structure is prevented from swaying, the *nt* analysis produces, for column A, P_{nt} = 200 kips and M_{nt} = 0 ft-kips. The lateral translation analysis produces, for column A, P_{lt} = 0 kips and M_{nt} = 300 ft-kips. Since there is no moment in the *nt* analysis, there are no P-δ effects (member effects) and no need to determine B_1. To assess the P-Δ effects (sway effects), B_2 will be determined. The 20 kip lateral load produces a drift calculated as for a cantilevered beam,

$$\Delta = \frac{HL^3}{3EI} = \frac{20\left(15\left(12\right)\right)^3}{3\left(29{,}000\right)\left(999\right)} = 1.34 \text{ in.}$$

The total gravity load on the structure is 400 kips. Half of this load is on the lateral load resisting column A and half is on the gravity only column. Thus,

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{tory}}\right) = 1 - 0.15\left(\frac{200}{400}\right) = 0.925$$

and

$$P_{e\ story} = R_M\,\frac{HL}{\Delta_H} = 0.925\frac{20\big(15(12)\big)}{1.34} = 2490 \text{ kips}$$

Thus,

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\ story}}} = \frac{1}{1 - \dfrac{1.0(400)}{2490}} = 1.19$$

Next, consider the limitations on use of the effective length method. This structure supports gravity loads through vertical columns so it meets the first limitation in Section 7.2.1. The second limitation requires that the second-order amplification, B_2, be less than or equal to 1.5. Since $B_2 = 1.19 \le 1.5$ the effective length method may be used for this frame.

The required strength, including second-order effects is found through Equations A-8-1 and A-8-2.

$$M_r = B_1 M_{nt} + B_2 M_{lt} = 0 + 1.19(300) = 357 \text{ ft-kips}$$

and

$$P_r = P_{nt} + B_2 P_{lt} = 200 + 1.19(0) = 200 \text{ kips}$$

The next step in the effective length method is determination of the effective length factor. The effective length factor for the flag pole column alone is K_x = 2.0. However, as discussed in Chapter 5, the inclusion of the gravity only column with load will increase the effective length of column A. Using the approach presented in Chapter 5, with the load on the moment frame column, P_{mf} = 200 kips and the load on the gravity only column, $P_{\text{grav only}}$ = 200 kips, the effective length factor is

- $K_x^* = K_x\sqrt{1 + P_{grav\ only}/P_{mf}} = 2.0\sqrt{1 + 200/200} = 2.83$

Assuming that the frame is braced out of the plane of the frame, K_y = 1.0.

The available strength of the W14×90 column can be determined from Table 6-2 for an unbraced length of the compression flange L_b = 15 ft < L_p = 15.1 ft, $\phi M_n = 574$ ft-kips. The controlling effective length is for *x*-axis buckling, thus $\left(L_{cx}\right)_{eff} = 2.83(15)/1.66 = 25.6$ ft and $\phi P_n = 720$ kips. With the required strength and available strength determined, the interaction equation can be checked.

First determine which interaction equation should be used. Since $P_r/\phi P_n = 200/720 = 0.278 > 0.2$ use Equation H1-1a, thus

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_r}{M_c}\right) = \frac{200}{720} + \frac{8}{9}\left(\frac{357}{574}\right) = 0.278 + 0.553 = 0.831 < 1.0$$

So the W14×90 is shown to be adequate by the effective length method.

First-Order Analysis Method: The first-order analysis method of Appendix Section 7.3 may be used for those structures that meet the limitations of Section 7.3.1. These limitations are the same as for the effective length method with the addition of the requirement that the columns behave elastically such that

$$\alpha P_r \le 0.5 P_{ns} \qquad \text{(AISC A-7-1)}$$

Since the W14×90 column does not have slender elements for compression, $P_{ns} = P_y = F_y A_g = 50(26.5) = 1330$ kips and for the frame of Figure 8.14 $\alpha P_r = 200$ kips. Thus $200 \le 0.5(1330) = 666$ kips and the first-order analysis method may be used.

The required strength for the first-order analysis method is determined from a first-order analysis that includes a notional load defined by Equation A-7-2 added to the lateral load in all load combinations. This notional load accounts for both the initial out-of-plumbness of the structure and second-order effects. Thus,

$$N_i = 2.1\alpha(\Delta/L)Y_i \ge 0.0042Y_i \qquad \text{(AISC A-7-2)}$$

For our structure, Δ = 1.34 in. as before and Y_i = 400 kips so the notional load for this load combination is

$$N_i = 2.1(1.0)\left(1.34/\left(15(12)\right)\right)(400) = 6.25 \text{ kips } \ge 0.0042(400) = 1.68 \text{ kips}$$

Thus, the lateral load in the analysis will be increased from 20 kips to 26.3 kips. The results of the first-order analysis for the determinate structure are $P_u = 200$ kips and $M_u = 26.3(15) = 395$ ft-kips. Although this is called the first-order analysis method, it does require that the moment be amplified by B_1 found using

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \ge 1.0 \qquad \text{(AISC A-8-3)}$$

This amplification addresses the member effect and is influenced by the buckling strength of the column as a pin ended column in a no sway condition, P_{e1}, and the equivalent uniform moment factor, C_m. Thus,

$$C_m = 0.6 - 0.4(M_1/M_2) = 0.6 - 0.4(0/395) = 0.6$$

and with *EI** = *EI*,

$$P_{e1} = \frac{\pi^2 EI^*}{L_{c1}^2} = \frac{\pi^2 (29,000)(999)}{\left(15(12)\right)^2} = 8830 \text{ kips}$$

which gives

$$B_1 = \frac{0.6}{1 - 200/8830} = 0.614 < 1.0$$

Therefore there is no amplification needed so $P_r = P_u = 200$ kips and $M_r = M_u = 395$ ft-kips.

The available moment strength of the W14×90 column determined previously from Table 6-2 is unchanged, thus $\phi M_n = 574$ ft-kips. The controlling effective length is for *y*-axis buckling, thus $L_{cy} = 15.0$ ft and $\phi P_n = 1000$ kips. With the required strength and available strength determined, the interaction equation can be checked.

First determine which interaction equation should be used. Since $P_r/\phi P_n = 200/1000 = 0.20 \le 0.2$ use Equation H1-1a, thus

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_r}{M_c}\right) = \frac{200}{1000} + \frac{8}{9}\left(\frac{395}{574}\right) = 0.200 + 0.612 = 0.812 < 1.0$$

So the W14×90 is shown to be adequate by the first order analysis method.

Direct Analysis Method: The third method to be considered is the direct analysis method of Chapter C. There are no limitations on the use of the direct analysis method like there are on the effective length or first-order analysis methods and second-order effects and initial out-of-plumbness must be accounted for as they were for the effective length method. The only new requirement is that the stiffness of all members that contribute to the lateral load resistance be reduced in the analysis to $EI^* = 0.8\tau_b EI$ and $EA^* = 0.8EA$. It is this stiffness reduction that permits the use of an effective length factor equal to one when using the direct analysis method. From the discussion of the effective length method it was seen that B_2 was less than 1.5 when using the unreduced stiffness thus the notional load to account for out-of-plumbness does not need to be added to the lateral load. Thus, from a first order analysis of the determinate structure, $P_u = 200$ kips and $M_u = 20.0(15) = 300$ ft-kips. As for the effective length method, $\tau_b = 1.0$ so that the flexural stiffness of column A will be taken as $EI^* = 0.8EI$. Thus the 20 kip lateral load produces a drift calculated as for a cantilevered beam,

$$\Delta = \frac{HL^3}{3EI^*} = \frac{20(15(12))^3}{3(0.8)(29{,}000)(999)} = 1.68 \text{ in.}$$

The total gravity load on the structure is 400 kips. Half of this load is on the lateral load resisting column A and half is on the gravity only column. Thus again,

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{tory}}\right) = 1 - 0.15\left(\frac{200}{400}\right) = 0.925$$

and

$$P_{e\ story} = R_M \frac{HL}{\Delta_H} = 0.925\frac{20(15(12))}{1.68} = 1980 \text{ kips}$$

Thus,

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\ story}}} = \frac{1}{1 - \dfrac{1.0(400)}{1980}} = 1.25$$

Note that the drift increased from what was calculated for the effective length method and therefore the second-order amplification increased.

The required strength, including second-order effects is found through Equations A-8-1 and A-8-2.

$$M_r = B_1 M_{nt} + B_2 M_{lt} = 0 + 1.25(300) = 375 \text{ ft-kips}$$

and

$$P_r = P_{nt} + B_2 P_{lt} = 200 + 1.25(0) = 200 \text{ kips}$$

The available moment strength of the W14×90 column determined previously from Table 6-2 is unchanged, thus $\phi M_n = 574$ ft-kips. The controlling effective length is for y-axis buckling, thus $L_{cy} = 15.0$ ft and $\phi P_n = 1000$ kips. With the required strength and available strength determined, the interaction equation can be checked.

First determine which interaction equation should be used. Since $P_r/\phi P_n = 200/1000 = 0.20 \le 0.2$ use Equation H1-1a, thus

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_r}{M_c}\right) = \frac{200}{1000} + \frac{8}{9}\left(\frac{375}{574}\right) = 0.200 + 0.581 = 0.781 < 1.0$$

So the W14×90 is adequate by the direct analysis method. Note that based on the results of the interaction equation, this approach is less conservative than the other two methods. Since the only new requirement of the direct analysis method is to use a reduced stiffness in calculating second-order effects and this permits the use of an effective length factor in the lateral load resisting direction of one, this is clearly the simplest and most direct method available.

Three methods of analysis are available and all three have their place in design. It is up to the user to determine when to use each approach most efficiently.

EXAMPLE 8.3a
Direct Analysis Method for Column Design by LRFD

Goal: Using the LRFD provisions and the results from a second-order direct analysis, determine if a W14×132, A992 member is adequate to carry the given loads and moments.

Given: The column has a length of 16 ft and is braced at the ends only. The results of the second-order direct analysis are P_u = 800 kips, M_{ux} = 300 ft kips, and M_{uy} = 76 ft kips.

SOLUTION

Step 1: Determine the required strength.

Since the given results are from a second-order analysis, there is no need to amplify forces and moments; thus

$$P_r = P_u = 800 \text{ kips}, \quad M_{rx} = M_{ux} = 300 \text{ ft kips}, \quad M_{ry} = M_{uy} = 76 \text{ ft kips}$$

Step 2: Determine the available compressive strength of the column.

Since the given results are from a direct analysis, K = 1.0; thus, from *Manual* Table 4-1 with L_c = 16.0 ft,

$$\phi P_n = 1440 \text{ kips}$$

Step 3: Determine the available strength for bending about the x-axis.

With an unbraced length L_b = 16 ft, from *Manual* Table 3-2,

$$\phi M_p = 878 \text{ ft kips}, \quad L_p = 13.3 \text{ ft}, \quad \phi BF = 7.74 \text{ kips}$$

and

$$\phi M_{nx} = \phi M_p - \phi BF\left(L_b - L_p\right) = 878 - 7.74\left(16.0 - 13.3\right) = 857 \text{ ft kips}$$

Step 4: Determine the available strength for bending about the y-axis.

From *Manual* Table 3-4,

$$\phi M_{ny} = 424 \text{ ft kips}$$

Step 5: Check the W14×132 for combined axial load and bending. To determine which equation to use, check

$$\frac{P_u}{\phi P_n} = \frac{800}{1440} = 0.556 \geq 0.2$$

Therefore, use Equation H1-1a.

$$\frac{P_r}{\phi P_n} + \frac{M_{rx}}{\phi M_{nx}} + \frac{M_{ry}}{\phi M_{ny}} \leq 1.0$$

$$0.556 + \frac{300}{857} + \frac{76}{424} = 1.09 > 1.0$$

Thus,

the W14×132 will not carry the given load.

EXAMPLE 8.3a
Direct Analysis Method for Column Design by ASD

Goal: Using the ASD provisions and the results from a second-order direct analysis, determine if a W14×132, A992 member is adequate to carry the given loads and moments.

Given: The column has a length of 16 ft and is braced at the ends only. The results of the second-order direct analysis are P_a = 530 kips, M_{ax} = 200 ft kips, and M_{ay} = 52 ft kips.

SOLUTION

Step 1: Determine the required strength.

Since the given results are from a second-order analysis, there is no need to amplify forces and moments. Thus,

$$P_r = P_a = 530 \text{ kips}, \quad M_{rx} = M_{ax} = 200 \text{ ft kips}, \quad M_{ry} = M_{ay} = 52 \text{ ft kips}$$

Step 2: Determine the available compressive strength of the column.

Since the given results are from a direct analysis, K = 1.0. Thus, from *Manual* Table 4-1 with L_c = 16.0 ft,

$$\frac{P_n}{\Omega} = 960 \text{ kips}$$

Step 3: Determine the available strength for bending about the x-axis.

With an unbraced length L_b = 16 ft, from *Manual* Table 3-2,

$$\frac{M_p}{\Omega} = 584 \text{ ft kips}, \quad L_p = 13.3 \text{ ft}, \quad \frac{BF}{\Omega} = 5.15 \text{ kips}$$

And

$$\frac{M_{nx}}{\Omega} = \frac{M_p}{\Omega} - \frac{BF}{\Omega}\left(L_b - L_p\right) = 584 - 5.15\left(16.0 - 13.3\right) = 570 \text{ ft kips}$$

Step 4: Determine the available strength for bending about the y-axis.
From *Manual* Table 3-4

$$\frac{M_{ny}}{\Omega} = 282 \text{ ft kips}$$

Step 5: Check the W14×132 for combined axial load and bending. To determine which equation to use, check

$$\frac{P_u}{\phi P_n} = \frac{530}{960} = 0.552 \;\geq\; 0.2$$

Therefore, use Equation H1-1a.

$$\frac{P_r}{P_n/\Omega} + \frac{M_{rx}}{M_{nx}/\Omega} + \frac{M_{ry}}{M_{ny}/\Omega} \leq 1.0$$

$$0.552 + \frac{200}{570} + \frac{52}{282} = 1.09 > 1.0$$

Thus,

the W14×132 will not carry the given load.

8.8 INITIAL BEAM-COLUMN SELECTION

Beam-column design is a trial-and-error process that requires that the beam-column section be known before any of the critical parameters can be determined for use in the appropriate interaction equations. There are numerous approaches to determining a preliminary beam-column size. Each incorporates its own level of sophistication and results in its own level of accuracy. Regardless of the approach used to select the trial section, one factor remains—the trial section must ultimately satisfy the appropriate interaction equation.

To establish a simple, yet useful, approach to selecting a trial section, Equation H1-1a is modified by multiplying each term by P_c which yields

$$P_r + \frac{8}{9}\frac{M_{rx}P_c}{M_{cx}} + \frac{8}{9}\frac{M_{ry}P_c}{M_{cy}} \le P_c \tag{8.9}$$

Then multiplying the third term by M_{cx}/M_{cx}, letting

$$m = \frac{8P_c}{9M_{cx}}$$

and

$$U = \frac{M_{cx}}{M_{cy}}$$

and substituting into Equation 8.9 yields

$$P_r + mM_{rx} + mUM_{ry} \le P_c \tag{8.10}$$

Because Equation 8.10 calls for the comparison of the left side of the equation to the column strength, P_c, Equation 8.10 can be thought of as an effective axial load; thus

$$P_{eff} = P_r + mM_{rx} + mUM_{ry} \le P_c \tag{8.11}$$

The accuracy used in the evaluation of m and U dictates the accuracy with which Equation 8.11 represents the strength of the column being selected. Because at this point in a design the actual column section is not known, exact values of m and U cannot be determined.

Past editions of the AISC *Manual* have presented numerous approaches to the evaluation of these multipliers. A simpler approach however, is more useful for preliminary design. If the influence of the length—that is, all buckling influence on P_c and M_{cx}—is neglected, the ratio, P_c/M_{cx}, becomes A/Z_x, and $m = 8A/9Z_x$. Evaluation of this m for all W6 to W14 shapes with the inclusion of a units correction factor of 12 results in the average m values given in Table 8.2. If the relationship between the area, A, and the plastic section modulus, Z_x, is established using an approximate internal moment arm of 0.89d, where d is the nominal depth of the member in inches, then $m = 24/d$. This value is also presented in Table 8.2. This new m is close enough to the average m that it may be readily used for preliminary design.

When bending occurs about the y-axis, U must be evaluated. A review of the same W6 to W14 shapes results in the average U values given in Table 8.2. However, an in-depth review of the U values for these sections shows that only the smallest sections for each nominal depth have U values appreciably larger than 3. Thus, a reasonable value of $U = 3.0$ can be used for the first trial.

More accurate evaluations of these multipliers, including length effects, have been conducted, but there does not appear to be a need for this additional accuracy in a preliminary design. Once the initial section is selected, however, the actual *Specification* provisions must be satisfied.

Table 8.2 Simplified Bending Factors

Shape	m_{avg}	$m = 24/d$	U_{avg}
W6	4.41	4.00	3.01
W8	3.25	3.00	3.11
W10	2.62	2.40	3.62
W12	2.08	2.00	3.47
W14	1.71	1.71	2.81

EXAMPLE 8.4a
Initial Trial Section Selection by LRFD

SOLUTION

Goal: Determine the initial trial section for a column.

Given: The loadings of Figure 8.11c are to be used. Assume the column is a W14 and use A992 steel. Also, use the simplified values of Table 8.2, $m = 24/d$.

Step 1: Obtain the required strength from Figure 8.11c. Use the first-order analysis results.

$$P_u = 453 \text{ kips}$$
$$M_u = 251 \text{ ft-kips}$$

Step 2: Determine the effective load by combining the axial force and the bending moment.

For a W14, $m = 1.71$, so

$$P_{eff} = 453 + 1.71(251) = 882 \text{ kips}$$

Step 3: Select a trial column size to carry the required force, P_{eff}.

Using an effective length $L_c = 12.5$ ft, from *Manual* Table 4-1, the lightest W14 to carry this load is

$$\text{W14} \times 90 \text{ with } \phi P_n = 1060 \text{ kips}$$

Example 8.2a showed that this column adequately carries the imposed load. Because the approach used here is expected to be conservative, it would be appropriate to consider the next smaller selection, a W14×82, and check it against the appropriate interaction equations.

EXAMPLE 8.4b
Initial Trial Section Selection by ASD

SOLUTION

Goal: Determine the initial trial section for a column.

Given: The loadings of Figure 8.11e are to be used. Assume the column is a W14, and use A992 steel. Also, use the simplified values of Table 8.2, $m = 24/d$.

Step 1: Obtain the required strength from Figure 8.11e. Use the first-order analysis results.

$$P_a = 342 \text{ kips}$$
$$M_a = 174 \text{ ft-kips}$$

Step 2: Determine the effective load by combining the axial force and the bending moment.

For a W14, $m = 1.71$; thus

$$P_{eff} = 342 + 1.71(174) = 640 \text{ kips}$$

Step 3: Select a trial column size to carry the required force, P_{eff}.

Using an effective length $L_c = 12.5$ ft, from *Manual* Table 4-1, the lightest W14 to carry this load is

$$\text{W14} \times 90 \text{ with } P_n/\Omega = 703 \text{ kips}$$

Example 8.2b showed that this column adequately carries the imposed load. Because the approach used here is expected to be conservative, it would be appropriate to consider the next smaller selection, a W14×82, and check it against the appropriate interaction equations.

Every column section selected must be checked through the appropriate interaction equations for the second-order forces and moments. Thus, the process for the initial selection should be quick and reasonable. The experienced designer will rapidly learn to rely on that experience rather than these simplified approaches.

8.9 BEAM-COLUMN DESIGN USING MANUAL PART 6

Manual Part 6, Design of Members Subject to Combined Loading contains Table 6-2 which includes the axial and flexural strength for all W-shapes. Although these tables are presented here as they relate to combined loading, they can also be used for compression only, bending only, tension only and shear. There is no information found in Table 6-2 that is not already included in other Parts of the *Manual* already discussed. The advantage for combined loading is that all of the available strength values needed are found in one location.

Figure 8.15 is a portion of *Manual* Table 6-2. It shows that the compressive strength for a given section is a function of the effective length about the weak axis of the member. The effective length is tabulated in the center of the table with the compressive strengths shown on the left portion of the table. This portion of the table is used in exactly the same way as the column tables in Part 4 of the *Manual*. The strong axis bending strength is a function of the unbraced length of the compression flange of the beam. Previously, this information was available only through the beam curves in Part 3 of the *Manual*. In Table 6-2 it is tabulated on the right portion of the table with the same column of lengths now defined as the unbraced length of the compression flange. Weak axis bending is not a function of length, so only one value is given for each shape. Although not used for beam-columns, when tension is combined with bending, the table also provides tension yield and rupture strength.

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

F_y = 50 ksi

Shape	W14×											
	109				99[f]				90[f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	1.04	0.694	1.86	1.23	1.15	0.764	2.07	1.38	1.26	0.839	2.33	1.55
11	1.14	0.761	1.86	1.23	1.26	0.838	2.07	1.38	1.38	0.920	2.33	1.55
12	1.16	0.774	1.86	1.23	1.28	0.853	2.07	1.38	1.41	0.937	2.33	1.55
13	1.19	0.789	1.86	1.23	1.31	0.869	2.07	1.38	1.44	0.955	2.33	1.55
14	1.21	0.805	1.87	1.25	1.33	0.887	2.08	1.38	1.47	0.975	2.33	1.55
15	1.24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.50	0.997	2.33	1.55
16	1.27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.53	1.02	2.35	1.57
17	1.30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.57	1.05	2.38	1.59
18	1.33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.62	1.08	2.42	1.61
19	1.37	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.66	1.11	2.45	1.63
20	1.41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.71	1.14	2.48	1.65
22	1.51	1.00	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.83	1.22	2.55	1.70
24	1.61	1.07	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.96	1.31	2.62	1.74
26	1.74	1.16	2.14	1.43	1.92	1.28	2.41	1.60	2.12	1.41	2.70	1.80
28	1.89	1.26	2.20	1.46	2.09	1.39	2.48	1.65	2.30	1.53	2.78	1.85
30	2.06	1.37	2.25	1.50	2.28	1.52	2.55	1.69	2.52	1.68	2.87	1.91
32	2.27	1.51	2.31	1.54	2.51	1.67	2.62	1.74	2.77	1.84	2.96	1.97
34	2.50	1.67	2.37	1.58	2.78	1.85	2.70	1.80	3.07	2.04	3.06	2.03
36	2.79	1.86	2.44	1.62	3.10	2.06	2.78	1.85	3.42	2.28	3.16	2.10
38	3.11	2.07	2.51	1.67	3.45	2.30	2.87	1.91	3.81	2.54	3.27	2.18
40	3.44	2.29	2.58	1.72	3.83	2.55	2.96	1.97	4.23	2.81	3.39	2.26
42	3.80	2.53	2.66	1.77	4.22	2.81	3.06	2.04	4.66	3.10	3.52	2.34
44	4.17	2.77	2.74	1.82	4.63	3.08	3.17	2.11	5.11	3.40	3.72	2.48
46	4.55	3.03	2.82	1.88	5.06	3.37	3.31	2.20	5.59	3.72	3.94	2.62
48	4.96	3.30	2.92	1.94	5.51	3.67	3.48	2.32	6.08	4.05	4.15	2.76
50	5.38	3.58	3.05	2.03	5.98	3.98	3.66	2.43	6.60	4.39	4.36	2.90

Other Constants and Properties

	109 ASD	109 LRFD	99[f] ASD	99[f] LRFD	90[f] ASD	90[f] LRFD
$b_y \times 10^3$, (kip-ft)$^{-1}$	3.84	2.56	4.29	2.85	4.90	3.26
$t_y \times 10^3$, (kips)$^{-1}$	1.04	0.694	1.15	0.764	1.26	0.839
$t_r \times 10^3$, (kips)$^{-1}$	1.28	0.855	1.41	0.940	1.55	1.03
r_x/r_y	1.67		1.66		1.66	
r_y, in.	3.73		3.71		3.70	

[f]Shape does not meet compact limit for flexure with F_y = 50 ksi.

Must Be Replaced

Figure 8.15 Combined Axial and Bending Strength for W-Shapes.

EXAMPLE 8.5a
Combined Strength Check Using Manual Part 6 and LRFD

Goal: Check the strength of a beam-column using *Manual* Part 6 and compare to the results of Example 8.2a.

Given: It has already been shown that the W14×90 column of Example 8.2a is adequate by LRFD. Use the required strength values given in Example 8.2a and recheck this shape using the values found in Figure 8.15 or *Manual* Table 6-2.

SOLUTION

Step 1: Determine the values needed from *Manual* Table 6-2 (Figure 8.15). The column is required to carry a compressive force with an effective length about the *y*-axis of 12.5 ft and an *x*-axis moment with an unbraced length of 12.5 ft. Thus, from Figure 8.15,

$$\phi P_n = 1060 \text{ kips}$$
$$\phi M_n = 574 \text{ ft-kips}$$

Step 2: Determine which interaction equation to use.

$$\frac{P_r}{\phi P_n} = \frac{459}{1060} = 0.433 > 0.2$$

Therefore, use Equation H1-1a.

$$\frac{200}{1060} + \frac{8}{9}\left(\frac{260}{574}\right) = 0.433 + 0.403 = 0.836 < 1.0$$

Therefore, as previously determined in Example 8.2a, the shape is adequate for this column and this load combination. The results from *Manual* Tables 6-2 are exactly the same as those determined from Table 4-1 for compression and Table 3-2 for bending.

EXAMPLE 8.5b
Combined Strength Check Using Part 6 and ASD

Goal: Check the strength of a beam-column using *Manual* Part 6 and compare to the results of Example 8.2b.

Given: It has already been shown that the W14×90 column of Example 8.2b is adequate by ASD. Use the required strength values given in Example 8.2b and recheck this shape using the values found in Figure 8.15 or *Manual* Table 6-2.

SOLUTION

Step 1: Determine the values needed from *Manual* Table 6-2 (Figure 8.15). The column is required to carry a compressive force with an effective length about the *y*-axis of 12.5 ft and an *x*-axis moment with an unbraced length of 12.5 ft. Thus, from Figure 8.15,

$$P_n/\Omega = 703 \text{ kips}$$
$$M_n/\Omega = 382 \text{ ft-kips}$$

Step 2: Determine which interaction equation to use.

$$\frac{P_r}{P_n/\Omega} = \frac{347}{703} = 0.494 > 0.2$$

Thus, use Equation H1-1a.

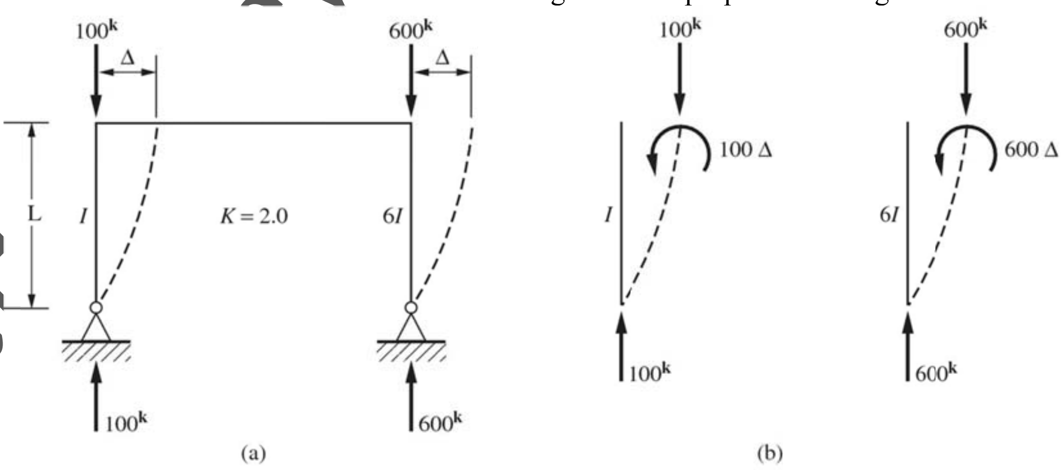
$$\frac{347}{703} + \frac{8}{9}\left(\frac{182}{382}\right) = 0.494 + 0.424 = 0.918 < 1.0$$

Thus, as previously determined in Example 8.2b, the shape is adequate for this column and this load combination. The results from *Manual* Tables 6-2 are exactly the same as those determined from Table 4-1 for compression and Table 3-2 for bending.

8.10 COMBINED SIMPLE AND MOMENT FRAMES

The practical design of steel structures often results in frames that combine segments of rigidly connected elements with segments that are pin connected as was the case in the example frame used in Section 8.7.5. If these structures rely on the moment frame to resist lateral load and to provide the overall stability of the structure, the rigidly connected columns are called upon to carry more load than appears to be directly applied to them. In these combined simple and moment frames, the simple columns "lean" on the moment frames in order to maintain their stability and thus are often called *leaning columns*. They are also called *gravity columns* which is a more appropriate term since they participate only in carrying gravity loads. These columns can be designed with an effective length factor, K = 1.0, regardless of the approach to analysis that has been taken. Because these gravity only columns have no lateral stability of their own, the moment frame columns must be designed to provide the lateral stability for the full frame. Although this combination of framing types makes design of a structure more complicated, it can also be economically advantageous, because the combination can reduce the number of moment connections for the full structure and thereby reduce overall cost.

Numerous design approaches have been proposed for consideration of the gravity only column and associated moment frame design.[3,4] Yura proposes to design columns that provide



[3] Yura, J. A., "The Effective Length of Columns in Unbraced Frames," *Engineering Journal*, AISC, Vol. 8, No. 2, 1971, pp. 37–42.
[4] LeMessurier, W. J., "A Practical Method of Second Order Analysis," *Engineering Journal*, AISC, Vol. 14, No. 2, 1977, pp. 49–67.

Figure 8.16 Pinned Base Unbraced Frame.

lateral stability for the total load on the frame at the story in question, whereas LeMessurier presents a modified effective length factor that accounts for the full frame stability. Perhaps the most straightforward approach is that presented by Yura, as follows.

The two-column frame shown in Figure 8.16a is a moment frame with pinned base columns and a rigidly connected beam. The column sizes are selected so that, under the loads shown, they buckle simultaneously in a sidesway mode, because their load is directly proportional to the stiffness of the members. Equilibrium in the displaced position is shown in Figure 8.16b. The lateral displacement of the frame, Δ, results in a moment at the top of each column equal to the load applied on the column times the displacement, as shown. These are the second-order effects discussed in Section 8.6. The total load on the frame is 700 kips, and the total $P\Delta$ moment is 700Δ, divided between the two columns based on the load that each carries.

If the load on the right-hand column is reduced to 500 kips, the column does not buckle sideways, because the moment at the top is now less than 600Δ. To reach the buckling condition, a horizontal force must be applied at the top of the column, as shown in Figure 8.17b. This force

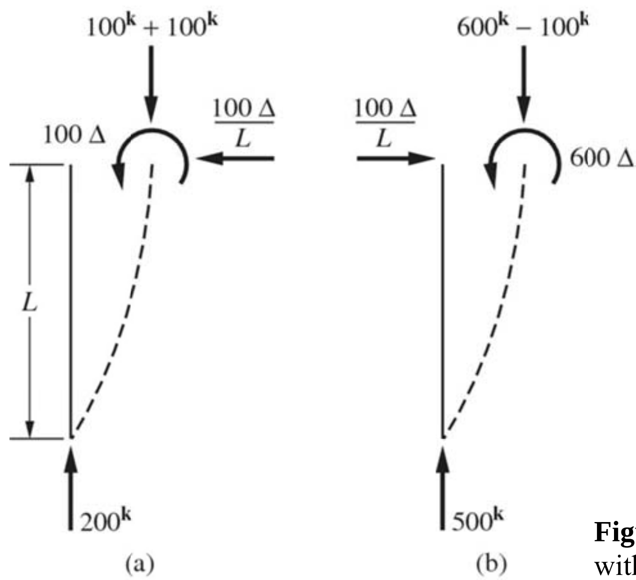


Figure 8.17 Columns from Unbraced Frame with Revised Loading.

can result only from action on the left column that is transmitted through the beam. Equilibrium of the left column, shown in Figure 8.17a, requires that an additional column load of 100 kips be applied to that column in order for the load on the frame to be in equilibrium in this displaced position. The total frame capacity is still 700 kips and the total second-order moment is still 700Δ.

The maximum load that an individual column can resist is limited to that permitted for the column in a braced frame for which $K = 1.0$. In this example case, the left column could resist 400 kips and the right column 2400 kips. This is an increase of four times the load originally on the column, because the effective length factor for each column would be reduced from 2.0 to 1.0. The additional capacity of the left column is only with respect to the bending axis. The column would have the same capacity about the other axis as it did prior to reducing the load on the right column.

The ability of one column to carry increased load when another column in the frame is called upon to carry less than its critical load for lateral buckling is an important characteristic. This allows a pin-ended column to lean on a moment frame column, provided that the total gravity load on the frame can be carried by the rigid frame.

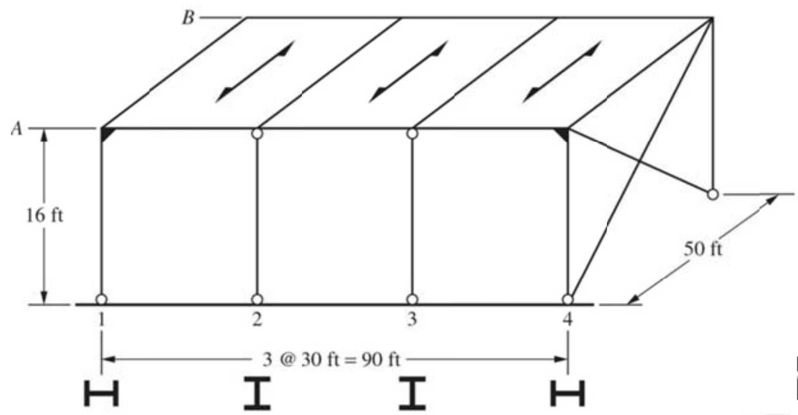


Figure 8.18 Frame Used in Example 8.6.

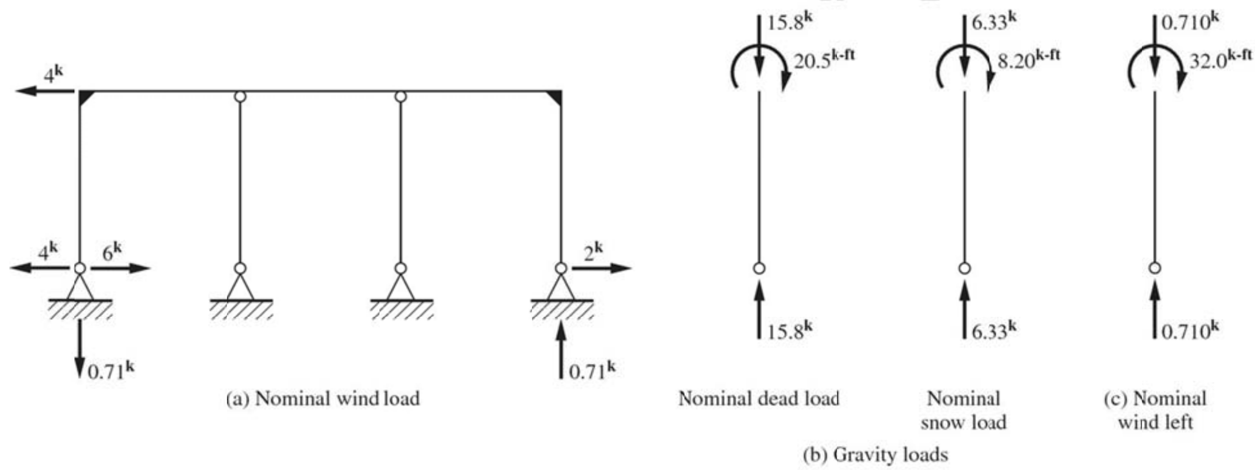


Figure 8.19 Nominal Wind Load, Snow Load, and Dead Load (Example 8.6).

EXAMPLE 8.6a ***Moment Frame Strength and Stability by LRFD***

Goal: Determine whether the structure shown in Figure 8.18 has sufficient strength and stability to carry the imposed loads.

Given: The frame shown in Figures 8.18 and 8.19 is similar to that in Example 8.1, except that the in-plane stability and lateral load resistance is provided by the moment frame action at the four corners. The exterior columns are W8×40, and the roof girder is assumed to be rigid. Out-of-plane stability and lateral load resistance is provided by X-bracing along column lines 1 and 4.

The loading is the same as that for Example 8.1: Dead Load = 50 psf, Snow Load = 20 psf, Roof Live Load = 10 psf, and Wind Load = 20 psf horizontal. Use A992 steel.

SOLUTION

Step 1: The analysis of the frame for gravity loads as given for Example 8.1 will be used. Because different load combinations may be critical, however, the analysis results for nominal Snow and nominal Dead Load are given in Figure 8.19b. The analysis results for nominal Wind Load acting to the left are given in Figure 8.19c.

Step 2: Determine the first-order forces and moments for the column on lines A-1.

For ASCE 7 load combination 3:

P_u = 1.2(15.8) + 1.6(6.33) + 0.5(0.710) = 29.1 + 0.355 = 29.5 kips

M_u = 1.2(20.5) + 1.6(8.20) + 0.5(32.0) = 37.7 + 16.0 = 53.7 ft-kips

For ASCE 7 load combination 4:

P_u = 1.2(15.8) + 0.5(6.33) + 1.0(0.710) = 22.1 + 0.710 = 22.8 kips

M_u = 1.2(20.5) + 0.5(8.20) + 1.0(32.0) = 28.7 + 32.0 = 60.7 ft-kips

Step 3: Determine the total story gravity load acting on one frame.

Dead = 0.05 ksf (90 ft)(50 ft)/2 frames = 113 kips

Snow = 0.02 ksf (90 ft)(50 ft)/2 frames = 45.0 kips

Step 4: Determine the second-order forces and moments for load combination 3. Gravity loads will be assumed to yield the no-translation effects, and wind load to yield the lateral translation effects.

From Step 2,

P_{nt} = 29.1 kips, P_{lt} = 0.355 kips, M_{nt} = 37.7 ft-kips, M_{lt} = 16.0 ft-kips

For the W8×40,

A = 11.7 in.^{2}, I_x = 146 in.^{4}, r_x = 3.53 in., r_x/r_y = 1.73

In the plane of the frame,

$$C_m = 0.6 = 0.4\left(M_1/M_2\right) = 0.6 - 0.4\left(\frac{0}{37.7}\right) = 0.6$$

$$P_{e1} = \frac{\pi^2 EI_x}{L_{c1}^2} = \frac{\pi^2(29{,}000)(146)}{(16.0(12))^2} = 1130 \text{ kips}$$

and with $P_r = P_{nt} + P_{lt}$ = 29.1 + 0.355 = 29.5

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{29.5}{1130}} = 0.616 < 1.0$$

Therefore, use B_1 = 1.0.

To determine the sway amplification, the total gravity load on the frame for this load combination from Step 3 is

P_{story} = 1.2(113) + 1.6(45.0) = 208 kips

A serviceability drift index of 0.003 is maintained under the actual wind loads. Therefore, H = 4.0 kips, and Δ/L = 0.003 is used to determine the sway amplification factor. If this limit is not met at the completion of the design, the second-order effects must be recalculated.

The sway amplification is given by

$$B_2 = \frac{1}{1-\left(\frac{\alpha P_{story}}{P_{e\ story}}\right)} > 1.0 \qquad \text{(AISC A-8-6)}$$

and

$$P_{e\ story} = R_M \frac{HL}{\Delta_H} \qquad \text{(AISC A-8-7)}$$

Since one third of the load is on the moment frame corner columns, Equation A-8-8 gives

$$R_M = 1 - 0.15\left(\frac{1}{3}\right) = 0.95$$

Thus, with $\alpha = 1.0$ for LRFD, Equation A-8-6 becomes

$$B_2 = \frac{1}{1-\left(\frac{\alpha P_{story}}{R_M H}\left(\frac{\Delta}{L}\right)\right)} = \frac{1}{1-\frac{1.0(208)}{0.95(4.0)}(0.003)} = 1.20$$

Thus, the second-order force and moment are

$M_r = 1.0(37.7) + 1.20(16.0) = 56.9$ ft-kips

$P_r = 29.1 + 1.20(0.355) = 29.5$ kips

Step 5: Determine whether the column satisfies the interaction equation.

Because the roof beam is assumed to be rigid in this example, use the recommended design value of $K = 2.0$ from Figure 5.17 case *f* in the plane of the frame, $L_{cx} = 2.0(16.0) = 32.0$ ft. Out of the plane of the frame, this is a braced frame where $K = 1.0$; thus, $L_{cy} = 16.0$ ft.

Determine the critical buckling axis.

$$\left(L_{cx}\right)_{eff} = \frac{L_{cx}}{r_x / r_y} = \frac{32.0}{1.73} = 18.5 \text{ ft} > L_{cy} = 16.0 \text{ ft}$$

Thus, from *Manual* Table 6-2, using $L_c = (L_{cx})_{eff} = 18.5$ ft,

$\phi P_n = 222$ kips

and from *Manual* Table 6-2 with an unbraced length of $L_b = 16$ ft

$\phi M_{nx} = 128$ ft-kips

Determine the appropriate interaction equation to use.

$$\frac{P_r}{\phi P_n} = \frac{29.5}{222} = 0.133 < 0.2$$

Therefore, use Equation H1-1b.

$$\frac{P_u}{2\phi P_n} + \frac{M_u}{\phi M_n} \leq 1.0$$

$$\frac{29.5}{2(222)} + \frac{56.9}{128} = 0.511 < 1.0$$

Thus, the column is adequate for this load combination.

Step 6: Determine the first-order forces and moments for load combination 4 with the same assumption as to translation and no-translation effects. From Step 2.

P_{nt} = 22.1 kips, P_{lt} = 0.710 kips, M_{nt} = 28.7 ft-kips, M_{lt} = 32.0 ft-kips

Step 7: Determine the second-order forces and moments.

In the plane of the frame, as in Step 4,

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4\left(\frac{0}{28.7}\right) = 0.6$$

$$P_{e1} = \frac{\pi^2 EI_x}{L_{c1}^2} = \frac{\pi^2(29,\ 000)(146)}{(16(12))^2} = 1130 \text{ kips}$$

and

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{22.8}{1130}} = 0.612 < 1.0$$

Therefore, use B_1 = 1.0.

To determine the sway amplification, the total gravity load on the frame is

P_u = 1.2(113) + 0.5(45.0) = 158 kips

Again, a serviceability drift index of 0.003 is maintained under the actual wind loads. Therefore, H = 4.0 kips, and Δ/L = 0.003 is used to determine the sway amplification factor. As before, R_M = 0.95 so

$$B_2 = \frac{1}{1 - \left(\dfrac{\alpha P_{story}}{R_M H}\left(\dfrac{\Delta}{L}\right)\right)} = \frac{1}{1 - \dfrac{1.0(158)}{0.95(4.0)}(0.003)} = 1.14$$

Thus, the second-order force and moment are

M_r = 1.0(28.7) + 1.14(32.0) = 65.2 ft-kips

P_r = 22.1 + 1.14(0.710) = 22.9 kips

Step 8: Determine whether the column satisfies the interaction equation.

Using the same strength values found in Step 5, determine the appropriate interaction equation.

$$\frac{P_r}{\phi P_n} = \frac{22.9}{222} = 0.103 < 0.2$$

Therefore, use Equation H1-1b.

$$\frac{P_u}{2\phi P_n} + \frac{M_u}{\phi M_n} \le 1.0$$

$$\frac{22.9}{2(222)} + \frac{65.2}{128} = 0.561 < 1.0$$

Thus, the column is adequate for this load combination also.

Step 9: The W8×40 is shown to be adequate for gravity and wind loads in combination. Now, check to see that these columns have sufficient capacity to brace the interior pinned columns for load combination 3, which will put the greatest load on the gravity only columns.

Step 10: For stability in the plane of the frame, using the Yura approach discussed in Section 8.10, the total load on the structure is to be resisted by the four corner columns; thus

Dead Load = 0.05 ksf (50 ft)(90 ft)/4 columns = 56.3 kips/column

Snow Load = 0.02 ksf (50 ft)(90 ft)/4 columns = 22.5 kips/column

Thus, for load combination 3

$P_u = 1.2(56.3) + 1.6(22.5) + 0.5(0.710) = 104$ kips

$M_u = 1.2(20.5) + 1.6(8.20) + 0.5(32.0) = 53.7$ ft-kips

Step 11: Determine the second-order amplification.

As before, for the length $L_x = 16.0$ ft, $P_{e1} = 1130$ kips, and $C_m = 0.6$, the second-order amplification for member effect is

$$B_1 = \frac{C_m}{1 - \frac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \frac{104}{1130}} = 0.66 < 1.0$$

Therefore, use $B_1 = 1.0$ and $P_r = P_u = 104$ kips.

Sway amplification will be the same as determined in step 4, since the gravity load is the same; thus $B_2 = 1.20$. Therefore

$M_r = M_u = 1.0(37.7) + 1.20(16.0) = 56.9$ ft-kips.

Step 12: Check the corner columns for interaction under these forces and moments.

As determined in Step 5 for in-plane buckling,

$\phi P_{nx} = 222$ kips

$\phi M_{nx} = 128$ ft-kips

Checking for the appropriate interaction equation,

$$\frac{P_u}{\phi P_n} = \frac{104}{222} = 0.468 > 0.2$$

Thus, use Equation H1-1a.

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi M_{nx}}\right) \le 1.0$$

$$\frac{104}{222} + \frac{8}{9}\left(\frac{56.9}{128}\right) = 0.864 < 1.0$$

Thus, the W8×40 is adequate for both strength under combined load and stability for supporting the gravity only columns.

EXAMPLE 8.6b Moment Frame Strength and Stability by ASD

Goal: Determine whether the structure shown in Figure 8.18 has sufficient strength and stability to carry the imposed loads.

Given: The frame shown in Figures 8.18 and 8.19 is similar to that in Example 8.1 except that the in-plane stability and lateral load resistance is provided by the rigid frame action at the four corners. The exterior columns are W8×40, and the roof girder is assumed to be rigid. Out-of-plane stability and lateral load resistance is provided by X-bracing along column lines 1 and 4.

The loading is the same as that for Example 8.1: Dead Load = 50 psf, Snow Load = 20 psf, Roof Live Load = 10 psf, and Wind Load = 20 psf horizontal. Use A992 steel.

SOLUTION

Step 1: The analysis of the frame for gravity loads as given for Example 8.1 will be used. Because different load combinations may be critical, however, the analysis results for nominal Snow and nominal Dead Load are given in Figure 8.19b. The analysis results for nominal Wind Load acting to the left are given in Figure 8.19c.

Step 2: Determine the first-order forces and moments for the column on lines A-1.

For ASCE 7 load combination 3:

$$P_a = (15.8) + (6.33) = 22.1 \text{ kips}$$
$$M_a = (20.5) + (8.20) = 28.7 \text{ ft-kips}$$

For ASCE 7 load combination 6:

$$P_a = (15.8) + 0.75(6.33) + 0.75(0.6(0.710)) = 20.9 \text{ kips}$$
$$M_a = (20.5) + 0.75(8.20) + 0.75(0.6(32.0)) = 41.1 \text{ ft-kips}$$

Step 3: Determine the total story gravity load acting on one frame.

Dead = 0.05 ksf (90 ft)(50 ft)/2 frames = 113 kips
Snow = 0.02 ksf (90 ft)(50 ft)/2 frames = 45.0 kips

Step 4: Determine the second-order forces and moments for load combination 3. Gravity loads will be assumed to yield the no-translation effects. With no wind load, there will be no lateral translation effects; thus

From Step 2:

$$P_{nt} = 22.1 \text{ kips},\ P_{lt} = 0 \text{ kips},\ M_{nt} = 28.7 \text{ ft-kips},\ M_{lt} = 0 \text{ ft-kips}$$

For the W8×40:

$$A = 11.7 \text{ in.}^2,\ I_x = 146 \text{ in.}^4,\ r_x = 3.53 \text{ in.},\ r_x/r_y = 1.73$$

In the plane of the frame:

$$C_m = 0.6 - 0.4\left(M_1/M_2\right) = 0.6 - 0.4\left(\frac{0}{28.7}\right) = 0.6$$

$$P_{e1} = \frac{\pi^2 EI_x}{L_{c1}^2} = \frac{\pi^2(29{,}000)(146)}{(16.0(12))^2} = 1130 \text{ kips}$$

and

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{1.6(22.1)}{1130}} = 0.619 < 1.0$$

Therefore, use B_1 = 1.0.

To determine the sway amplification, even though there are no lateral translation forces or moments for this combination, the total gravity load on the frame for this load combination from Step 3 is
P_{story} = (113) + (45.0) = 158 kips

A serviceability drift index of 0.003 is maintained under the actual wind loads. Therefore, H = 4.0 kips, and Δ/L = 0.003 is used to determine the sway amplification factor. If this limit is not met at the completion of the design, the second-order effects must be recalculated.

The sway amplification is given by

$$B_2 = \frac{1}{1 - \left(\dfrac{\alpha P_{story}}{P_{e\ story}}\right)} > 1.0 \qquad \text{(AISC A-8-6)}$$

and

$$P_{e\ story} = R_M \frac{HL}{\Delta_H} \qquad \text{(AISC A-8-7)}$$

Since one third of the load is on the moment frame corner columns, Equation A-8-8 gives

$$R_M = 1 - 0.15\left(\frac{1}{3}\right) = 0.95$$

Thus, with α = 1.6 for ASD, Equation A-8-6 becomes

$$B_2 = \frac{1}{1 - \left(\dfrac{\alpha P_{story}}{R_M H}\left(\dfrac{\Delta}{\mathrm{L}}\right)\right)} = \frac{1}{1 - \dfrac{1.6(158)}{0.95(4.0)}(0.003)} = 1.25$$

Thus, the second-order force and moment are
M_r = 1.0(28.7) + 1.25(0) = 28.7 ft-kips
P_r = 22.1 + 1.25(0) = 22.1 kips

Step 5: Determine whether the column satisfies the interaction equation.

Because the roof beam is assumed to be rigid in this example, use the recommended design value of $K = 2.0$ from Figure 5.17 case *f* in the plane of the frame, $L_{cx} = 2(16.0) = 32.0$ ft. Out of the plane of the frame, this is a braced frame where $K = 1.0$; thus, $L_{cy} = 16.0$ ft.

Determining the critical buckling axis.

$$\left(L_{cx}\right)_{eff} = \frac{L_{cx}}{r_x/r_y} = \frac{32}{1.73} = 18.5 \text{ ft} > L_{cy} = 16.0 \text{ ft}$$

Thus, from *Manual* Table 6-2, using $L_c = (L_{cx})_{eff} = 18.5$ ft,

$$P_n/\Omega = 148 \text{ kips}$$

and from *Manual* Table 6-2, with an unbraced length of $L_b = 16$ ft,

$$M_{nx}/\Omega = 84.9 \text{ ft-kips}$$

Determine the appropriate interaction equation to use.

$$\frac{P_r}{P_n/\Omega} = \frac{22.1}{148} = 0.149 < 0.2$$

Therefore, use Equation H1-1b.

$$\frac{P_a}{2\left(P_n/\Omega\right)} + \frac{M_a}{\left(M_n/\Omega\right)} \leq 1.0$$

$$\frac{22.1}{2(148)} + \frac{28.7}{84.9} = 0.413 < 1.0$$

Thus, the column is adequate for this load combination.

Step 6: Determine the first-order forces and moments for load combination 6. Gravity loads will be assumed to yield the no-translation effects, and wind load will yield the lateral translation effects. From Step 2.

$$P_{nt} = 20.5 \text{ kips},\ P_{lt} = 0.320 \text{ kips},\ M_{nt} = 26.7 \text{ ft-kips},\ M_{lt} = 14.4 \text{ ft-kips}$$

Step 7: Determine the second-order forces and moments.

In the plane of the frame, as in Step 4,

$$C_m = 0.6 - 0.4\left(M_1/M_2\right)0.6 - 0.4\left(\frac{0}{26.7}\right) = 0.6$$

$$P_{e1} = \frac{\pi^2 EI_x}{L_{c1}^2} = \frac{\pi^2(29{,}000)(146)}{(16(12))^2} = 1130 \text{ kips}$$

and

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} = \frac{0.6}{1 - \dfrac{1.6(20.8)}{1130}} = 0.618 < 1.0$$

Therefore, use $B_1 = 1.0$.

To determine the sway amplification, the total gravity load on the frame is

$$P_{story} = (113) + 0.75(45.0) = 147 \text{ kips}$$

Again, a serviceability drift index of 0.003 is maintained under the actual wind loads. Therefore, H = 4.0 kips and Δ/L = 0.003 is used to determine the sway amplification factor. As before, R_M = 0.95 so

$$B_2 = \frac{1}{1 - \left(\frac{\alpha P_{story}}{R_M H}\left(\frac{\Delta}{\mathrm{L}}\right)\right)} = \frac{1}{1 - \frac{1.6(147)}{0.95(4.0)}(0.003)} = 1.23$$

Thus, the second-order force and moment are

$$M_r = 1.0(26.7) + 1.23(14.4) = 44.4 \text{ ft-kips}$$

and, adding in the lateral load effect amplified by B_2,

$$P_r = 20.5 + 1.23(0.320) = 20.9 \text{ kips}$$

Step 8: Determine whether the column satisfies the interaction equation.

Using the same values found in Step 5, determine the appropriate interaction equation.

$$\frac{P_r}{P_n/\Omega} = \frac{20.9}{148} = 0.141 < 0.2$$

Therefore, use Equation H1-1b.

$$\frac{P_a}{2(P_n/\Omega)} + \frac{M_a}{(M_n/\Omega)} \le 1.0$$

$$\frac{20.9}{2(148)} + \frac{44.4}{84.9} = 0.594 < 1.0$$

Thus, the column is adequate for this load combination also.

Step 9: The W8×40 is shown to be adequate for gravity and wind loads in combination. Now, check to see that these columns have sufficient capacity to brace the interior pinned columns for gravity load only. This load combination puts the greatest load in the gravity only columns.

Step 10: For stability in the plane of the frame, using the Yura approach discussed in Section 8.10, the total load on the structure is to be resisted by the four corner columns; thus

Dead Load = 0.05 ksf (50 ft)(90 ft)/4 columns = 56.3 kips
Snow Load = 0.02 ksf (50 ft)(90 ft)/4 columns = 22.5 kips

Thus, for load combination 3,

P_a = (56.3) + (22.5) = 78.8 kips
M_a = (20.5) + (8.20) = 28.7 ft-kips

Step 11: Determine the second-order amplification

As before, for the length L_x = 16.0 ft, P_{e1} = 1130 kips and C_m = 0.6, the second-order amplification for member effect is

$$B_1 = \frac{C_m}{1-\dfrac{\alpha P_r}{P_1}} = \frac{0.6}{1-\dfrac{1.6(78.8)}{1130}} = 0.675 < 1.0$$

Therefore, use B_1 = 1.0 and $P_r = P_a$ = 78.8 kips

Sway amplification will be the same as determined in step 4, since the gravity load is the same; thus, B_2 =1.25. Therefore

$$M_r = 1.0(28.7) + 1.25(0) = 28.7 \text{ ft-kips}$$

Step 12: Check the corner columns for interaction under this force and moment.

As determined in Step 5 for in-plane buckling,

$$P_n/\Omega = 148 \text{ kips}$$
$$M_{nx}/\Omega = 84.9 \text{ ft-kips}$$

Checking for the appropriate interaction equation,

$$\frac{P_u}{P_n/\Omega} = \frac{78.8}{148} = 0.532 > 0.2$$

Thus, use Equation H1-1a.

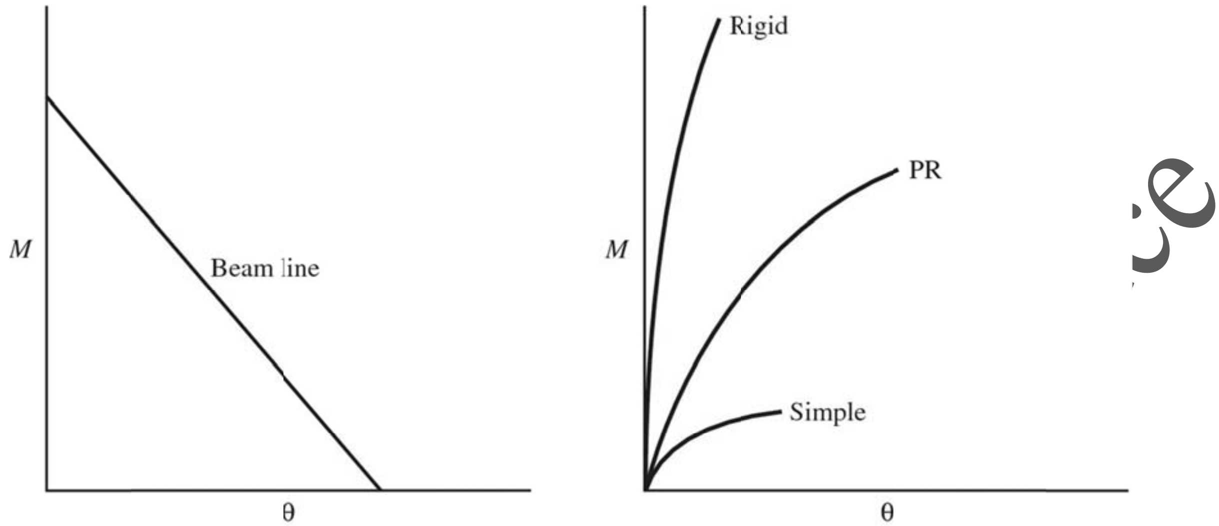
$$\frac{P_a}{P_n/\Omega} + \frac{8}{9}\left(\frac{M_{ax}}{M_{nx}/\Omega}\right) \leq .0$$

$$\frac{78.8}{148} + \frac{8}{9}\left(\frac{28.7}{84.9}\right) = 0.833 < 1.0$$

Thus, the W8×40 is adequate for both strength under combined load and stability for supporting the gravity only columns.

8.11 PARTIALLY RESTRAINED FRAMES

The beams and columns in the frames considered up to this point have all been connected with moment-resisting fully restrained (FR) connections or simple pinned connections. These latter simple connections are defined in *Specification* Section B3.4a. Partially restrained connections,



(a) Beam line (b) Connections

Figure 8.20 Moment Rotation Curves for Uniformly Loaded Beam and Typical Connections.

defined in *Specification* Section B3.4b along with FR connections, have historically been referred to as *semirigid connections*. When these PR connections are included as the connecting elements in a structural frame, they influence both the strength and stability of the structure.

Before considering the partially restrained frame, it will be helpful to look at the partially restrained beam. The relationship between the end moment and end rotation for a symmetric, uniformly loaded prismatic beam can be obtained from the well-known slope deflection equation as

$$M = -2\frac{EI\theta}{L} + \frac{WL}{12} \tag{8.12}$$

This equation is plotted in Figure 8.20a and labeled as the *beam line*.

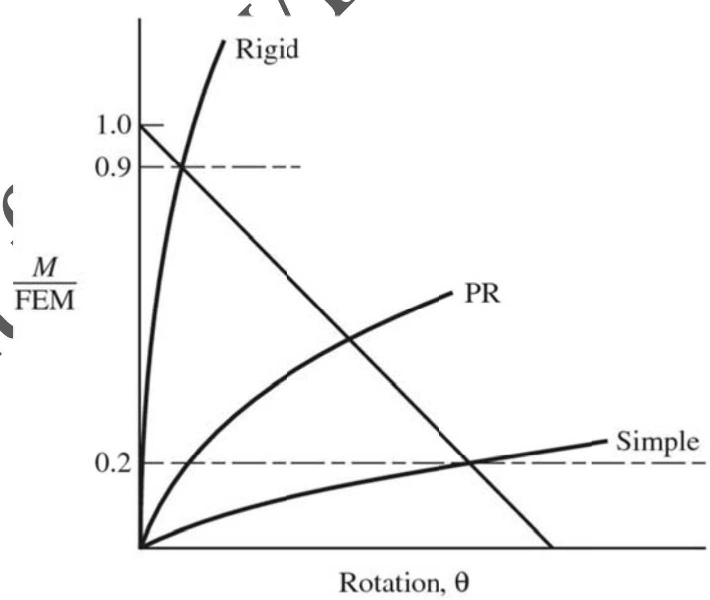


Figure 8.21 Beam Line and Connection Curves.

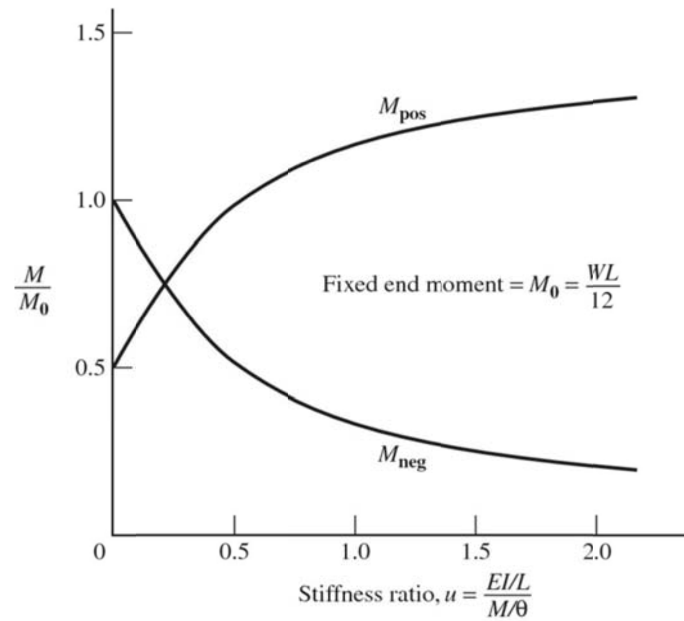


Figure 8.22 Influence of the PR Connection on the Maximum Positive and Negative Moments of a Beam.

All PR connections exhibit some rotation as a result of an applied moment. The moment-rotation characteristics of these connections are the key to determining the type of connection and thus the behavior of the structure. Moment-rotation curves for three generic connections are shown in Figure 8.20b and are labeled rigid, simple, and PR. Numerous research studies have been conducted in an effort to identify the moment-rotation curves for real connections. Two compilations of these curves have been published.[5,6]

The relationship between the moment-rotation characteristics of a connection and a beam can be seen by plotting the *beam line* and *connection curve* together, as shown in Figure 8.21. Equilibrium is obtained when the beam line and the connection curve intersect. Normal engineering practice treats connections capable of resisting at least 90 percent of the fixed-end moment as rigid and those capable of resisting no more than 20 percent of the fixed-end moment as simple. All connections that exhibit an ability to resist moment between these limits must be treated as partially restrained connections, accounting for their true moment-rotation characteristics.

The influence of the PR connection on the maximum positive and negative moments on the beam is seen in Figure 8.22. Here, the ratio of positive or negative moment to the fixed-end moment is plotted against the ratio of beam stiffness, *EI/L*, to a linear connection stiffness, M/θ. The moment for which the beam must be designed ranges from 0.75 times the fixed-end moment to 1.5 times the fixed-end moment, depending on the stiffness of the connection.

When PR connections are used to connect beams and columns to form PR frames, the analysis becomes much more complex. The results of numerous studies dealing with this issue have been reported. Although some practical designs have been carried out, widespread practical design of PR frames is not common. In addition to the problems associated with modeling a particular connection, the question of loading sequence arises. Because real, partially restrained connections behave nonlinearly, the sequence of applied loads influences the structural response. The approach to load application may have more significance than the accuracy of the connection model used in the analysis.

[5]Goverdhan, A. V., *A Collection of Experimental Moment Rotation Curves and Evaluation of Prediction Equations for Semi-Rigid Connections*, Master of Science Thesis, Vanderbilt University, Nashville, TN, 1983.
[6]Kishi, N., and Chen, W. F,. *Data Base of Steel Beam-to-Column Connections*, CE-STR-86-26, West Lafayette, IN: Purdue University, School of Engineering, 1986.

Although a complete, theoretical analysis of a partially restrained frame may currently be beyond the scope of normal engineering practice, a simplified approach exists that is not only well within the scope of practice, but also commonly carried out in everyday design and has been for over half a century. This approach can be referred to as *Flexible Moment Connections*. It has historically been called *Type 2 with Wind*. The Flexible Moment Connection approach relies heavily on the nonlinear moment-rotation behavior of the PR connection although the actual curve is not used. In addition, it relies on a phenomenon called *shake-down*, which shows that the connection, although exhibiting nonlinear behavior initially, behaves linearly after a limited number of applications of lateral load.[7]

The moment-rotation curve for a typical PR connection is shown in Figure 8.23a along with the beam line for a uniformly loaded beam. The point labeled 0 represents equilibrium for the applied gravity loads. The application of wind load produces moments at the beam ends that add to the gravity moment at the leeward end of the beam and subtract from the windward end. Because moment at the windward end is being removed, the connection behaves elastically with a stiffness close to the original connection stiffness, whereas at the leeward end, the connection continues to move along the nonlinear connection curve. Points labeled 1 and 1′ in Figure 8.23b represent equilibrium under the first application of wind to the frame.

When the wind load is removed, the connection moves from points 1 and 1′ to points 2 and 2′, as shown in Figure 8.23c. The next application of a wind load that is larger than the first

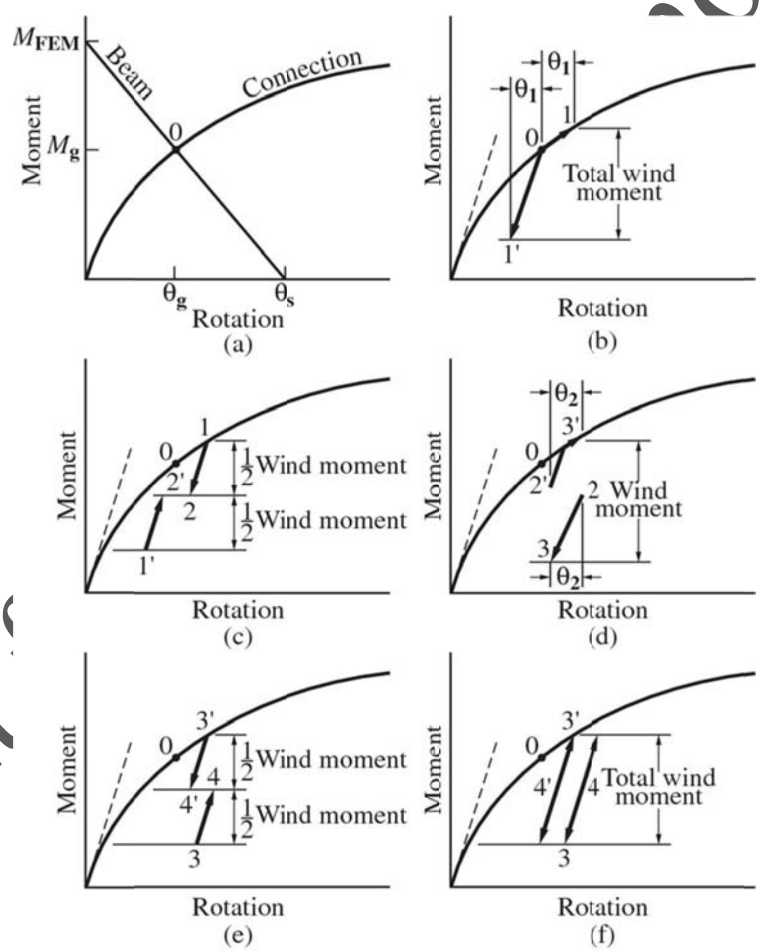


Figure 8.23 Moment-Rotation Curves Showing Shake-Down

[7]Geschwindner, L. F., and Disque, R. O., "Flexible Moment Connections for Unbraced Frames Subject to Lateral Forces—A Return to Simplicity." *Engineering Journal*, AISC, Vol. 42, No. 2, 2005, pp 99–112.

and in the opposite direction will see the connection behavior move to points 3 and 3′. Note that on the windward side, the magnitude of this applied wind moment dictates whether the connection behaves linearly or follows the nonlinear curve, as shown in Figure 8.23d. Removal of this wind load causes the connection on one end to unload and on the other end to load, both linearly as shown in Figure 8.23e. Any further application of wind load, less than the maximum already applied, will see the connection behave linearly. In addition, the maximum moment on the connection is still close to that applied originally from the gravity load. Thus, the condition described in Figure 8.23f shows that shake-down has taken place and the connection now behaves linearly for both loading and unloading.

The design procedure used to account for this shake-down is straight forward. All beams are designed as simple beams using the appropriate load combinations. This assures that the beams are adequate, regardless of the actual connection stiffness, as was seen in Figure 8.22. Wind load moments are determined through a modified portal analysis where the leeward column is assumed not to participate in the lateral load resistance. Connections are sized to resist the resulting moments, again for the appropriate load combinations. In addition, it is particularly important to provide connections that have sufficient ductility to accommodate the large rotations that will occur, without overloading the bolts or welds under combined gravity and wind.

Columns must be designed to provide frame stability under gravity loads as well as gravity plus wind. The columns may be designed using the approach that was presented for columns in moment frames, but with two essential differences from the conventional rigid frame design:

1. Because the gravity load is likely to load the connection to its plastic moment capacity, the column can be restrained only by a girder on one side and this girder will act as if it is pinned at its far end. Therefore, in computing the girder stiffness rotation factor, I_g/L_g, for use in the effective length alignment chart, the girder length should be doubled.

2. One of the external columns, the leeward column for the wind loading case, cannot participate in frame stability, because it will be attached to a connection that is at its plastic moment capacity. The stability of the frame may be assured, however, by designing the remaining columns to support the total frame load.

For the exterior column, the moment in the beam to column joint is equal to the capacity of the connection. It is sufficiently accurate to assume that this moment is distributed one-half to the upper column and one-half to the lower column. For interior columns, the greatest realistically possible difference in moments resulting from the girders framing into the column should be distributed equally to the columns above and below the joint.

EXAMPLE 8.7a
Column Design with Flexible Wind Connections by LRFD

Goal: Select girders and columns for a building with flexible wind connections.

Given: An intermediate story of a three-story building is given in Figure 8.24. Story height is 12 ft. The frame is braced in the direction normal to that shown. Use the LRFD provisions and A992 steel.

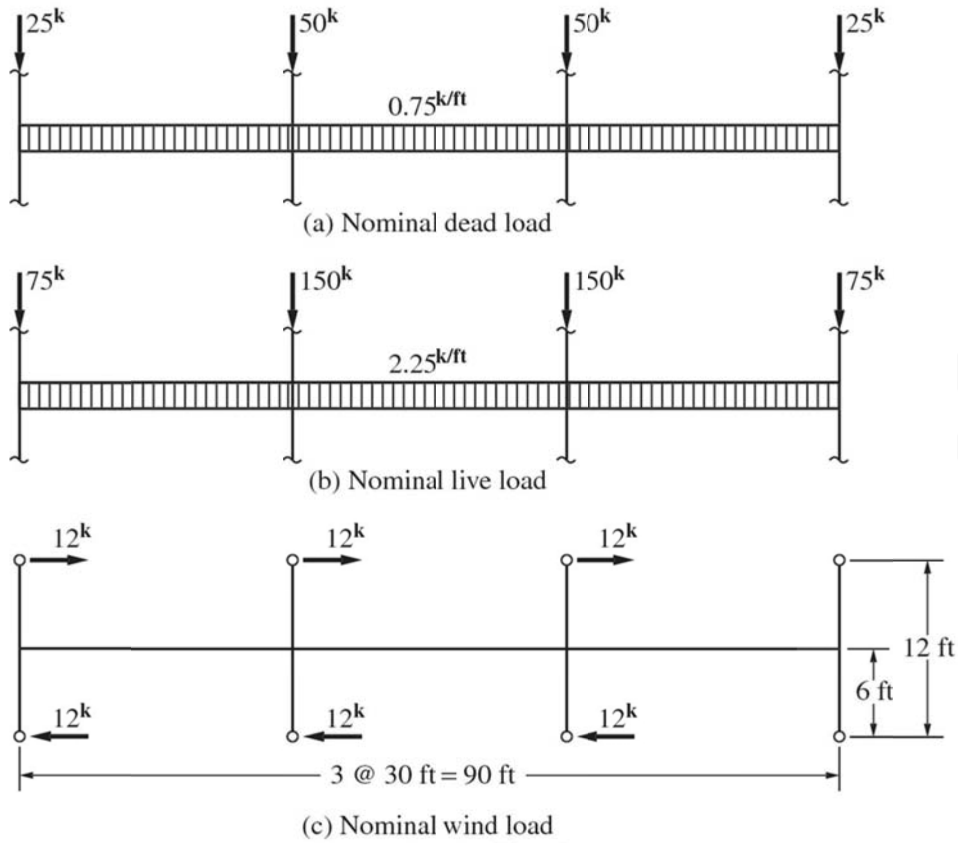


Figure 8.24 Intermediate Story of a Three-Story Building (Example 8.7).

SOLUTION

Step 1: Determine the required forces and moments for the load combination

12.D + 0.5L + 1.0W.

The loads shown in Figure 8.24 are the code-specified nominal loads. The required forces are calculated using tributary areas as follows.

Gravity loads on exterior columns.

1.2D = 1.2(25 kips + 0.75 kips/ft (15 ft)) = 43.5 kips
0.5L = 0.5 (75 kips + 2.25 kips/ft (15 ft)) = 54.4 kips
Total =97.9 kips

Gravity loads on interior columns

1.2D = 1.2 (50 kips + 0.75 kips/ft (30 ft)) = 87.0 kips
0.5L = 0.5 (150 kips + 2.25 kips/ft (30 ft)) = 109 kips
Total= 196 kips

Gravity load on girders for the worst case 1.2D + 1.6L:

1.2D = 1.2 (0.75 kips/ft (30 ft)) = 27.0 kips
1.6L = 1.6 (2.25 kips/ft (30 ft)) = 108 kips
Total = 135 kips

Step 2: Design the girder for the simple beam moment assuming full lateral support using *Manual* Table 3-2 or 6-2.

$$M_u = WL/8 = 135(30.0)/8 = 506 \text{ ft-kips}$$

Therefore use

$$\text{W21×62 } (\phi M_n = 540 \text{ ft-kips}, I_x = 1330 \text{ in.}^4)$$

Step 3: Design the columns for the gravity load on the interior column using *Manual* Table 4-1.

For buckling out of the plane in a braced frame, $K = 1.0$ and $L_{cy} = 12.0$

Thus, with $P_u = 196$ kips ,try

$$\text{W14×43, } (\phi P_n = 371 \text{ kips}, I_x = 428 \text{ in.}^4, r_x/r_y = 3.08)$$

Step 4: To check the column for stability in the plane, determine the effective length factor from the alignment chart with

$$G_{top} = G_{bottom} = \frac{\Sigma(I/L)_c}{\Sigma(I/L)_g} = \frac{2\left(\dfrac{428}{12.0}\right)}{\left(\dfrac{1330}{2(30.0)}\right)} = 3.21$$

Note that only one beam is capable of restraining the column and that the beam is pinned at its far end; thus the effective beam length is taken as twice its actual length.

Considering the stress in the column under load, the stiffness reduction factor can be determined.

$$P_u/A = 196/12.6 = 15.6 \text{ ksi}$$

Thus, since $P_u/A = 15.6 < 0.5F_y$, the stiffness reduction factor from Manual Table 4-13 is $\tau_b = 1.00$. The stiffness ratio then remains

$$G_{top} = G_{bottom} = 3.21$$

which yields, from the alignment chart, Figure 5.20

$$K = 1.87$$

Step 5: Determine the effective length in the plane of bending.

$$(L_{cx})_{eff} = \frac{L_{cx}}{r_x/r_y} = \frac{1.87(12.0)}{3.08} = 7.29 \text{ ft}$$

Step 6 Determine the column compressive strength from *Manual* Table 4-1 or 6-2 with $L_c = 7.29$ ft.

$$\phi P_n = 484 \text{ kips}$$

Step 7: Determine the second-order moment.

The applied wind moment is M_u = 1.0(6.0)(12.0) = 72.0 ft-kips and the applied force is P_u = 196 kips.

Considering all the moment as a translation moment and using Commentary equation C-A-8-1

$$P_{e\ story} = \frac{\pi^2 EI}{(K_2 L)^2} = \frac{\pi^2 (29{,}000)(428)}{(1.87(12.0)(12))^2} = 1690 \text{ kips}$$

Therefore, for all three columns,

$$B_2 = \frac{1}{1 - \left(\dfrac{\alpha P_{story}}{P_{e\ story}}\right)} = \frac{1}{1 - \dfrac{1.0(3(196))}{3(1690)}} = 1.13$$

and

$$M_r = 1.13\ (72.0) = 81.4 \text{ ft-kips}$$

Step 8: Determine whether the column satisfies the interaction equation

$$\frac{P_u}{\phi P_n} = \frac{196}{484} = 0.405 > 0.2$$

Therefore, use Equation H1-1a, ϕM_n = 222, from *Manual* Table 3-10 or 6-2, which results in

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\left(\frac{M_u}{\phi M_n}\right) \le 1.0$$

$$0.405 + \frac{8}{9}\left(\frac{81.4}{222}\right) = 0.731 < 1.0$$

This indicates that the W14×43 is adequate for stability. The members can then be used as a starting point in a more rigorous analysis.

EXAMPLE 8.7b
Column Design with Flexible Wind Connections by ASD

Goal: Select girders and columns for a building with flexible wind connections.

Given: An intermediate story of a three-story building is given in Figure 8.24. Story height is 12 ft. The frame is braced in the direction normal to that shown. Use the ASD provisions and A992 steel.

Step 1: Determine the required forces and moments for the load combination
D + 0.75L +0.75(0.6W).

The loads shown in Figure 8.24 are the code-specified nominal loads. The required forces are calculated using tributary areas as follows.

Gravity loads on exterior columns

D = (25 k + 0.75 k/ft (15 ft)) = 36.3 kips
0.75L = 0.75 (75 k + 2.25 k/ft (15 ft)) = 81.6 kips
Total = 118 kips

Gravity loads on interior columns

D = (50 k + 0.75 k/ft (30 ft)) = 72.5 kips
0.75L = 0.75 (150 k + 2.25 k/ft (30 ft)) = <u>163 kips</u>
Total = 236 kips

Gravity load on girders for the worst case, D + L
D = (0.75 k/ft (30 ft)) = 22.5 kips
L = (2.25 k/ft (30 ft)) = <u>67.5 kips</u>
Total = 90.0 kips

Step 2: Design the girder for the simple beam moment assuming full lateral support using *Manual* Table 3-2 or 6-2.

$$M_a = WL/8 = 90.0(30.0)/8 = 338 \text{ ft-kips}$$

Therefore use

W21×62 (M_n/Ω = 359 ft-kips, I_x = 1330 in.^{4})

Step 3: Design the columns for gravity load on the interior column using *Manual* Table 4-1 or 6-2.

For buckling out of the plane in a braced frame, K = 1.0 and L_{cy} = 12.0

Thus, with P_a = 236 kips
try W14×43 (P_n/Ω = 247 kips, I_x = 428 in.^{4}, r_x/r_y = 3.08)

Step 4: To check the column for stability in the plane, determine the effective length factor from the alignment chart with

$$G_{top} = G_{bottom} = \frac{\Sigma(I/L)_c}{\Sigma(I/L)_g} = \frac{2\left(\frac{428}{12.0}\right)}{\left(\frac{1330}{2(30.0)}\right)} = 3.21$$

Note that only one beam is capable of restraining the column and that that beam is pinned at its far end; thus the effective beam length is taken as twice its actual length.

Considering the stress in the column under load, the stiffness reduction factor can be determined.

$$\frac{P_a}{A} = \frac{236}{12.6} = 18.7 \text{ ksi}$$

Thus, from the *Manual* Table 4-13, the stiffness reduction factor τ_b = 0.960. The inelastic stiffness ratio then becomes

$$G_{top} = G_{bottom} = 0.960(3.21) = 3.08$$

which yields, from the alignment chart, Figure 5.20

$$K = 1.84$$

Step 5: Determine the effective length in the plane of bending.

$$(L_{cx})_{eff} = \frac{L_{cx}}{r_x/r_y} = \frac{1.84(12.0)}{3.08} = 7.17 \text{ ft}$$

Step 6: Determine the column compressive strength from *Manual* Table 4-1 or 6-2 with L_c = 7.17 ft.

$$P_n/\Omega = 324 \text{ kips}$$

Step 7: Determine the second-order moment.

The applied wind moment is M_a = 0.75(0.6)(6.0(12.0)) = 32.4 ft-kips and the applied force is P_a = 236 kips.

Considering all the moment as a translation moment and using Commentary equation C-A-8-1

$$P_{e\ story} = \frac{\pi^2 EI}{(K_2 L)^2} = \frac{\pi^2(29{,}000)(428)}{(1.84(12)(12))^2} = 1740 \text{ kips}$$

$$\alpha P_a = 1.6(236) = 378 \text{ kips}$$

Therefore, for all three columns,

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\ story}}} = \frac{1}{1 - \dfrac{3(378)}{3(1740)}} = 1.28$$

and

$$M_r = 1.28(32.4) = 41.5 \text{ ft-kips}$$

Step 8: Determine whether the column satisfies the interaction equation

$$\frac{P_r}{P_n/\Omega} = \frac{236}{324} = 0.728 < 0.2$$

Therefore, use Equation H1-1a, M_n/Ω = 148, from *Manual* Table 3-10 or 6-2, which results in

$$\frac{P_a}{(P_n/\Omega)} + \frac{8}{9}\left(\frac{M_a}{(M_n/\Omega)}\right) \le 1.0$$

$$0.728 + \frac{8}{9}\left(\frac{41.5}{148}\right) = 0.98 < 1.0$$

This indicates that the W14×43 is adequate for stability. These members can then be used as a starting point in a more rigorous analysis.

After an acceptable column is selected, the lateral displacement of the structure must be checked. Coverage of drift in wind moment frames is beyond the treatment intended here, but is covered in the Geschwindner and Disque paper already referenced.

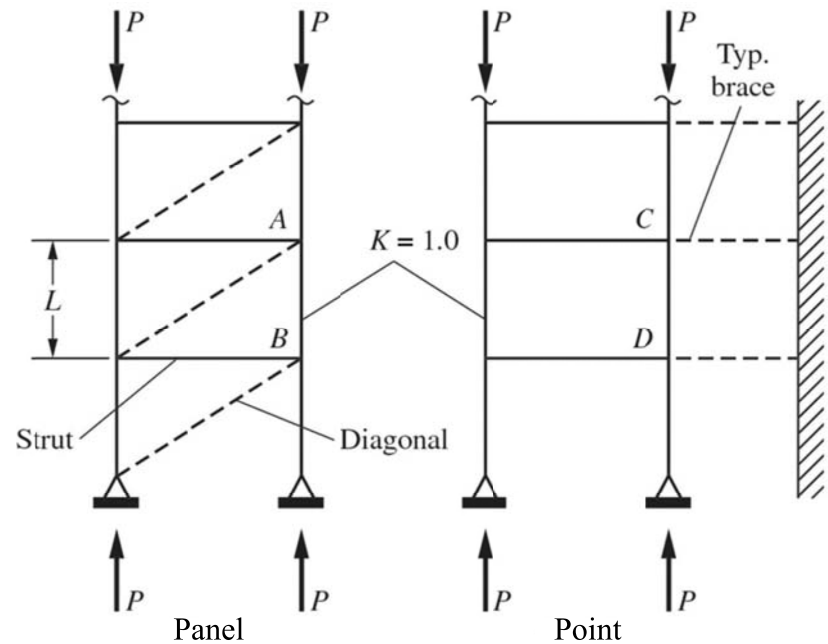


Figure 8.25 Definitions of Bracing Types. Copyright © American Institute of Steel Construction, Inc. Reprinted with Permission. All rights reserved.

8.12 STABILITY BRACING DESIGN

Braces in steel structures are used to reduce the effective length of columns, reduce the unbraced length of beams, and provide overall structural stability. The discussion of columns in Chapter 5 showed how braces could be effective in reducing effective length and thereby increasing column strength. Chapter 6 demonstrated how the unbraced length of a beam influenced its strength, and earlier in this chapter the influence of sway on the stability of a structure was discussed. Every case assumed that the given bracing requirements were satisfied, but nothing was said about the strength or stiffness of the required braces. For cases when braces are not specifically included in the second-order analysis, design of braces will follow the provisions of Appendix 6 of the *Specification*. Appendix 6 treats bracing for columns and beams similarly, although the specific requirements are different. Two types of braces are defined: point bracing and panel bracing.

Point bracing controls the movement of a point on the member without interaction with any adjacent braced points. These braces would be attached to the member and then to a fixed support, such as the abutment shown in Figure 8.25b.

Panel bracing relies on other braced points of the structure to provide support. A diagonal brace within a frame would be a panel brace, as shown in Figure 8.25a. In this case, the axial deformation of the diagonal brace is a function of the displacement at each end of the brace. Because the horizontal strut is usually a part of a very stiff floor system that has significant strength in its plane, the strength and stiffness of the diagonal element usually controls the overall behavior of this braced system.

The brace requirements of the *Specification* are intended to enable the members' being designed to reach their maximum strength based on the length between bracing points and an effective length factor, $K = 1.0$. A brace has two requirements: strength and stiffness. A brace that is inadequate in either of these respects is not sufficient to enable the member it is bracing to perform as it was designed.

8.12.1 Column Bracing

For column panel bracing, the required shear strength of the bracing system is

$$V_{br} = 0.005P_r \qquad \text{(AISC A-6-1)}$$

and the required shear stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{2P_r}{L_{br}}\right)\text{(LRFD)} \qquad \beta_{br} = \Omega\left(\frac{2P_r}{L_{br}}\right)\text{(ASD)} \qquad \text{(AISC A-6-2)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

L_{br} = unbraced length of the panel under consideration
P_r = required strength of the column within the panel under consideration for ASD or LRFD as appropriate for the design method being used.

For a column point brace, the required brace strength is

$$P_{br} = 0.01\,P_r \qquad \text{(AISC A-6-3)}$$

and the required brace stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{8P_r}{L_{br}}\right)\text{(LRFD)} \qquad \beta_{br} = \Omega\left(\frac{8P_r}{L_{br}}\right)\text{(ASD)} \qquad \text{(AISC A-6-4)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

L_{br} = laterally unbraced length adjacent to the point brace
P_r = required strength for ASD or LRFD as appropriate for the design method being used.

It should be noted that the requirements for point braces are significantly greater than those for panel braces. Thus, if a panel bracing system can be developed, it has the potential to be the more economical approach.

8.12.2 Beam Bracing

For a beam panel brace, the required shear strength of the bracing system is

$$V_{br} = 0.01\left(\frac{M_r C_d}{h_o}\right) \qquad \text{(AISC A-6-5)}$$

and the required panel brace stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{4M_r C_d}{L_{br} h_o}\right)\text{(LRFD)} \qquad \beta_{br} = \Omega\left(\frac{4M_r C_d}{L_{br} h_o}\right)\text{(ASD)} \qquad \text{(AISC A-6-6)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

h_o = distance between flange centroids
C_d = 1.0 for single curvature bending and 2.0 for the brace closest to the inflection point for double curvature bending
L_{br} = laterally unbraced length within the panel under consideration
M_r = the largest required flexural strength of the beam within the unbraced lengths adjacent to the point being braced

For a beam point brace, the required strength of the brace is

$$P_{br} = 0.02 M_r C_d / h_o \qquad \text{(AISC A-6-7)}$$

and the required brace stiffness is

$$\beta_{br} = \frac{1}{\phi}\left(\frac{10 M_r C_d}{L_{br} h_o}\right) \text{(LRFD)} \qquad \beta_{br} = \Omega\left(\frac{10 M_r C_d}{L_{br} h_o}\right) \text{(ASD)} \qquad \text{(AISC A-6-8)}$$

$$\phi = 0.75 \text{ (LRFD)} \qquad \Omega = 2.00 \text{ (ASD)}$$

where

h_o = distance between flange centroids
C_d = 1.0 for single curvature and 2.0 for double curvature as above
L_{br} = laterally unbraced length adjacent to the point brace
M_r = the largest required flexural strength of the beam within the unbraced lengths adjacent to the point being braced

As with column bracing, the requirements for point braces are greater than those for panel braces.

8.12.3 Frame Bracing

Frame bracing and column bracing are accomplished by the same panel and point braces and may be designed using the same stiffness and strength equations. However, the most direct approach to bracing design for frames is to include the braces in the model when a second-order analysis is carried out. When that is the case, the provisions of Appendix 6 do not need to be checked.

EXAMPLE 8.8a
Bracing Design by LRFD

Goal: Determine the required bracing for a braced frame to provide stability for the gravity load.

Given: Using the LRFD requirements, select a rod to provide the point bracing shown in the center panel of the three-bay frame of Figure 8.9a to provide stability for a total gravity dead load of 113 kips and live load of 45 kips.

SOLUTION

Step 1: Determine the required brace stiffness for gravity load.

For the gravity load, the required brace stiffness is based on $1.2D + 1.6L$.

$$P_r = 1.2(113) + 1.6(45.0) = 208 \text{ kips}$$

and from Equation A-6-4

$$\beta_{br} = \frac{1}{\phi}\left(\frac{8P_r}{L_{br}}\right) = \frac{1}{0.75}\left(\frac{8(208)}{16.0}\right) = 139 \text{ kips/ft}$$

Step 2: Determine the required brace area based on required stiffness and accounting for the angle of the brace.

Based on the geometry of the brace from Figure 8.9, where θ is the angle of the brace with the horizontal and L_r = 34.0 ft is the length of the brace,

$$\beta_{br} = \frac{A_{br} E}{L_r} \cos^2 \theta = 139 \text{ kips/ft}$$

This results in a required brace area

$$A_{br} = \frac{\beta_{br} L_r}{E \cos^2 \theta} = \frac{139(34.0)}{29{,}000\left(\dfrac{30}{34}\right)^2} = 0.209 \text{ in.}^2$$

Step 3: Determine the required brace force for gravity load. The required horizontal brace force for a point brace given by Equation A-6-3 is

$$P_{br} = 0.01 P_r = 0.01(208) = 2.08 \text{ kips}$$

which gives a force in the member of

$$P_{br(angle)} = 2.08(34/30) = 2.36 \text{ kips}$$

and a required area, assuming F_y = 36 ksi for a rod, of

$$A_{br} = \frac{P_{br(angle)}}{\phi F_y} = \frac{2.36}{0.9(36)} = 0.0728 \text{ in.}^2$$

Step 4: For the dead plus live load case,

$$A_{\min} = 0.209 \text{ in.}^2$$

Step 5: Select a rod to meet the required area for the controlling case of stiffness for the dead plus live load case where $A_{\min} = 0.209$ in.^{2}

use a 5/8-in. rod with A = 0.307 in.^{2}

EXAMPLE 8.8b
Bracing Design by ASD

Goal: Determine the required bracing for a braced frame to provide stability for the gravity load.

Given: Using the ASD requirements, select a rod to provide the point bracing shown in the center panel of the three-bay frame of Figure 8.9a to provide stability for a total gravity dead load of 113 kips and live load of 45 kips.

SOLUTION

Step 1: Determine the required brace stiffness for gravity load.

For the gravity load, the required brace stiffness is based on $D + L$.

$$P_r = 113 + 45.0 = 158 \text{ kips}$$

and from Equation A-6-4

$$\beta_{br} = \Omega\left(\frac{8P_r}{L_{br}}\right) = 2.00\left(\frac{8(158)}{16.0}\right) = 158 \text{ kips/ft}$$

Step 2: Determine the required brace area based on required stiffness and accounting for the angle of the brace.

Based on the geometry of the brace from Figure 8.9, where θ is the angle of the brace with the horizontal and L_r =34.0 ft is the length of the brace.

$$\beta_{br} = \frac{A_{br}E}{L_r}\cos^2\theta = 158 \text{ kips/ft}$$

This results in a required brace area

$$A_{br} = \frac{\beta_{br}L_r}{E\cos^2\theta} = \frac{158(34.0)}{29{,}000\left(\frac{30}{34}\right)^2} = 0.238 \text{ in.}^2$$

Step 3: Determine the required brace force for gravity load.

The required horizontal brace force for a point brace given by Equation A-6-3 is

$$P_{br} = 0.01P_r = 0.01(158) = 1.58 \text{ kips}$$

which gives a force in the member of

$$P_{br(angle)} = 1.58(34/30) = 1.79 \text{ kips}$$

and a required area, assuming F_y = 36 ksi for a rod, of

$$A_{br} = \frac{P_{br(angle)}}{F_y/\Omega} = \frac{1.79}{(36/1.67)} = 0.0830 \text{ in.}^2$$

Step 4: For the dead plus live load case,

$$A_{\min} = 0.238 \text{ in.}^2$$

Step 5: Select a rod to meet the required area for the controlling case of stiffness for the dead plus live load case, $A_{\min}$ = 0.238 in.^{2}.

use a 5/8-in. rod with A = 0.307 in.^{2}

8.13 TENSION PLUS BENDING

Throughout this chapter, the case of combined compression plus bending has been treated. That is the most common case of combined loading in typical building structures. However, the *Specification* also has provisions, in Section H1.2, for combining flexure and tension. The addition of a tension force to a member already undergoing bending may be beneficial.

The interaction equations for combined tension and flexure are the same as those already discussed and given as Equations H1-1a and H1-1b. However, if the flexural strength is controlled by the limit state of lateral-torsional buckling, the addition of a tension force can increase bending strength. This is accounted for in the *Specification* by the introduction of a modification factor to be applied to C_b. Thus, for doubly symmetric members, C_b in Chapter F can be multiplied by $\sqrt{1+\alpha P_r/P_{ey}}$ for axial tension that acts concurrently with flexure, where $P_{ey} = \pi^2 EI_y / L_b^2$ and α = 1.0 for LRFD and 1.6 for ASD, as before. The limit that M_n cannot exceed M_p still must be satisfied as it was for beam design discussed in Chapter 6.

EXAMPLE 8.9a
Combined Tension and Bending by LRFD

Goal: Check the given W-shape beam for combined tension and bending

Given: A W16×77 beam spans 25 ft and carries a uniform dead load of 0.92 kips/ft and a uniform live load of 2.79 kips/ft. It also carries a tension live load of 62.5 kips. The member is braced at the ends only for lateral-torsional buckling. Use A992 steel.

SOLUTION

Step 1: Determine the required moment strength

$$w_u = 1.2(0.92) + 1.6(2.79) = 5.57 \text{ kips/ft}$$

$$M_u = \frac{5.57(25)^2}{8} = 435 \text{ ft-kips}$$

Step 2: Determine the required tension strength

$$T_u = 1.6(62.5) = 100 \text{ kips}$$

Step 3: Determine the available moment strength. With L_b = 25 ft and C_b = 1.14, from Manual Table 6-2

$$\phi M_n = 1.14(382) = 435 \text{ ft-kips} < \phi M_p = 563 \text{ ft-kips}$$

Step 4: Determine the available tension strength for the limit state of yielding. Connections at the end of the member are at a location of zero moment so tension rupture will not be a factor for interaction with bending. From Table 6-2

$$\phi T_n = 1020 \text{ kips}$$

Step 5: Determine the increase to be applied to C_b when tension is applied in conjunction with moment strength determined for the lateral-torsional buckling limit state.

$$P_{ey} = \frac{\pi^2 EI}{L_b^2} = \frac{\pi^2(29{,}000)(138)}{\left(25(12)\right)^2} = 439 \text{ kips}$$

$$\sqrt{1 + \frac{\alpha P_r}{P_{ey}}} = \sqrt{1 + \frac{1.0(100)}{439}} = 1.11$$

Step 6: Moment strength when considered in combination with tension

$$\phi M_n = 1.11(435) = 483 \text{ ft-kips} < \phi M_p = 563 \text{ ft-kips}$$

Step 7: Determine the interaction equation to use

$$\frac{P_r}{P_c} = \frac{100}{1020} = 0.098 < 0.2$$

Step 8: Use Equation H1-1b

$$\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{0.098}{2} + \frac{435}{483} = 0.049 + 0.901 = 0.95 < 1.0$$

So the beam is adequate to carry the bending moment and tension force.

Step 9: Check the beam for bending alone, in case the tension force were not there. Use the available moment strength from Step 3.

$$\frac{M_r}{M_c} = \frac{435}{435} = 1.0 \le 1.0$$

So the beam would just be adequate. In cases where the application of the tension force increases interaction strength and that force may not actually occur, it is important to check the member for flexure alone.

The W16×77 is adequate to carry the applied loads.

EXAMPLE 8.9b
Combined Tension and Bending by ASD

Goal: Check the given W-shape beam for combined tension and bending

Given: A W16×77 beam spans 25 ft and carries a uniform dead load of 0.92 kips/ft and a uniform live load of 2.79 kips/ft. It also carries a tension live load of 62.5 kips. The member is braced at the ends only for lateral-torsional buckling. Use A992 steel.

SOLUTION

Step 1: Determine the required moment strength

$$w_a = 0.92 + 2.79 = 3.71 \text{ kips/ft}$$

$$M_a = \frac{3.71(25)^2}{8} = 290 \text{ ft-kips}$$

Step 2: Determine the required tension strength

$$T_a = 62.5 \text{ kips}$$

Step 3: Determine the available moment strength. With L_b = 25 ft and C_b = 1.14, from Manual Table 6-2

$$\frac{M_n}{\Omega} = 1.14(254) = 290 \text{ ft-kips} < \frac{M_p}{\Omega} = 374 \text{ ft-kips}$$

Step 4: Determine the available tension strength for the limit state of yielding. Connections at the end of the member are at a location of zero moment so tension rupture will not be a factor for interaction with bending. From Table 6-2

$$\frac{T_n}{\Omega} = 677 \text{ kips}$$

Step 5: Determine the increase to be applied to C_b when tension is applied in conjunction with moment strength determined for the lateral-torsional buckling limit state.

$$P_{ey} = \frac{\pi^2 EI}{L_b^2} = \frac{\pi^2(29{,}000)(138)}{\left(25(12)\right)^2} = 439 \text{ kips}$$

$$\sqrt{1 + \frac{\alpha P_r}{P_{ey}}} = \sqrt{1 + \frac{1.6(62.5)}{439}} = 1.11$$

Step 6: Moment strength when considered in combination with tension

$$\frac{M_n}{\Omega} = 1.11(290) = 322 \text{ ft-kips} < \frac{M_p}{\Omega} = 374 \text{ ft-kips}$$

Step 7: Determine the interaction equation to use

$$\frac{P_r}{P_c} = \frac{62.5}{677} = 0.092 < 0.2$$

Step 8: Use Equation H1-1b

$$\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{0.092}{2} + \frac{290}{322} = 0.046 + 0.901 = 0.95 < 1.0$$

So the beam is adequate to carry the bending moment and tension force.

Step 9: Check the beam for bending alone, in case the tension force were not there. Use the available moment strength from Step 3.

$$\frac{M_r}{M_c} = \frac{290}{290} = 1.0 \le 1.0$$

So the beam would just be adequate. In cases where the application of the tension force increases interaction strength and that force may not actually occur, it is important to check the member for flexure alone.

The W16×77 is adequate to carry the applied loads.

8.14 PROBLEMS

Unless noted otherwise, all columns should be considered pinned in a braced frame out of the plane being considered in the problem with bending about the strong axis.

1. Determine whether a W14×90, A992 column with a length of 12.5 ft is adequate in a braced frame to carry the following loads from a first-order analysis: a compressive dead load of 100 kips and live load of 300 kips, a dead load moment of 30 ft-kips and live load moment of 70 ft-kips at one end, and a dead load moment of 15 ft-kips and a live load moment of 35 ft-kips at the other. The member is bending in reverse curvature about the strong axis. Determine by (a) LRFD and (b) ASD.

2. A W12×58, A992 is used as a 14 ft column in a braced frame to carry a compressive dead load of 60 kips and live load of 120 kips. Will this column be adequate to carry a dead load moment of 30 ft-kips and live load moment of 60 ft-kips at each end, bending the column in single curvature about the strong axis? The analysis results are from a first-order analysis. Determine by (a) LRFD and (b) ASD.

3. Determine whether a W12×190, A992 column with a length of 22 ft is adequate in a braced frame to carry the following loads from a first-order analysis: a compressive dead load of 300 kips and live load of 500 kips, a dead load moment of 50 ft-kips and live load moment of 100 ft-kips at one end, and a dead load moment of 25 ft-kips and a live load moment of 50 ft-kips at the other. The member is bending in reverse curvature about the strong axis. Determine by (a) LRFD and (b) ASD.

4. A W10×60, A992 is used as a 13 ft column in a braced frame to carry a compressive dead load of 74 kips and live load of 120 kips. Will this column be adequate to carry a dead load moment of 30 ft-kips and live load moment of 45 ft-kips at each end, bending the column in single curvature about the strong axis? The analysis results are from a first-order analysis. Determine by (a) LRFD and (b) ASD.

5. Given a W14×500, A992 42 ft column in a braced frame with a compressive dead load of 90 kips and live load of 270 kips. Maintaining a live load to dead load ratio of 3, determine the maximum live and dead load second-order moments that can be applied about the strong axis on the upper end when the lower end is pinned by (a) LRFD and (b) ASD.

6. Given a W14×132, A992 15 ft column in a braced frame with a compressive dead load of 350 kips and live load of 350 kips, and maintaining a live load to dead load ratio of 1, determine the maximum live and dead load second-order moments that can be applied about the strong axis on the upper end when the lower end is pinned by (a) LRFD and (b) ASD.

7. Reconsider the column and loadings in Problem 1 if that column were bent in single curvature by (a) LRFD and (b) ASD.

8. Reconsider the column and loadings in Problem 2 if that column were bent in reverse curvature by (a) LRFD and (b) ASD.

9. Reconsider the column and loadings in Problem 3 if that column were bent in single curvature by (a) LRFD and (b) ASD.

10. Reconsider the column and loadings in Problem 4 if that column were bent in reverse curvature by (a) LRFD and (b) ASD.

11. A 14 ft pin-ended column in a braced frame must carry a compressive dead load of 85 kips and live load of 280 kips, along with a uniformly distributed transverse dead load of 0.4 kips/ft and live load of 1.3 kips/ft. Will a W14×68, A992 member be adequate if the transverse load is applied to put bending about the strong axis? Determine by (a) LRFD and (b) ASD.

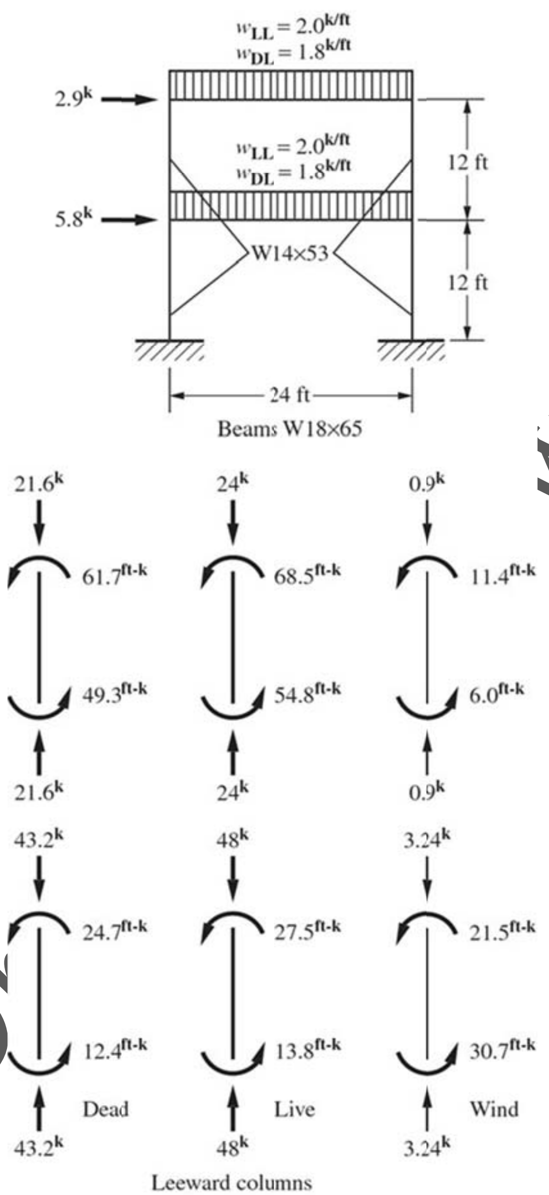
12. A pin-ended chord of a truss is treated as a member in a braced frame. Its length is 12 ft. It must carry a compressive dead load of 90 kips and live load of 170 kips, along with a uniformly distributed transverse dead load of 1.1 kips/ft and live load of 2.3 kips/ft. Will a W8×58, A992 member be adequate if the transverse load is applied to put bending about the strong axis? Determine by (a) LRFD and (b) ASD.

13. A moment frame is designed so that under a service lateral load H= 150 kips, the frame drifts no more than L/400. There are a total of 15 columns in this frame, so P_{story} is 15 times the load on this column. A 13 ft, W14×120, A992 column is to be checked. Analysis results are from a first-order analysis. The column is called upon to carry a compressive dead load of 100 kips and live load of 300 kips. This load will be taken as coming from a no-translation analysis. The top of the column is loaded with no-translation dead load moment of 25 ft-kips and a no-translation live load moment of 80 ft-kips. The translation moments applied to that column end are a dead load moment of 35 ft-kips and a live load moment of 100 ft-kips. The lower end of the column feels half of these moments. The column is bending in reverse curvature about the strong axis. Will the W14×120, A992 member be adequate to carry this loading? Analysis shows that the effective length factor in the plane of bending is 1.66. Determine by (a) LRFD and (b) ASD.

14. A W14×193, A992 member is proposed for use as a 12.5 ft column in an moment frame. The frame is designed so that under a service lateral load H= 120 kips, the frame drifts no more than L/500. The total story load, P_{story}, is 20 times the individual column load. Analysis results are from a first-order analysis. Will this member be adequate to carry a no-translation compressive dead load of 160 kips and live load of 490 kips? The top of the column is loaded with a no-translation dead load moment of 15 ft-kips and a no-translation live load moment of 30 ft-kips. The translation moments applied to that column end are a dead load moment of 80 ft-kips and a live load moment of 250 ft-kips. The column is bending about the strong axis and, the lower end of the column is considered pinned, and the effective length factor is taken as 1.5. Determine by (a) LRFD and (b) ASD.

15. Will a W14×48 be adequate as a 14 ft column in a moment frame with a compressive dead load of 35 kips and live load of 80 kips? One half of this compressive load is taken as a no-translation load and one half as a translation load. The top and bottom of the column are loaded with a no-translation dead load moment of 20 ft-kips and a no-translation live load moment of 55 ft-kips. The translation moments applied to the column ends are a dead load moment of 10 ft-kips and a live load moment of 50 ft-kips. Analysis results are from a first-order analysis. The frame is designed so that under a service lateral load H= 50 kips, the frame drifts no more than $L/300$. The total story load, P_{story}, is eight times the individual column load. The column is bent in reverse curvature about the strong axis, and K_x = 1.3. Determine by (a) LRFD and (b) ASD.

16. Determine whether a 10 ft braced frame W14×43, A992 column can carry a compressive dead load of 35 kips and live load of 80 kips along with a dead load moment of 20 ft-kips and live load moment of 40 ft-kips. One half of these moments are applied at the other end, bending it in single curvature.



17. A two-story single bay frame is shown in Figure P8.17. The uniform live and dead loads are indicated along with the wind load. A first-order elastic analysis has yielded the results shown in the figure for the given loads and the appropriate notional loads. Assuming that the story drift is limited to height/300 under the given wind loads, determine whether the first- and second-story columns are adequate. The gravity loads will produce the no-translation results and the wind load will produce the translation results. The members are shown and are all A992 steel. Determine by (a) LRFD and (b) ASD.

18. Determine whether the columns of the two-bay unbraced frame shown in Figure P8.18 are adequate to support the given loading. Results for the first-order analysis are provided. The gravity loads will produce the no-translation results and the wind load will produce the translation results. Assume that the lateral drift under the given wind load will be limited to a maximum of 0.5 in. All members are A992 steel and the sizes are as shown. Determine by (a) LRFD and (b) ASD.

P8.17

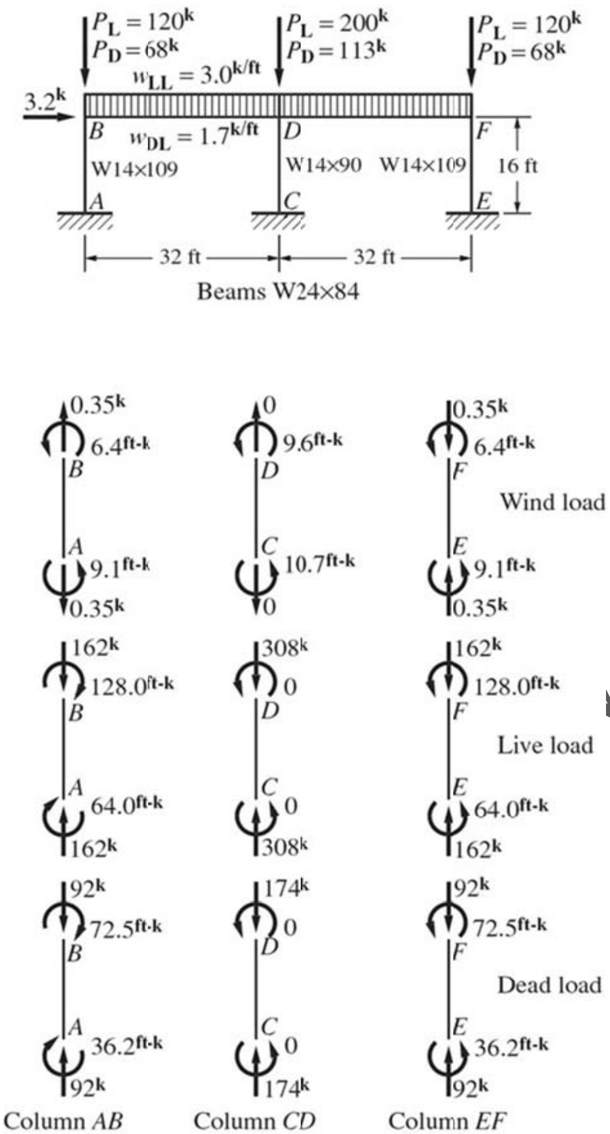
19. A nonsymmetric two-bay unbraced frame is required to support the live and dead loads given in Figure P8.19. Using the results from the first-order elastic analysis provided, assuming the axial forces provided are from the no-translation analysis and the lateral drift due to a 5 kip force is limited to 0.4 in., determine whether each column will be adequate. All members are A992 steel and the sizes are as shown. Determine by (a) LRFD and (b) ASD.

P8.18

20. A 14 ft column in a moment frame must carry a compressive load of 540 kips and a moment about the strong axis of 135 ft-kips from an LRFD second-order direct analysis. Will a W14×74, A992 member be adequate if the moment is applied to put bending about the strong axis?

21. A 14 ft column in a moment frame must carry a compressive load of 360 kips and a moment about the strong axis of 90 ft-kips from an ASD second-order direct analysis. Will a W14×74, A992 member be adequate if the moment is applied to put bending about the strong axis?

22. A 28 ft column in a moment frame must carry a compressive load of 110 kips and moment about the strong axis of 100 ft-kips from an LRFD second-order direct analysis. Will a W10×60, A992 member



be adequate if the moment is applied to put bending about the strong axis?

23. A 28 ft column in a moment frame must carry a compressive load of 73 kips and moment about the strong axis of 67 ft-kips from an LRFD second-order direct analysis. Will a W10×60, A992 member be adequate if the moment is applied to put bending about the strong axis?

24a. Select a W-shape for a column with a length of 15 ft. The results of a second-order direct analysis indicate that the member must carry a force of 700 kips and a moment of 350 ft-kips. Design by LRFD.

24b. Select a W-shape for a column with a length of 15 ft. The results of a second-order direct analysis indicate that the member must carry a force of 467 kips and a moment of 230 ft-kips. Design by ASD.

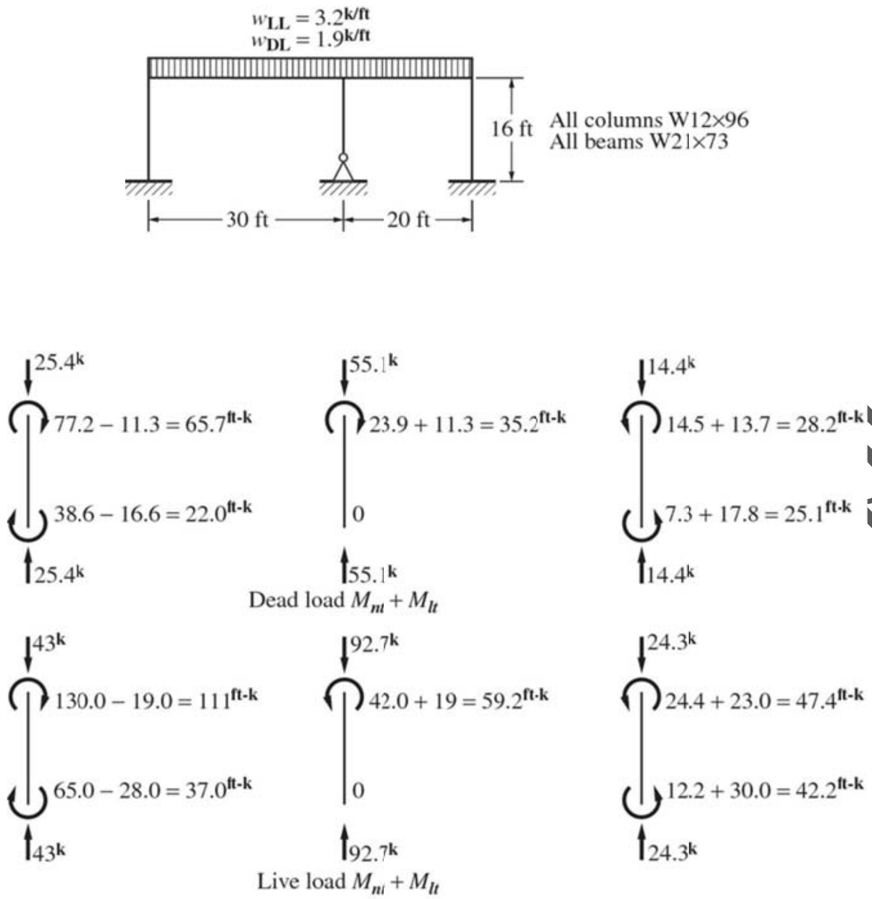
25a. Select a W-shape for a column with a length of 28 ft. The results of a second-order direct analysis indicate that the member must carry a force of 1100 kips and a moment of 170 ft-kips. Design by LRFD.

25b. Select a W-shape for a column with a length of 28 ft. The results of a second-order direct analysis indicate that the member must carry a force of 730 kips and a moment of 110 ft-kips. Design by ASD.

26a. Select a W-shape for a column with a length of 14 ft. The results of a second-order direct analysis indicate that the member must carry a force of 350 kips and a moment of 470 ft-kips. Design by LRFD.

26b. Select a W-shape for a column with a length of 14 ft. The results of a second-order direct analysis indicate that the member must carry a force of 230 kips and a moment of 320 ft-kips. Design by ASD.

27a. Select a W-shape for a column with a length of 16 ft. The results of a second-order direct analysis indicate that the member must carry a force of 1250 kips and a moment of 450 ft-kips. Design by LRFD.



P8.19

27b. Select a W-shape for a column with a length of 16 ft. The results of a second-order direct analysis indicate that the member must carry a force of 830 kips and a moment of 300 ft-kips. Design by ASD.

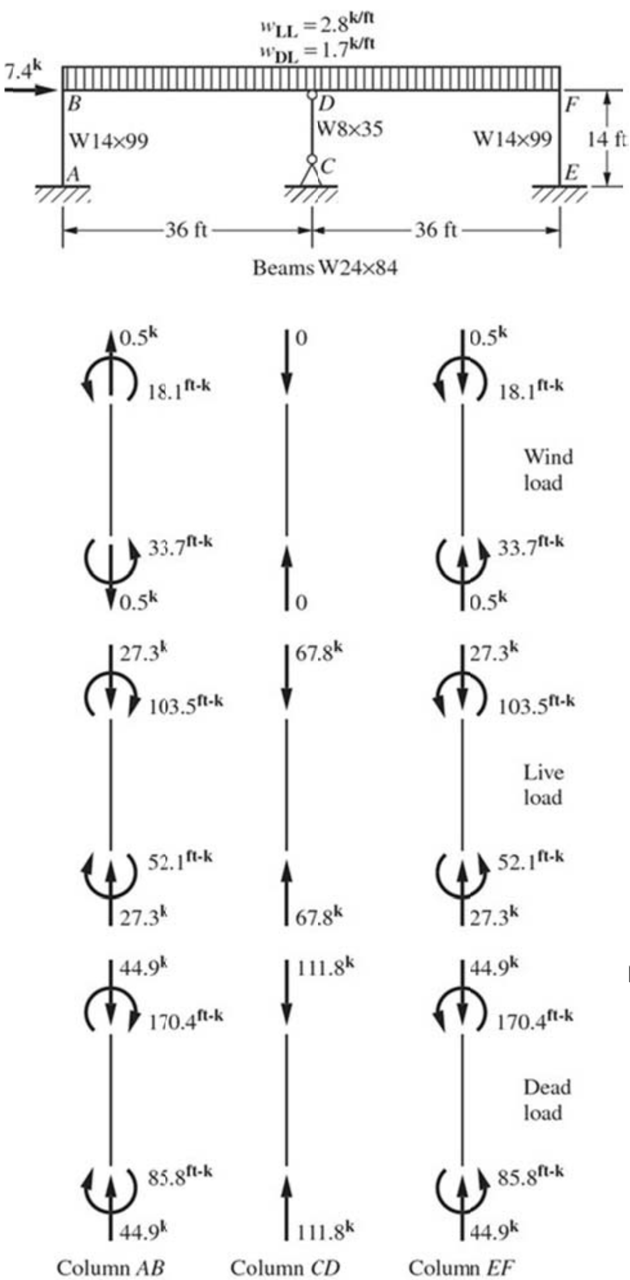
28. The two-bay moment frame shown in Figure P8.28 contains a single leaning column. The results of a first-order elastic analysis for each load are given. Determine whether the exterior columns are adequate to provide stability for the frame under dead and live load. All W-shapes are given and the steel is A992. Determine by (a) LRFD and (b) ASD.

29. The two-story frame shown in Figure P8.29 relies on the left-hand columns to provide stability. Using the first-order analysis results shown, determine whether the given structure is adequate if the steel is A992.

30. The two-bay, two-story frame shown in Figure P8.30 is to be designed. Using the Live, Dead, Snow, and Wind Loads given in the figure, design the columns and beams to provide the required strength and stability by (a) LRFD and (b) ASD.

31. Select an A36 rod to provide the point bracing shown in the center panel of the three-bay frame of Figure 8.9a to provide stability for a total gravity dead load of 150 kips and live load of 60 kips. Design by (a) LRFD and (b) ASD.

32. Select an A36 rod to provide the point bracing shown in the center panel of the three-bay frame of Figure 8.9a to provide stability for a total gravity dead load of 180 kips and live load of 95 kips. Design by (a) LRFD and (b) ASD.

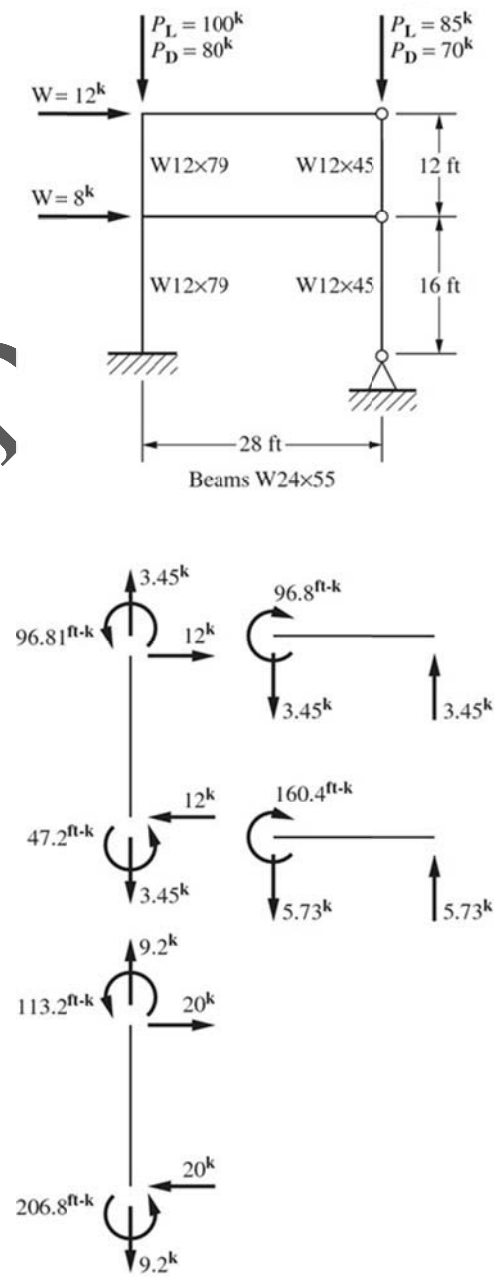


P8.28

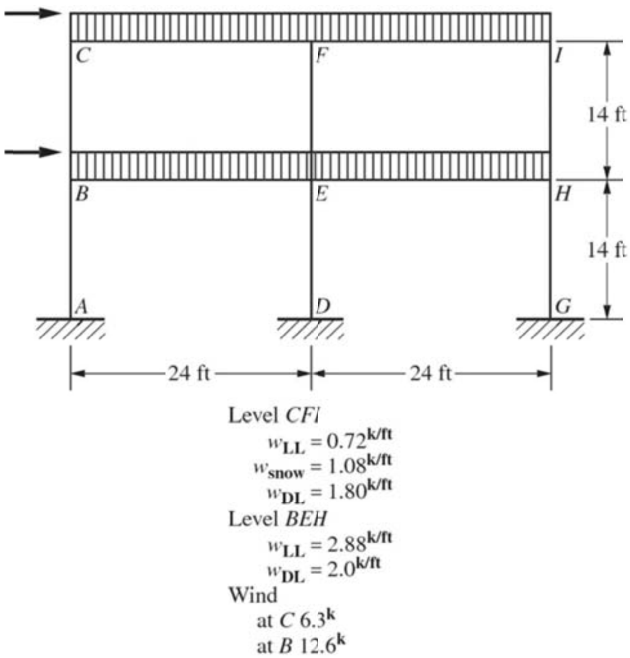
33. A 30 ft simply supported W24×68 beam has point bracing at third points. The beam supports a uniformly distributed dead load of 1.2 kips/ft and a uniformly distributed live load of 2.0 kips/ft. Determine the required point brace strength and the required point brace stiffness by (a) LRFD and (b) ASD.

34. A 30 ft simply supported W24×68 beam has panel bracing in three equal panels over its length. The beam supports a uniformly distributed dead load of 1.2 kips/ft and a uniformly distributed live load of 2.0 kips/ft. Determine the required shear strength of the bracing system and the required panel brace stiffness by (a) LRFD and (b) ASD.

35. A simply supported W18×86 beam spans 30 ft and carries a uniformly distributed dead load of 0.8 kips/ft and a uniformly distributed live load of 2.4 kips/ft. It also carries a tension live load of 18 kips. The member is fully braced for lateral-torsional buckling. Use A992 steel. Determine if the W18×86 is adequate for the combined tension and bending by (a) LRFD and (b) ASD.



P8.29



P8.30

36. A simply supported W27×84 beam spans 30 ft and carries a uniformly distributed dead load of 1.2 kips/ft and a uniformly distributed live load of 2.5 kips/ft. It also carries a tension live load of 30 kips. The member is braced at third points for lateral-torsional buckling. Use A992 steel. Determine if the W27×84 is adequate for the combined tension and bending by (a) LRFD and (b) ASD.

37. Integrated Design Project – Effective Length Method

Lateral load resistance in the east-west direction is provided by two perimeter moment frames as seen in Figure 1.24. Before the forces in these members can be determined, the specified wind load must be determined. At this stage in the design, a simplified approach to wind load calculation similar to that used in Chapter 4 might yield the following loads at each level:

Roof	32.0 kips
4th Floor	59.0 kips
3rd Floor	54.0 kips
2nd Floor	50.0 kips
Total Wind Load	195.0 kips

The moment frames will share equally in carrying these loads. They will be designed using the effective length method of Appendix 7.2, and second-order effects will be incorporated using the amplified first-order analysis method of Appendix 8; thus, superposition may be used.

Before an analysis may be carried out, preliminary member sizes must be obtained. Using the gravity loads calculated in Chapter 2, select preliminary column and beam sizes without concern for the frame behavior of the structure.

With these member sizes, the analysis is to be carried out for dead load, live load, roof load, and wind load. Members are to be selected for the gravity plus wind load combination, so there should be no need to include notional loads.

Design the columns and beams for the resulting load effects and redo the analysis to check the strength of these new members and the drift of the structure. Confirm that the effective length method may be used.

38. Integrated Design Project – Direct Analysis Method

Lateral load resistance in the east-west direction is provided by two perimeter moment frames as seen in Figure 1.24. Before the forces in these members can be determined, the specified wind load must be determined. At this stage in the design, a simplified approach to wind load calculation similar to that used in Chapter 4 might yield the following loads at each level:

Roof	32.0 kips
4th Floor	59.0 kips
3rd Floor	54.0 kips
2nd Floor	50.0 kips
Total Wind Load	195.0 kips

The moment frames will share equally in carrying these loads. They will be designed using the Direct Analysis Method from Chapter C.

Before an analysis may be carried out, preliminary member sizes must be obtained. Using the gravity loads calculated in Chapter 2, select preliminary column and beam sizes without concern for the frame behavior of the structure.

With these member sizes, the analysis is to be carried out for dead load, live load, roof load, and wind load, following the general analysis requirements of Section C2. Members are to be selected for the gravity plus wind load combination, so there should be no need to include notional loads.

Design the columns and beams for the resulting load effects and redo the analysis to check the strength of these new members and the drift of the structure.