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## Pre-programmed Failure Behavior using Biology-Inspired Structures

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### ABSTRACT

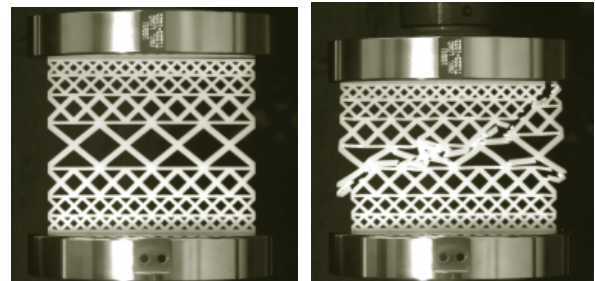
Core (filler) materials are key components of the sandwich panel and box-beams that are used in the design of lightweight structures. They perform a variety of elastic-range functions such as transferring and supporting working stresses and energy and collapse management. There is an increasing demand, however, for post-yield performance characteristics such as buckling control, impact toughness, and maintenance of component strength after damage. Low density is also an important consideration, as overall component mass is critical in most applications. These cellular solids need to perform well under normal working stress conditions, yet still resist damage from simple and unavoidable low velocity impacts.

A new design approach is suggested by biological systems that have evolved for toughness and damage tolerance (bones, trees, plants, corals, etc.). These systems share the relatively low density cellular arrangements of common synthetic core materials, but also exhibit variable density gradients within the core. (Figures 1 and 2) This paper describes engineering design methods that are inspired by such biology. The result is that a design's failure modes can be more effectively "designed-in", controlling locations and amounts of failure.

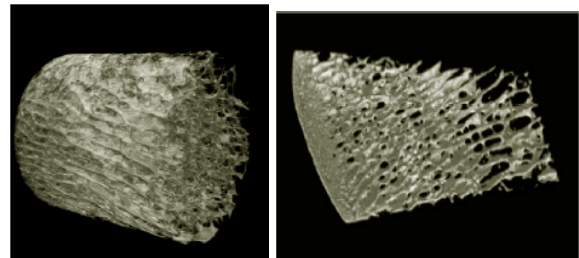
### INTRODUCTION

Biology serves as a very capable inspiration for the design of mechanical structures. Inspired by such biology, we are using interactive graphics and visualization methods to evaluate non-homogeneous microtruss structures. We are developing numerical approaches with spatial mapping of density variations and optimization strategies. Figure 2 shows volume renderings of trabecular bone. We have fabricated those volumes on a layered manufacturing system to

test and observe their micro-behavior under stress. (Figure 3) We are now mimicking that inhomogeneity using novel distribution functions, such as the radial density functions shown here, and a variety of interactive graphics techniques. The result will be novel arrangements of microtruss core designs that optimize the damage tolerance and toughness of core-filler structures.



**Figure 1:** Unstressed and stressed variable density structures.



**Figure 2:** A: a core of trabecular bone. B: slab extracted from the core.

### PREVIOUS WORK

Despite proven strength-to-weight and stiffness-to-weight characteristics, lightweight sandwich panels suffer from sensitivity to low-velocity impact damage. Yet impact is unavoidable in the vehicles and structures that can potentially make the best use of them. Simple tool drops and runway debris plague airframe applications, low-speed impact with docks

and floating debris is common for watercraft, and any light-weight structure will encounter impacts during assembly and service. Panels designed for the most demanding performance applications are generally composed of fiber reinforced polymer facesheets bonded to a core of honeycomb or structural foam. Two related issues are of general interest: 1) impact resistance (limiting damage induced in the facesheet and/or core), and 2) compression-after-impact strength (residual strength maintained after a damaging impact event).

Many studies have made small alterations to the facesheet and/or core in order to improve impact resistance. For example, altering the weave (twill versus plain) of facesheet carbon and glass fabric has been shown to improve impact resistance.<sup>26</sup> Altering fiber type (glass vs. carbon) and polymer matrix (polyester vs. epoxy) also affects impact resistance.<sup>53</sup> Compression after impact strength can be improved by varying the facesheet and core thicknesses in carbon/aluminum honeycomb systems.<sup>19</sup>

Other techniques of improving damage resistance have involved more significant changes to the standard facesheet/core system. Partially filling the cells of a paper fiber honeycomb core can increase the absorption of impact energy.<sup>60</sup> Compliant or compressible sub-layers inserted between the facesheets and core has been shown to make watercraft more tolerant to typical impacts.<sup>57</sup> The core can also be partitioned, either through insertion of tough material layers within the core<sup>27</sup>, or by creating a multi-core construct with periodic insertion of additional facesheets.<sup>62</sup>

Many methods, both analytical and experimental, have been developed to evaluate sandwich panel performance. Numerical simulation using finite element analysis, with nonlinear geometry, delamination, and fiber/matrix failures have been conducted.<sup>46,50,67</sup> Ability to accurately predict damage zones and energy absorption has been verified.<sup>45</sup> Experimental investigations have included examination of core yielding and facesheet wrinkling<sup>48</sup>, the relationship between damage and dent-depth<sup>61</sup>, the effects of indenter shape and loading rate<sup>1</sup>, damage area and energy absorption for curved versus flat plates<sup>35</sup>, damage detection with embedded optical fibers<sup>59</sup>, and fracture after impact in aluminum foam core structures.<sup>36</sup>

Structural optimization using traditional gradient-based methods has been used in numerous settings

related to sandwich panels. Many applications seek a specific component shape that will maintain elastic performance with minimum mass<sup>52</sup>, or the optimal dimensions of structural elements in truss assemblies.<sup>47</sup> Other research has sought component shapes or structural configurations that optimize failure response with elastoplastic materials<sup>44</sup>, or in the presence of nonlinear geometric behavior.<sup>66</sup> Composite panels have been extensively studied for optimization of stiffener dimensions and spacing for elastic performance<sup>22</sup>, resistance to buckling<sup>63</sup>, and ability to withstand impact damage.<sup>64</sup> Lightweight truss core panels have been subjected to optimization of truss element and facesheet dimensions.<sup>42</sup> Some attempts have also been made to optimize material characteristics, either by altering material elastic symmetry<sup>23</sup>, or by varying material property distributions in the context of component shape.<sup>49</sup>

In addition to traditional optimization techniques, evolutionary strategies have been exploited to a limited extent for structural mechanics problems. Component shape optimization has been conducted by combining genetic algorithms with both finite element and boundary element representations, for both single<sup>2,37</sup> and multiple<sup>43</sup> objective functions. Large-scale modular framework structures and trusses have also been subjected to optimization by evolutionary techniques.<sup>20</sup> Stochastic algorithms, which share characteristics with the mutation operations in evolutionary strategies, have been employed in topology optimization.<sup>38</sup> An evolutionary strategy has been employed to perform identification of structural parameters from experimental data.<sup>21</sup>

## Inspiration from Biological Systems

In isolation from other design criteria, toughness and damage tolerance are not particularly difficult to achieve, they simply demand more material, with the associated increases in cost and decrease in other performance characteristics. But cost is always an issue, and the greater weight and bulk of simply adding more material is acceptable only to a degree. The real difficulty, and an enduring challenge of both structural materials development and mechanical design, is to increase the toughness of lighter, more agile and less bulky systems. For this, natural systems provide strong inspiration.

Toughness is, perhaps, the most outstanding mechanical feature of natural systems.<sup>18</sup> Despite the existence of many synthetic alternatives, for example,

leather and wood are still extremely useful materials, largely because of their ability to tolerate damage. Insect cuticle provides tough armor for some of the most successful groups of animals on the planet. The shells of marine mollusks are extremely tough and provide survivability in mechanically demanding settings such as inter-tidal zones. Mineralized bones in the many animals that utilize them provide mechanically efficient yet durable structural support.

But it is a mistake to view natural materials in too simple of a light. Many have searched in vain for simple, dense, monotonic materials with equivalent properties. The distinguishing features of natural materials are the complex micro-architectures and hierarchical arrangements that they exhibit. Compositing of various kinds, layering, gradations in composition and density, macroscopic porosity, discrete and anisotropic arrangements of small-scale structural arrangements – all can be found in tough natural systems. In fact simple materials, despite a strong tendency toward economy in evolved systems, do not seem to exist in structural roles within the natural world.

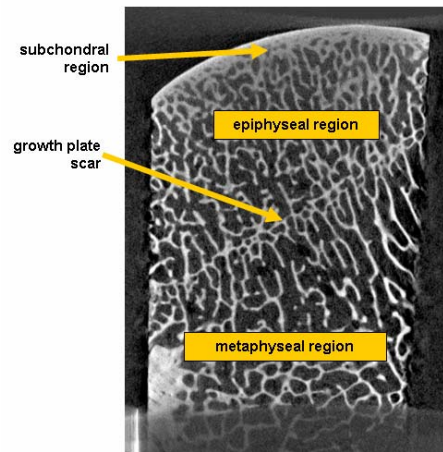
A critical aspect of using biological systems as inspiration for synthetic systems is identification of the key contributors to toughness and damage tolerance since the full complexity of natural materials cannot be duplicated with synthetic production methods. In the case of trabecular bone (the highly porous tissue “core” of human and animal bones), there are many levels of spatial hierarchy that could contribute to material toughness. In this proposal we are evaluating the highest level: the micro-architectural arrangement of struts and plates that compose the tissue. It is not that organization at finer spatial scales (lamellar arrangements within individual struts, collagen/mineral compositing within the lamellae) is unimportant. But the microarchitectural scale of trabecular bone has already been clearly associated with macroscopic component behavior<sup>8,9,16,17,58,65</sup>, and it is the most readily controllable in current prototyping systems and potentially in production systems. We also have considerable experience with modeling micromechanical behavior at this spatial scale.<sup>39-41</sup>

This approach is distinct from the many attempts to optimize traditional sandwich and stiffened panel constructs based on manipulations of facesheet, core, and stiffener characteristics. These attempts adjust dimensions and component material properties, but do not exploit inhomogeneity within any given

component. We believe that this constraint is in fact why damage tolerance has been resistant to improvement even in “optimized” systems. The optimization has an implied constraint of homogeneity that significantly influences system performance. We believe fundamentally improved characteristics will emerge by freeing this implied constraint.

### Damage Tolerance Features of Bones

A closer look at a typical skeletal component clearly illustrates the complexity of structural biological systems. In a study related to the treatment of osteoporosis, co-author Bay performed detailed imaging studies of the sheep proximal humerus (the forelimb bone at the shoulder joint). In quadrupeds the shoulder joints support approximately 60% of body weight and are therefore major structural components of the skeleton. The imaging was performed using high-resolution x-ray tomography equipment<sup>10</sup> capable of resolving in detail the microarchitecture of the high-porosity trabecular bone tissue within the joint.



**Figure 3:** Vertical section of a sheep proximal humerus, from x-ray microtomography, showing variations in density and orientation of the microarchitectural elements

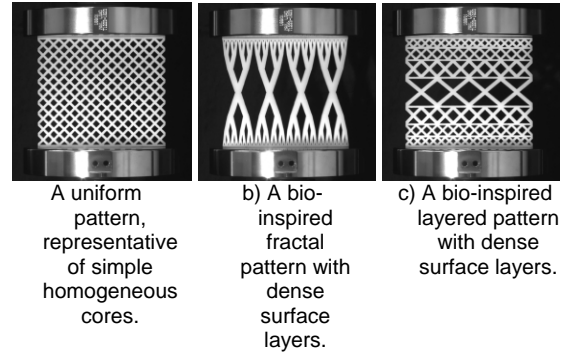
Distinct regions are identifiable in the tomographic image (Figure 3)). Deep in the joint surface at the top of the image is a subchondral (“beneath the cartilage”) region of very dense trabecular bone supporting an extremely thin bone tissue shell. The epiphyseal (“upon the growth plate”) region exhibits a gradual, fractal-like transition to a more open, less dense microarchitecture. The growth plate itself, no longer active and appearing as a “scar” in mature animals, is a plane of tissue forming a discrete boundary with the deep metaphyseal (“adjacent to the growth plate”)

region. Here the tissue is much more open, with long, slender trabecular elements and relatively few lateral connections.

It is difficult to rationalize these variations on the basis of efficient performance in the elastic range. Without consideration of damaging loads at the joint surface, there is no reason for trabecular bone tissue to appear different from large-cell foam core materials typical of synthetic sandwich assemblies.<sup>12,13</sup> A uniform tissue could have evolved with adequate strength to support anticipated working loads and minimum weight. Development and maintenance of such a tissue would presumably be more efficient. But as with synthetic components designed on this basis, unanticipated impacts would produce localized damage at the surface of the component. The resulting shape alterations and reduced strength are known to cause rapid degeneration of joint function in a process known as post-traumatic arthritis.

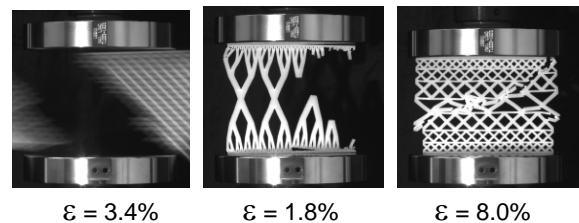
#### Microarchitecture has a strong influence on failure characteristics of low-density materials

With elastic-range performance considerations alone, there is little benefit to an inhomogeneous distribution of material within the core. An experiment was conducted to evaluate whether inhomogeneous distributions could substantially affect failure and post-yield characteristics for a fixed amount of material. Three microarchitectural patterns were developed using standard computer-aided design tools (Pro/Engineer, PTC Corp) and translated into testable samples via rapid prototyping (Dimension BST, Stratasys Inc.). The first pattern was a uniform orthogonal grid, representative of homogeneous core materials (Figure 4a). The other patterns embodied specific features inspired by detailed observations of bone tissue: 1) fractal branching from a dense surface layer (Figure 4b), and 2) discrete layering with transitions to regions of different densities (Figure 4c). The samples were 120 mm wide x 120 mm tall, and 12 mm deep. They were produced in polystyrene by fused deposition modeling, and all had identical mass. Testing was conducted on a standard screw-driven mechanical testing system (Instron model 5567, Instron Corp.). Samples were compressed in the vertical direction at a rate of 0.1 mm/sec (quasi-static loading) until loss of structural integrity. Load and deflection were recorded throughout, and digital images were acquired approximately every 30 seconds during loading.



**Figure 4.** Uniform and bio-inspired samples generated by rapid prototyping for mechanical testing.

The samples demonstrated three distinct behaviors (Figure 5). The uniform sample had relatively low strength and failed in a rapid, catastrophic manner. A single strut failure initiated near one corner, and immediately propagated along a plane of high shear stress. The fractal sample was 2.5 times stronger than the uniform sample, but it also failed in an abrupt manner, with a large piece breaking out of one corner. The layered sample had strength similar to the uniform sample, but failed in a much more controlled manner. An initial strut failure near the upper-right corner developed, but was diverted into the low-density middle layer. This layer collapsed in a controlled manner. The sample was damaged, but remained intact and continued to support load far beyond the initial failure. These samples clearly demonstrate that, for a given mass of material, very distinct failure and collapse behaviors can emerge with different microarchitectures. This proposal aims to optimize these patterns for damage tolerance and post-yield performance criteria.



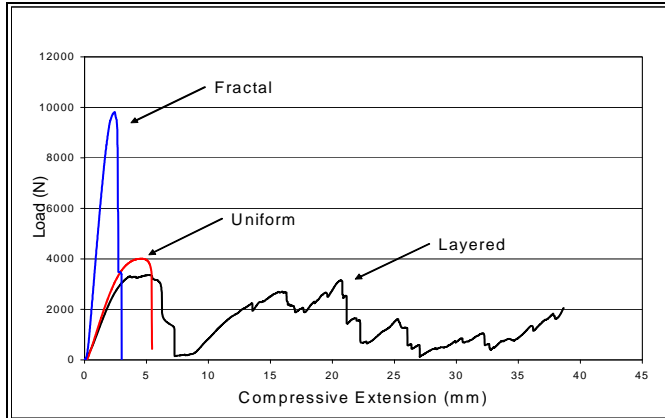


Figure 5. Load-deflection plots for the three samples

### Geometric mapping with simple control parameters to generate pattern variations

We have approached this in two steps:

#### 1. Use volume visualization to examine and understand the varying density of bone structure.

To accurately mimic biological structures we must first understand them. This is difficult because bone structures are typically scanned as uniform rectilinear volumes which are less straightforward to graphically display than line and polygon data. We have developed a volume exploration program known as *vx* (Figure 6) which runs on a PC with special graphics hardware.<sup>5</sup> *vx* provides a number of interactive controls for the display of volumes. Because it is hardware accelerated, it performs this interaction in real-time and allows true stereo viewing with shutter glasses, dual projectors, and a stereo mirror system.

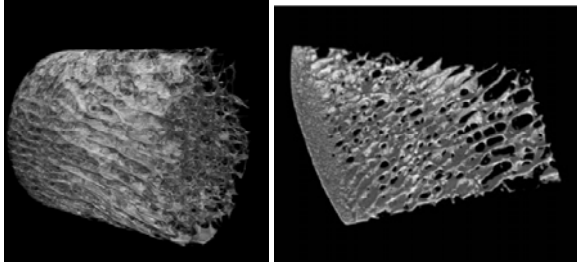


Figure 6: L: A trabecular bone core, R: A slab extracted from the core

Even complex image content, such as trabecular bone, displays accurately and rapidly in *vx* (Figure 6). This example of a bone core from the sheep proximal humerus (upper forelimb) consists of 156 million (500 x 500 x 624) individual volume elements, or voxels. Through *vx*, it is displayed at an interactive rate of 10-20 frames per second. In the bone image, some of the data values, such as the scanning background noise, have been interactively eliminated. Also, lighting

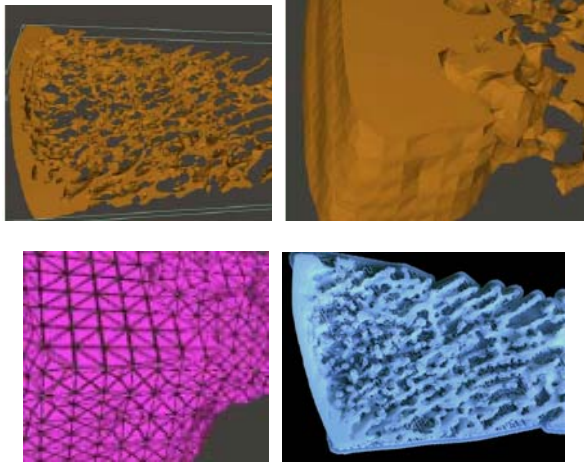
parameters have been adjusted to better reveal the surface detail.

#### 2. Fabricate test samples from actual bone data and test for toughness and damage transmission.

Isosurfaces are a common visualization display technique used to identify the locus where a particular scalar value exists within a volume dataset. When requesting an isosurface, a single scalar value,  $S^*$  must be given. A manufacturable *isovolume* must be a legal solid, which means that it must be faceted on all sides.<sup>4</sup> In transforming a volume dataset into an isovolume in manufacturable form, two scalar values,  $S^*_{min}$  and  $S^*_{max}$  must be specified. “Manufacturable form”, in this sense, means converting the desired portion of the volume into an STL file. The STL file format is an industry-wide *de facto* standard for communicating geometry to rapid prototyping machines. It is essentially a list of 3D triangles. Rapid prototyping machines are based on layer-by-layer or drop-by-drop methods. They are too slow for mass production, but their simplicity of use is ideal for “one-of” part production, such as we are doing here.

Thus, as part of this project, actual bone microarchitectures have been converted into physical polystyrene models that we can test and observe. Converting a volume dataset, such as the bone structure (Figure 7a), and the two bounding isovalues into a manufacturable isovolume is a two-step process: 1) compute the isosurface corresponding to each isovalue, and 2) at the boundaries of the volume, cap the gaps between the isosurfaces.<sup>3,6</sup> This STL file consists of approximately 100,000 triangles, although that number of triangles could easily be made larger through adaptive subdivision. Zooming in on a corner of this model (Figures 7b and 7c) shows how it has been triangulated for prototyping. It is crucial that each triangle edge exactly matches up with a complete edge from an adjacent triangle so that there will be no cracks in the model. If just the inside or just the outside is desired, the value of  $S^*_{min}$  can be set to  $-\infty$  or the value of  $S^*_{max}$  can be set to  $+\infty$ .

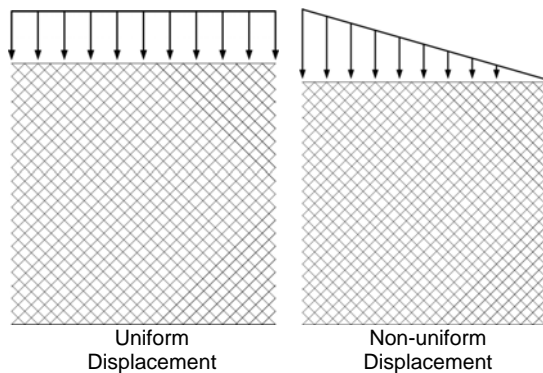




**Figure 7, Top-to-Bottom, Left-to-Right:** **A:** slab of trabecular bone as a manufacturable subvolume, **B:** detail of the subvolume in solid shaded form, **C:** details of the surface triangularization, **D:** fabricated plastic model

## Results

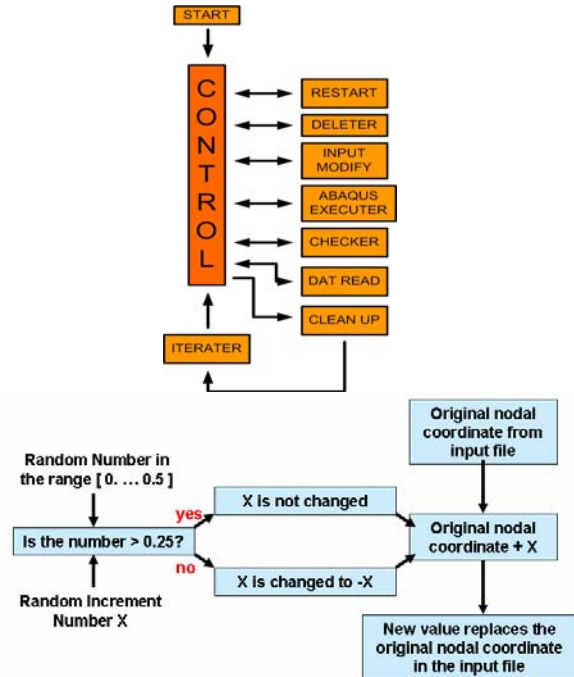
In order to investigate the feasibility of directing material architecture development using genetic algorithms we created simple uniform truss models and applied compressive loading in two modes (Figure 8): uniform and linearly varying (to simulate indentation). Load levels were great enough to induce substantial plastic strain in truss elements near the surface of the model. A steel plasticity representation and 2-node beam elements were used for the model which was evaluated under material and geometric nonlinear conditions.



**Figure 8:** Imposed Displacement Model Configurations

We then developed a set of scripts (Figure 9) that evaluated the models, made random modifications to nodal positions to create offspring, selected among the offspring for the lowest total plastic strain, then repeated the process until model change became

negligible. The scripts were all written in Perl and run on a small UNIX cluster. The commercial finite element code ABAQUS (Hibbit, Karlsson & Sorensen, Inc.) was used for model evaluation.

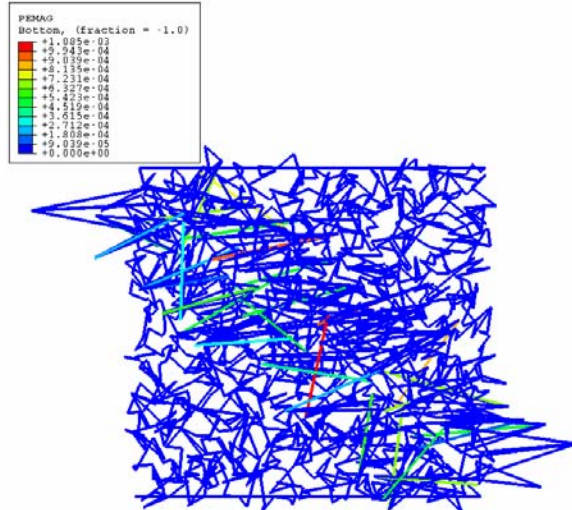


**Figure 9.** Two of the key scripts used for evolution of the truss networks through random mutation: (top) Process control script, (bottom) Generation of offspring through random modification of node locations.

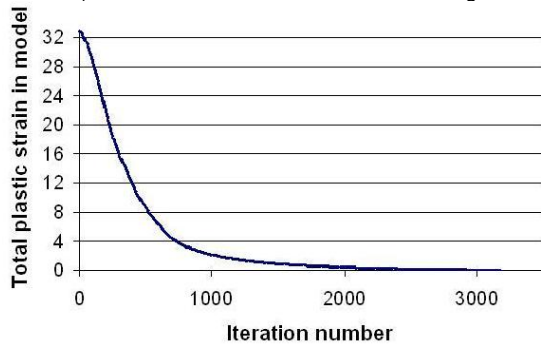
Both trusses changed substantially and achieved configurations that dramatically lowered total plastic strain in the models (Figures 10 and 11). The uniform load case produced in effect a shear band from the top-left to bottom-right corners where the individual truss elements elongated and established preferential horizontal alignments. The non-uniform loading case, which simulates an indentation, showed two responses. Near the loading surface the truss elements preferentially aligned perpendicular to the surface while maintaining near original lengths. Deep to the loading surface a discrete zone of elongated, horizontally oriented truss elements formed.

These particular truss configurations are not practical from a manufacturing standpoint. But they do clearly illustrate the main contention of this work that evolutionary algorithms can direct, through random modification and selection based on post-yield characteristics, the development of novel material microarchitectures. Implementation of these algorithms is practical on computer clusters of current

capacity, particularly if we implement our truss shape function mapping. This will greatly reduce the degrees of freedom available for optimization without sacrificing pattern generality, with the added benefit of producing more manufacturable patterns.



a) Final deformed mesh for uniform loading.



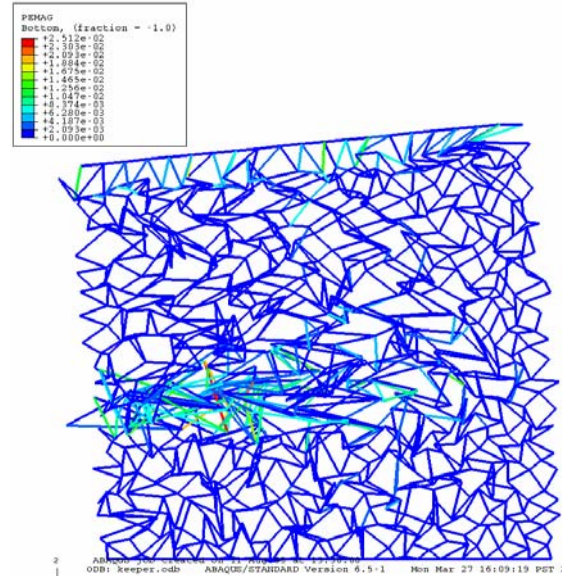
b) Evolution of total plastic strain for uniform loading (mm/mm)

**Figure 10.** Evolution of the truss architecture for the uniform loading case

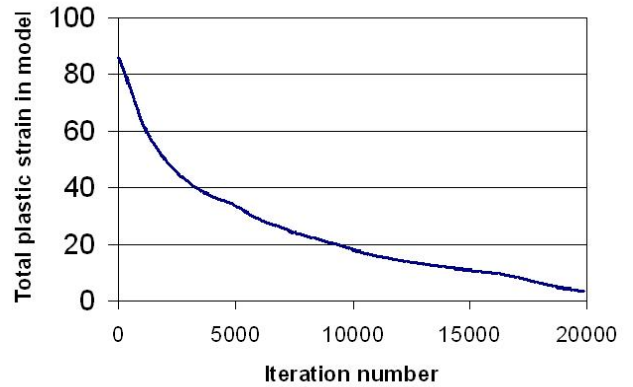
### Future Work: Mathematical Microarchitecture Functions

In addition to evaluating naturally occurring bone microarchitectures, we are generating artificial microarchitecture patterns based on geometric transformations of uniform truss structures. Our strategy is to use a small number of geometry-controlling parameters to create structural meshes with spatially varying densities and microarchitectural orientations (Figure 12). These parameters are being optimized to produce the most damage-resistant designs. One approach is based on defining “hotspots” that attract nearby portions of the mesh. The location and amount of attraction, or influence, can be varied. We have been experimenting with several mapping

functions that will give us this density-variation behavior. In addition, we can vary the resultant meshes by altering the initial mesh configuration, with cubic, rectangular, triangular, and hexagonal patterns all possibilities.

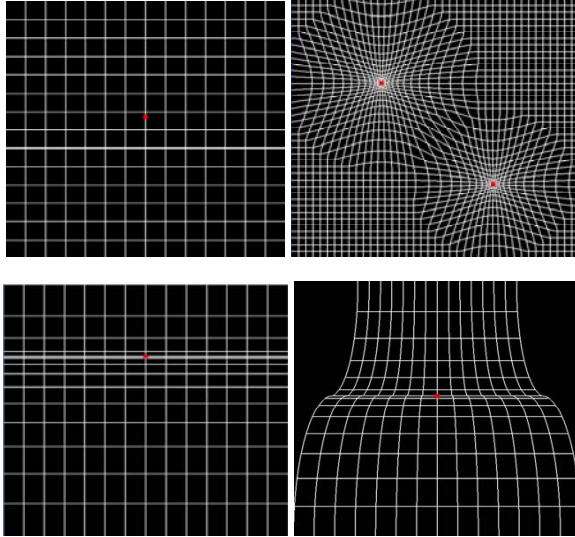


a) Final deformed mesh for non-uniform loading



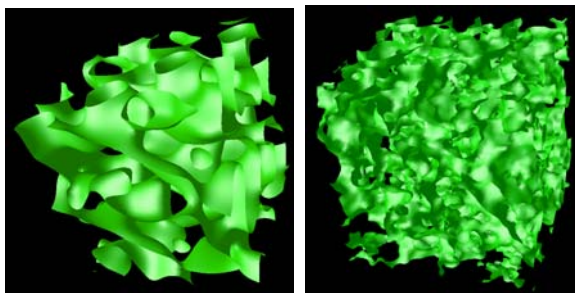
b) Evolution of total plastic strain for non-uniform loading (mm/mm)

**Figure 11.** Evolution of the truss architecture for the non-uniform loading case



**Figure 12:** (UL) an unaltered mesh in a rectangular grid pattern, (LL) linear density influence, (UR) radial density influence, (LR) cubic Hermite influence

Another approach is based on a 3D computer graphics “noise” (controlled randomness) function. Figure 13 shows two renditions of such a function, with different densities, but still mimicking the honeycomb structure of bone. (Compare these images with Figures 3 and 6b). We are experimenting with using isovolumes<sup>4</sup> to create manufacturable models from this function distribution. This will give us manufactured models that can be analyzed and tested. Figure 13 shows one of our tests of isosurfaces through the volume, both without and with higher-frequency noise components. By strategically manipulating both the isovalues and the number of octaves as a function of location, we will be able to recreate geometry distributions similar to what we see in actual bone samples.



**Figure 13:** Isosurfaces through a noise volume. From left to right, one level of noise, eight levels of noise

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