CS 261 – Data Structures

AVL Trees

Binary Search Tree

- Complexity of BST operations:
  - proportional to the length of the path from a node to the root
- Unbalanced tree: operations may be O(n)
  - E.g.: adding elements in a sorted order

Balanced Binary Search Tree

- Balanced tree: the length of the longest path is roughly $\log n$

- BALANCE IS IMPORTANT!

Complete Binary Tree is Balanced

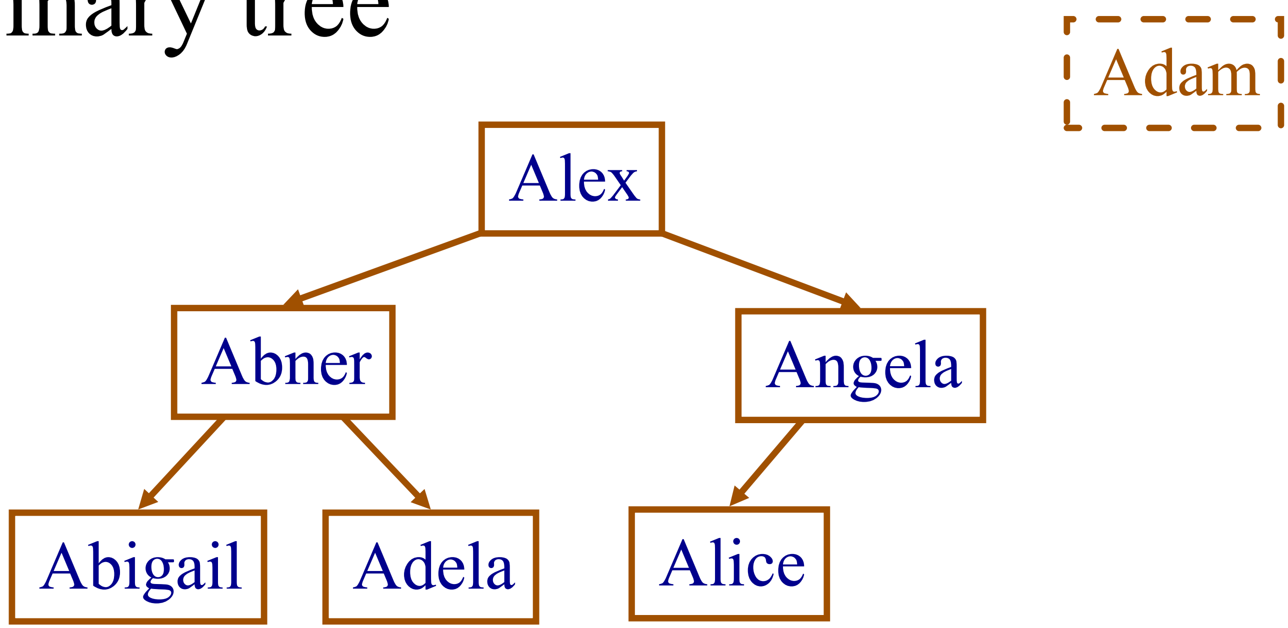
- Has the smallest height for any binary tree with the same number of nodes

- The longest path guaranteed to be $\leq \log n$

- => Keep the tree complete

Requiring Complete Trees

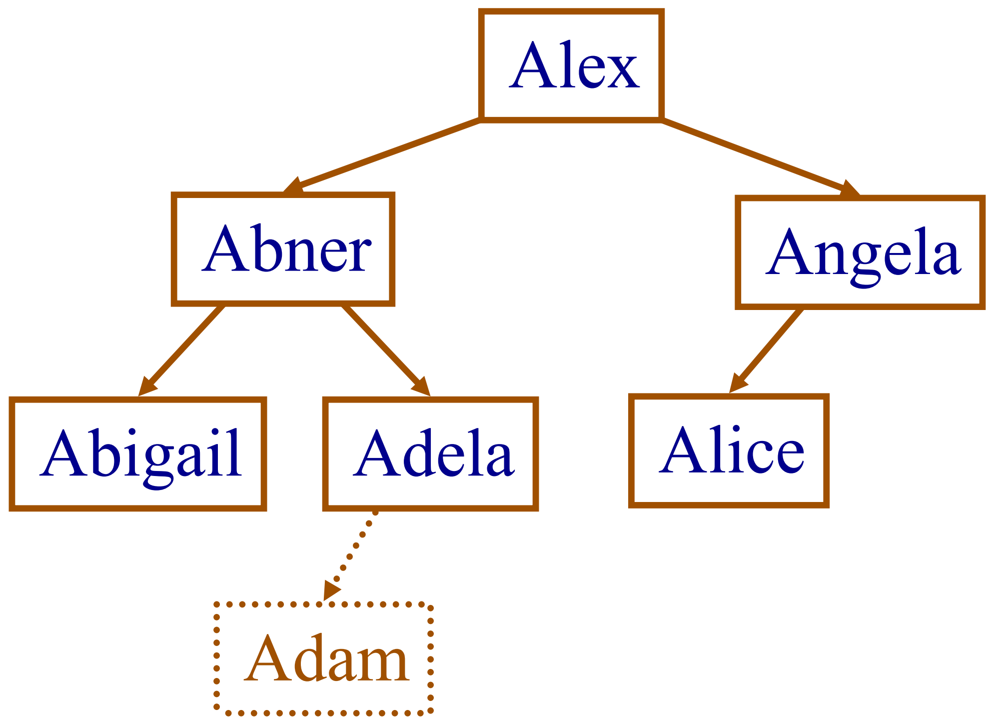
- However, it is very costly to maintain a complete binary tree



Add to tree

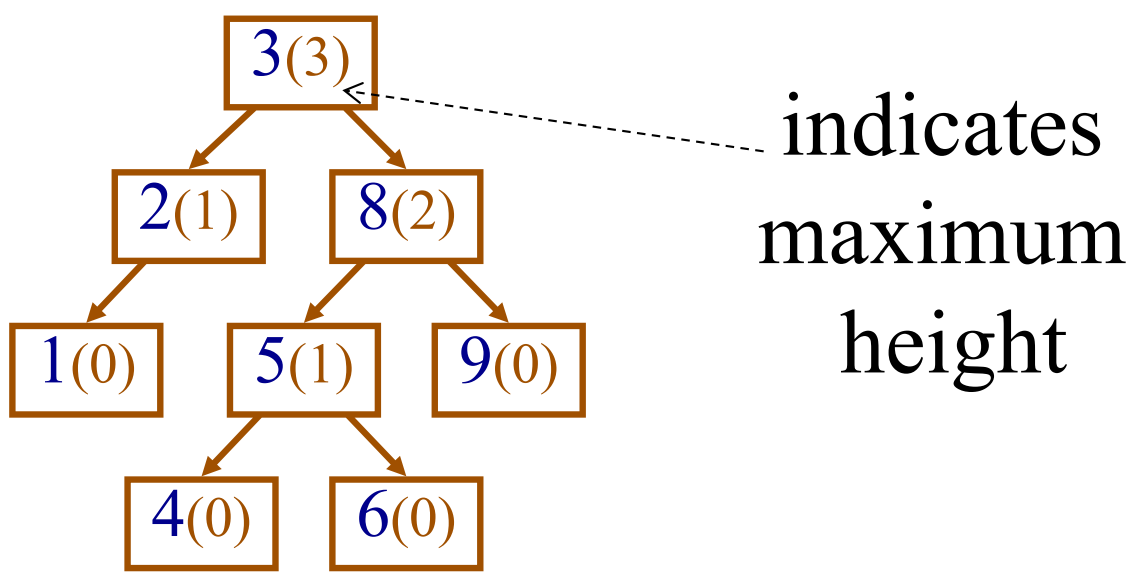
Requiring Complete Trees

- However, it is very costly to maintain a complete binary tree



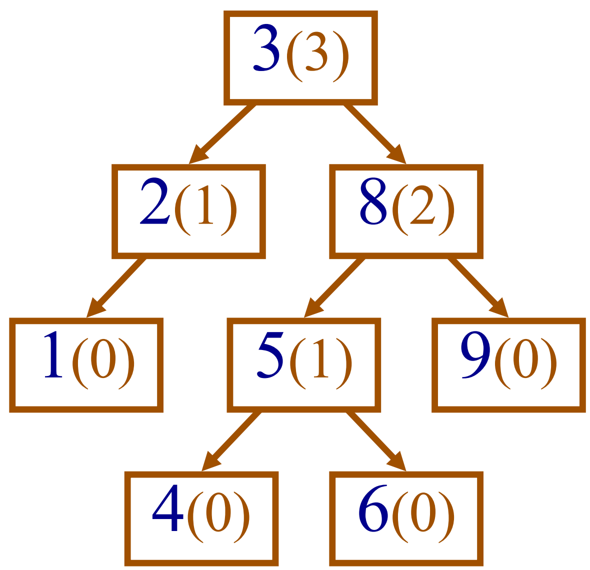
Height-Balanced Trees

- For each node, the height difference between the left and right subtrees is ≤ 1



Height-Balanced Trees

- Are locally balanced, but globally (slightly) unbalanced



Height-Balanced Trees

- Mathematically, the longest path has been shown to be, at worst, 44% longer than $\log n$

- Algorithms that run in time proportional to the path length are still $O(\log n)$
  - Why?

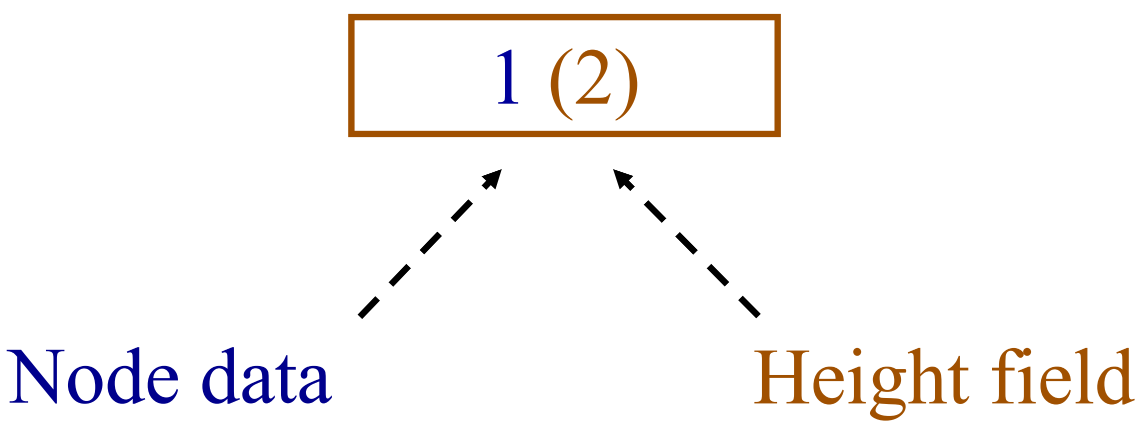
AVL Trees

- Named after the inventors' initials:
  - Adelson-Velskii and Landis

- Maintain the height balanced property of Binary Search Trees

AVL Trees

- Add an integer height field to each node:
  - Null child has a height of –1
  - A node is ***unbalanced*** when the absolute height difference between the left and right subtrees is ***greater than one***



AVL Implementation

```
struct AVLNode {
  TYPE            val;
  struct AVLNode *left;
  struct AVLNode *rght;
  int             hght;  /* Height of node*/
};
```

Get Height

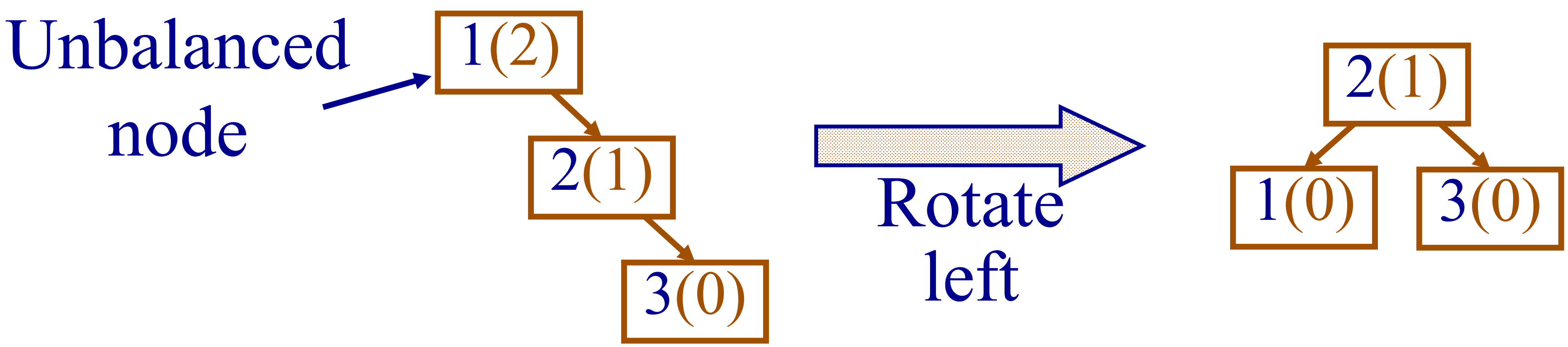
```
int _height(struct AVLNode *cur)
{
    if(cur == 0)
       return -1
    else return cur->hght;
}
```

Compute Height

```
void _setHeight(struct AVLNode *cur) {
    int lh = _height(cur->left);
    int rh = _height(cur->rght);
    if(lh < rh)
        cur->hght = 1 + rh;
    else
        cur->hght = 1 + lh;
}
```

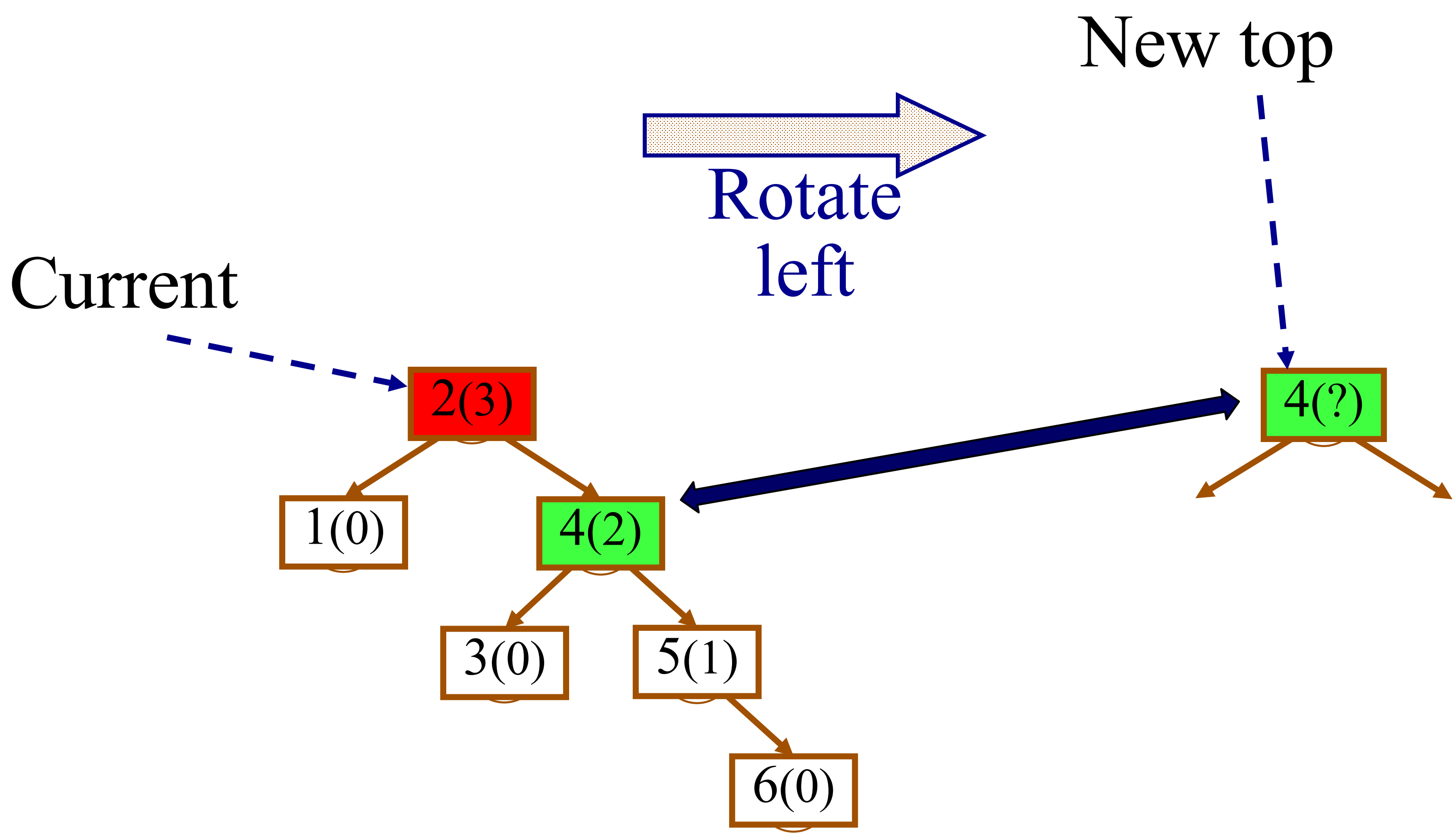
Maintaining the Height Balanced Property

- When unbalanced, perform a "rotation" to balance the tree



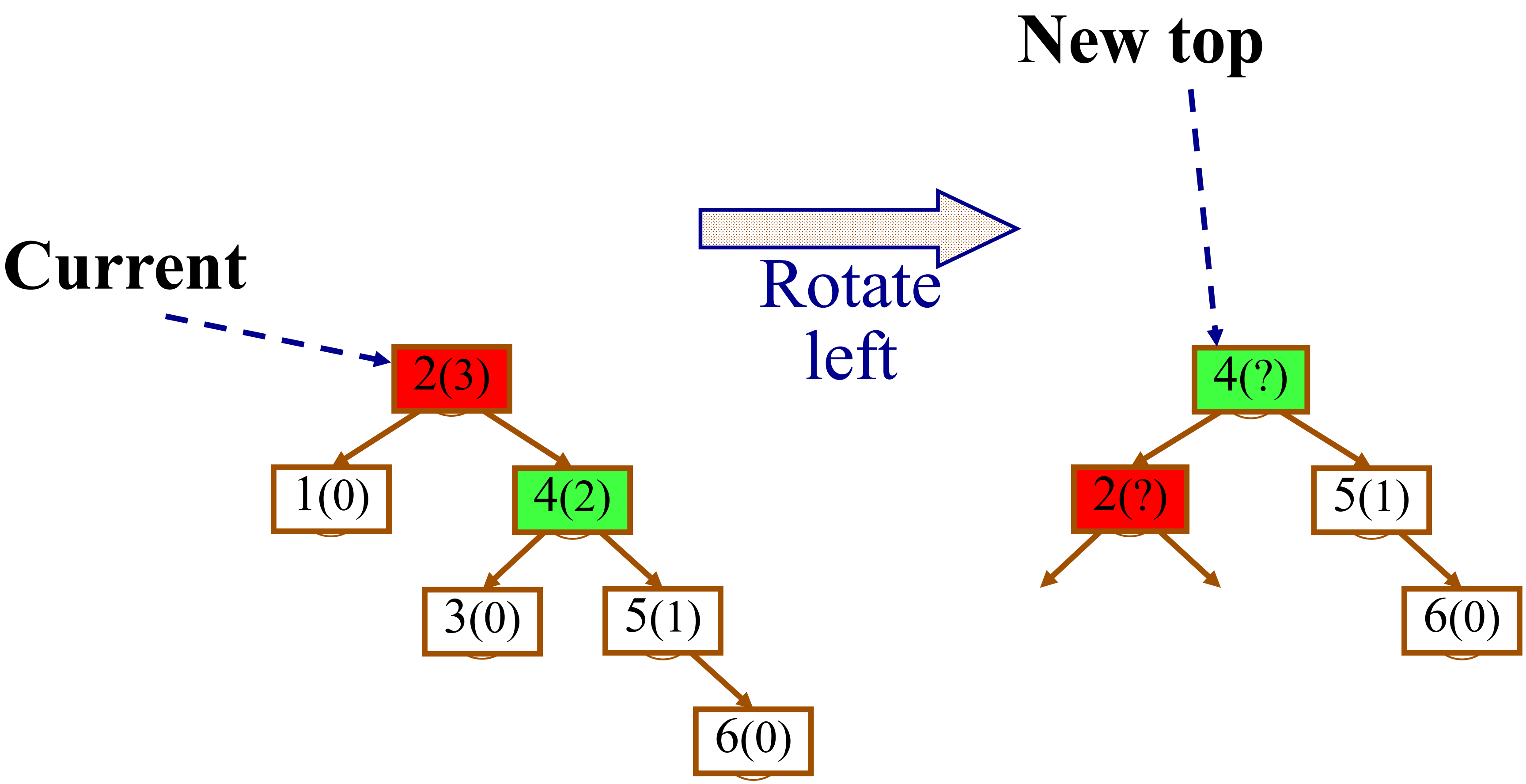
Left Rotation

1. Input: **current**
2. New top = **current's right child**



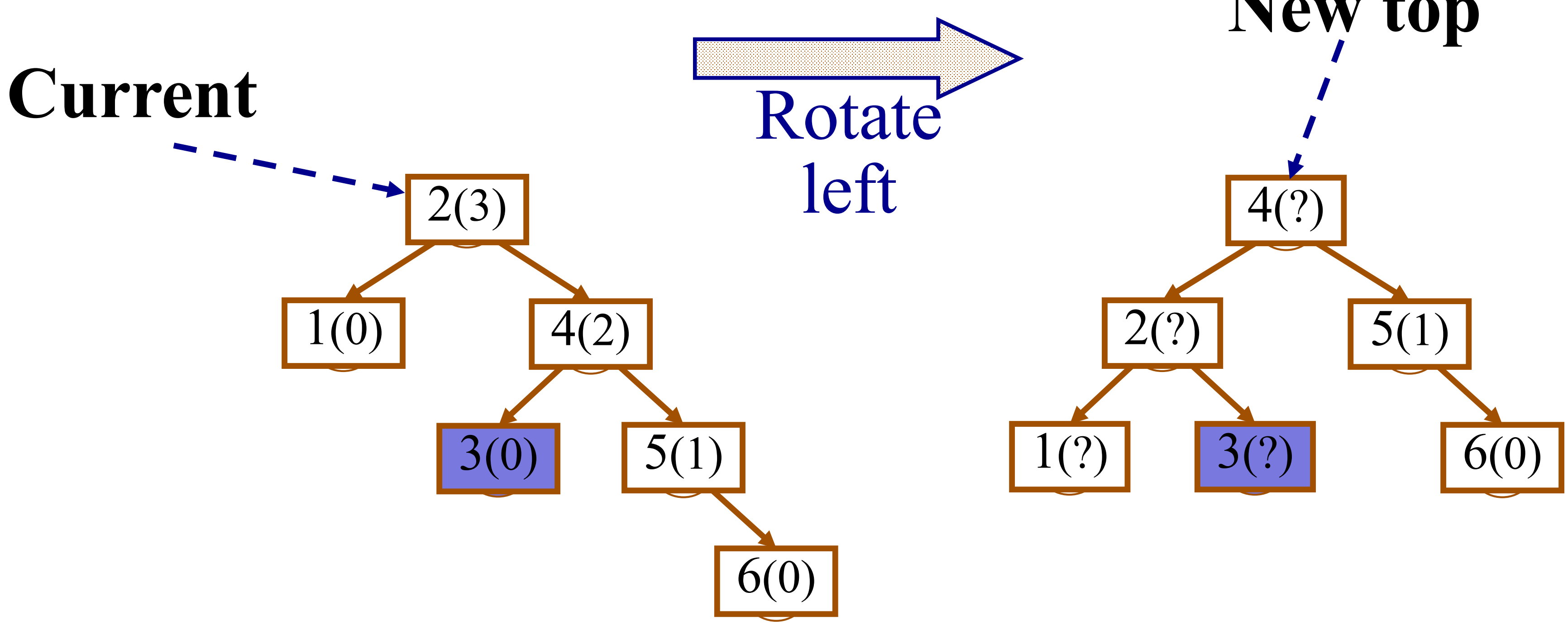
Left Rotation

1. Input: `current`
2. New top = `current`**'s right child**
3. New top's new left child = `current`



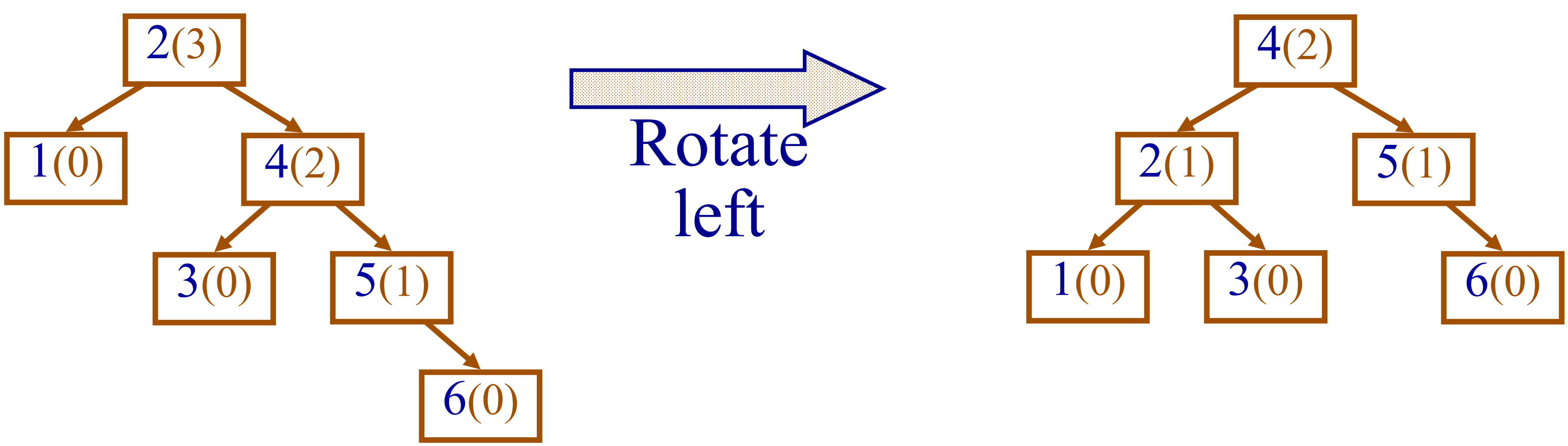
Left Rotation

1. Input: `current`
2. New top = **`current`'s right child**
3. New top's new left child = `current`
4. Current's new right child = **new top's left child**

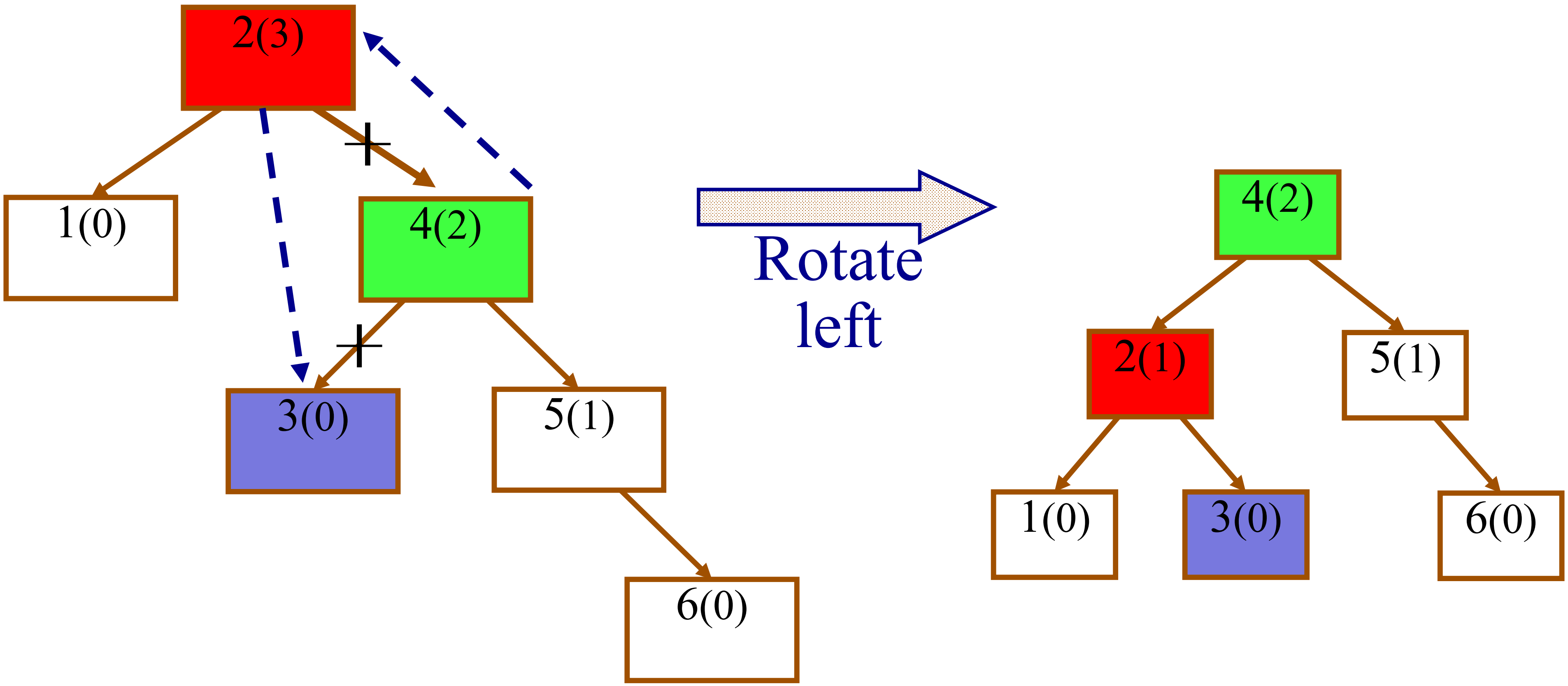


Left Rotation

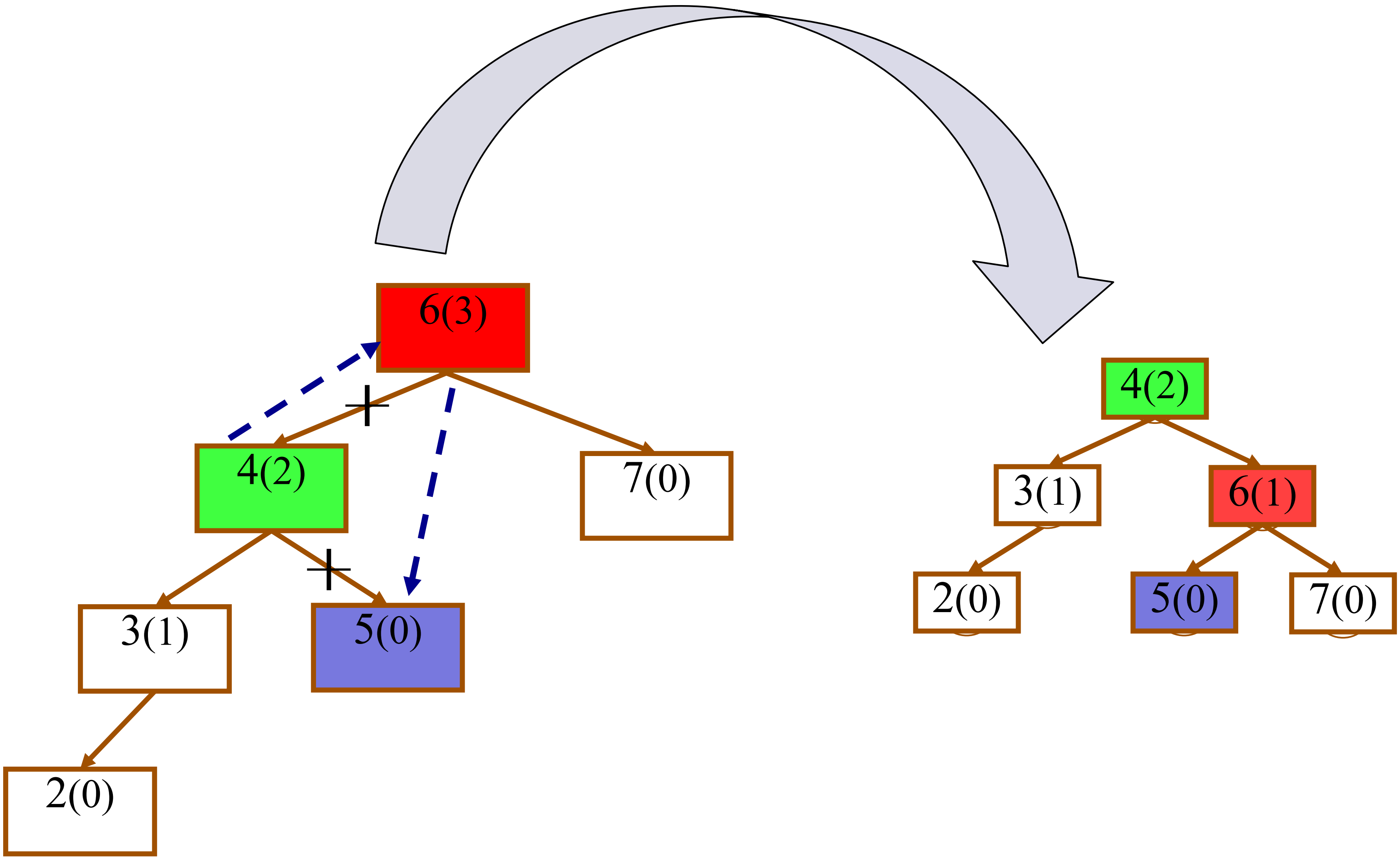
1. Input: `current`
2. New top = `current`'s right child
3. New top's new left child = `current`
4. Current's new right child = new top's left child
5. Set height of `current`
6. Set height of new top node



Left Rotation

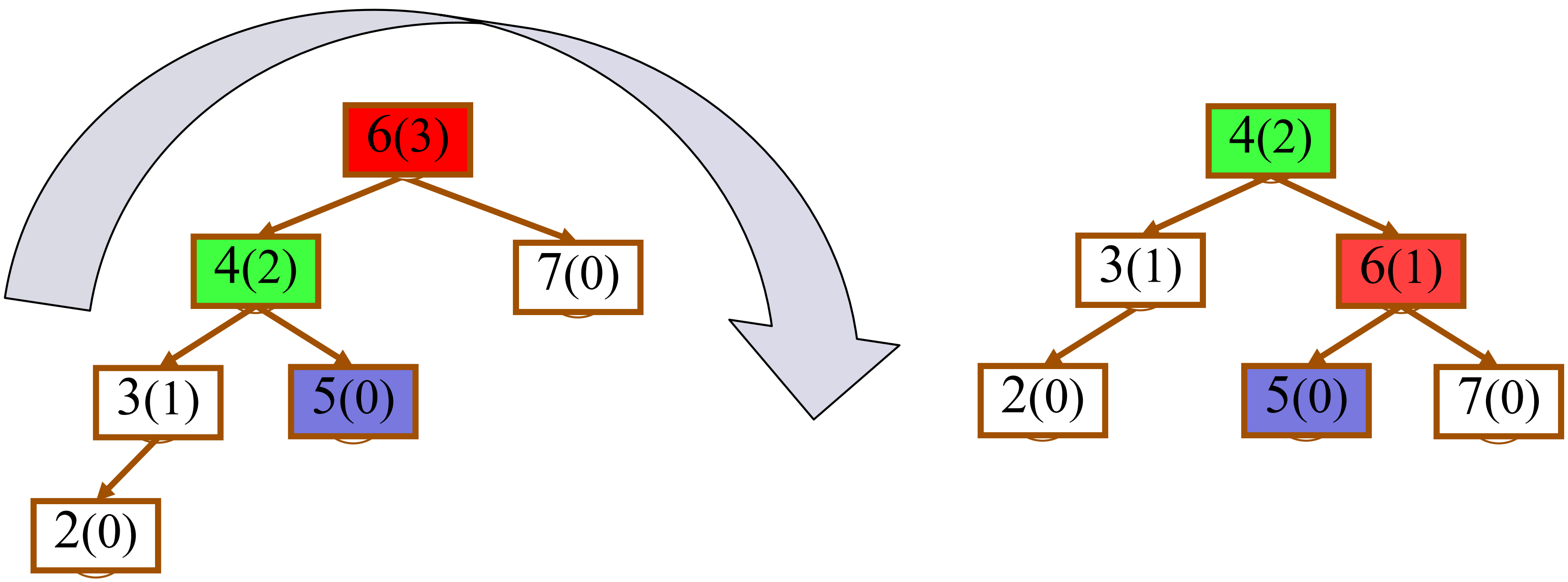


Right Rotation



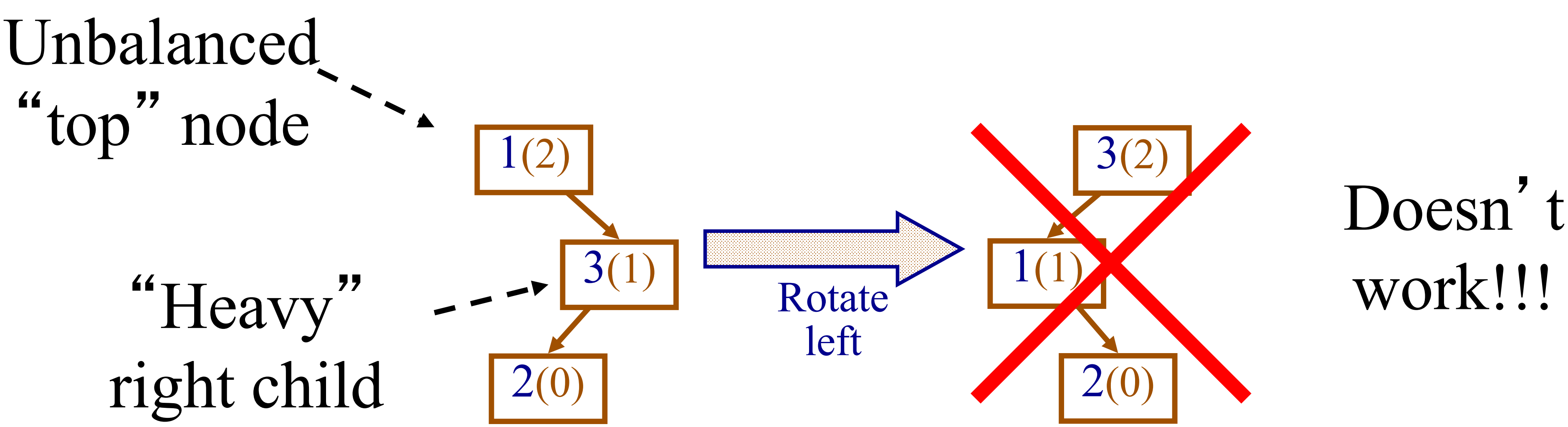
Right Rotation

1. Input: **`current`**
2. New top = **`current`'s left child**
3. New top's right child = **`current`**
4. Current's new left child = **new top's right child**
5. Set height of `current`
6. Set height of new top node



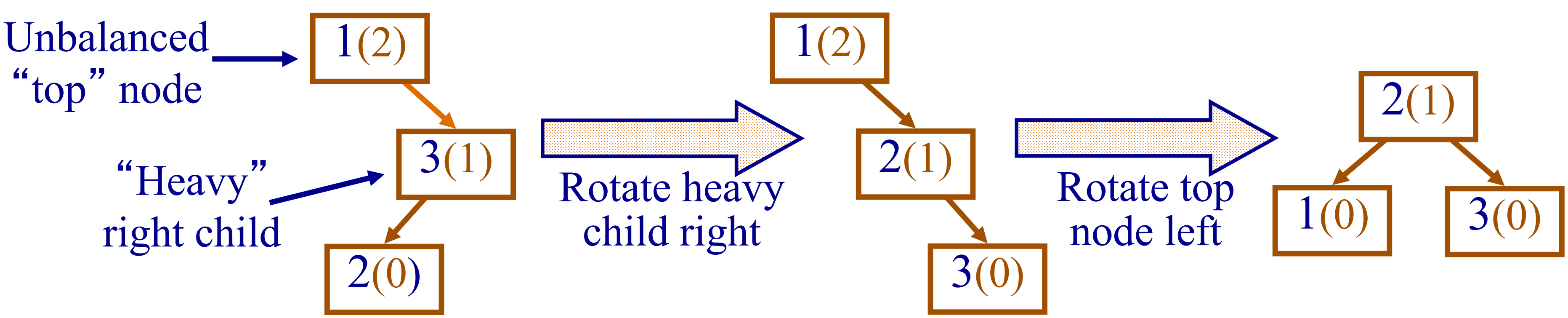
Double Rotation Left

- A single rotation may not fix the problem:
  - When the **right** child is **heavy,** i.e.,
    - its parent is unbalanced
    - has only a right subtree



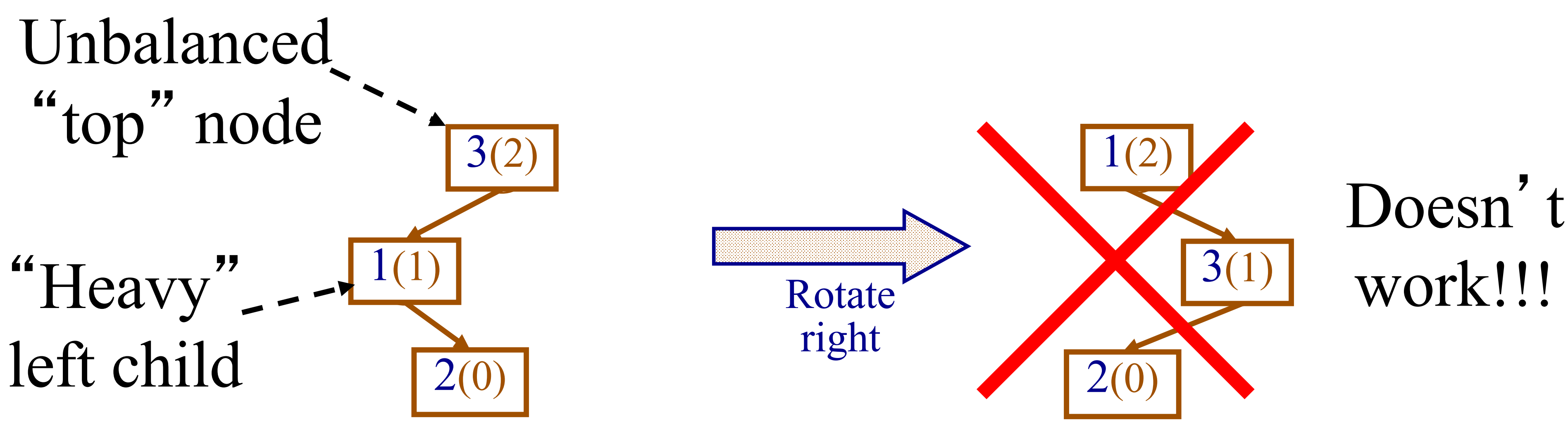
Double Rotation Left

- R*otate the child* before the regular rotation:
  1. Rotate the heavy right child to the **right**
  2. Rotate the "top" node to the **left**



Double Rotation

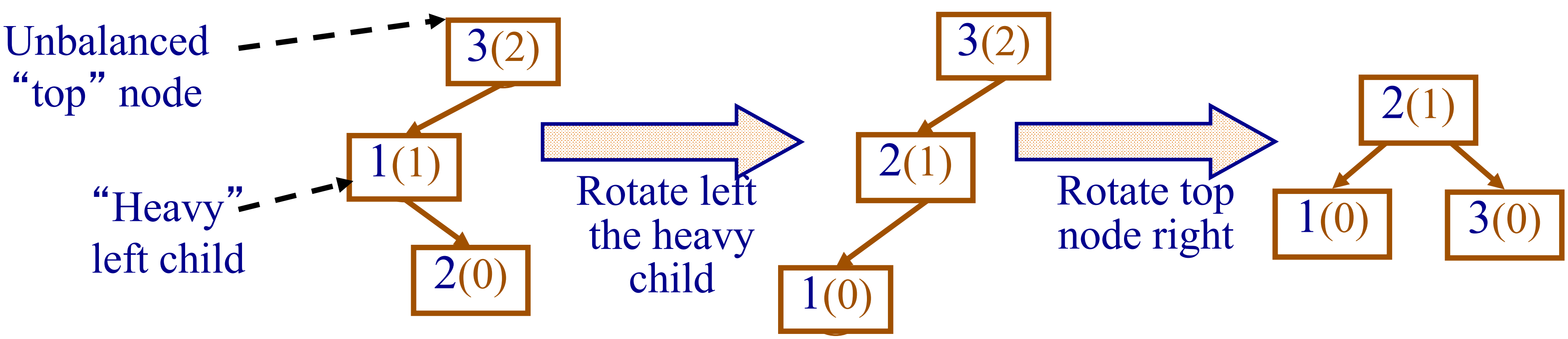
- A single rotation may not fix the problem:
  - When the **left** child is **heavy,** i.e.,
    - its parent in unbalanced from the left
    - has only a left subtree



Double Rotation Right

- This case requires *rotating the child* before the regular rotation:

1. Rotate the heavy left child to the **left**
2. Rotate the "top" node to the **right**

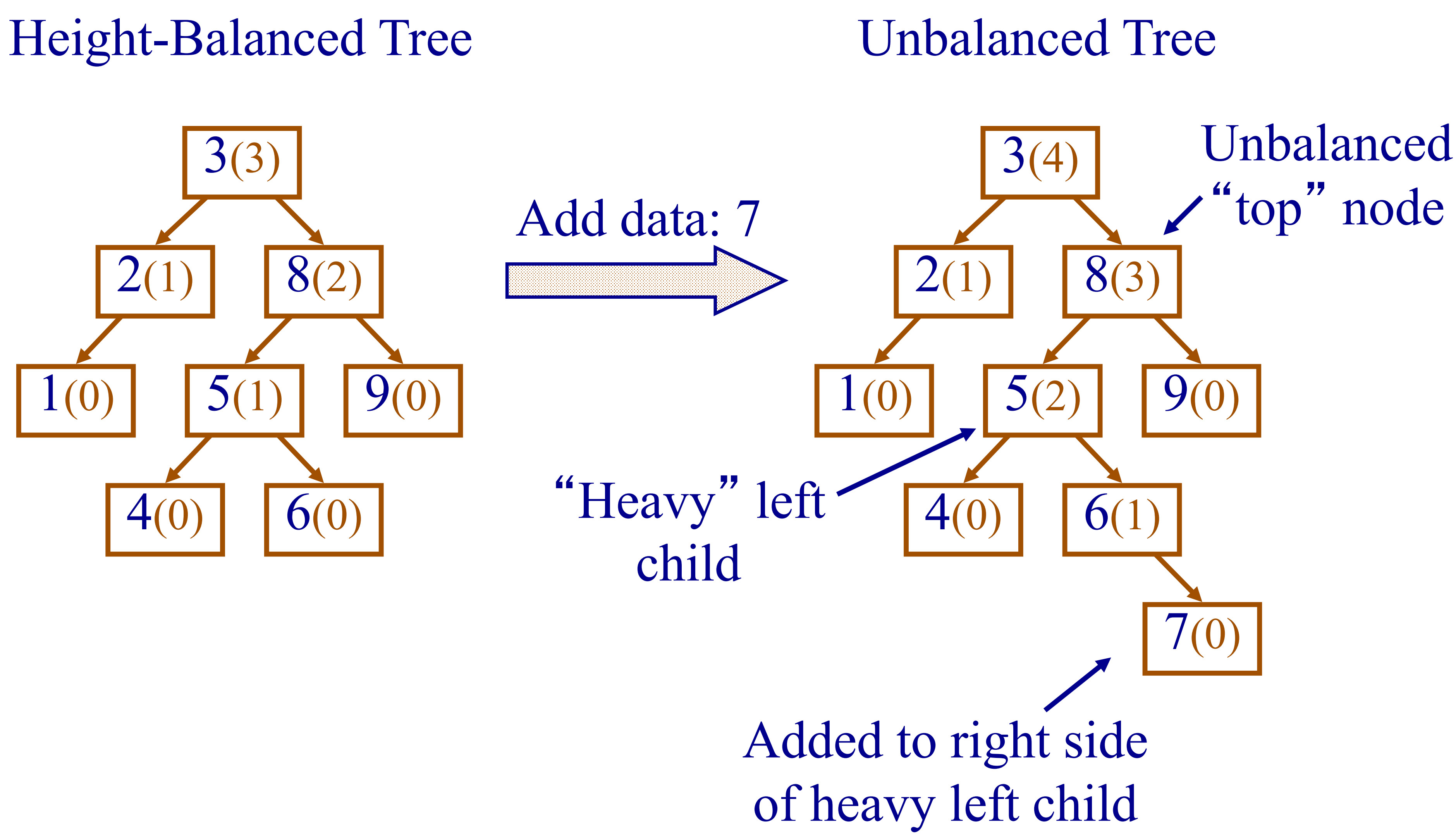


Balancing an Unbalanced Node

```
If left child is taller than right child{/* Rotation right */
        If left child is heavy{/* Double rotation right*/
             Rotate left the heavy left child
        }
        Rotate right the node
}else{ /* Rotation left */
        If right child is heavy {/* Double rotation left */
             Rotate right the heavy right child
        }
        Rotate left the node

}
Return node
```

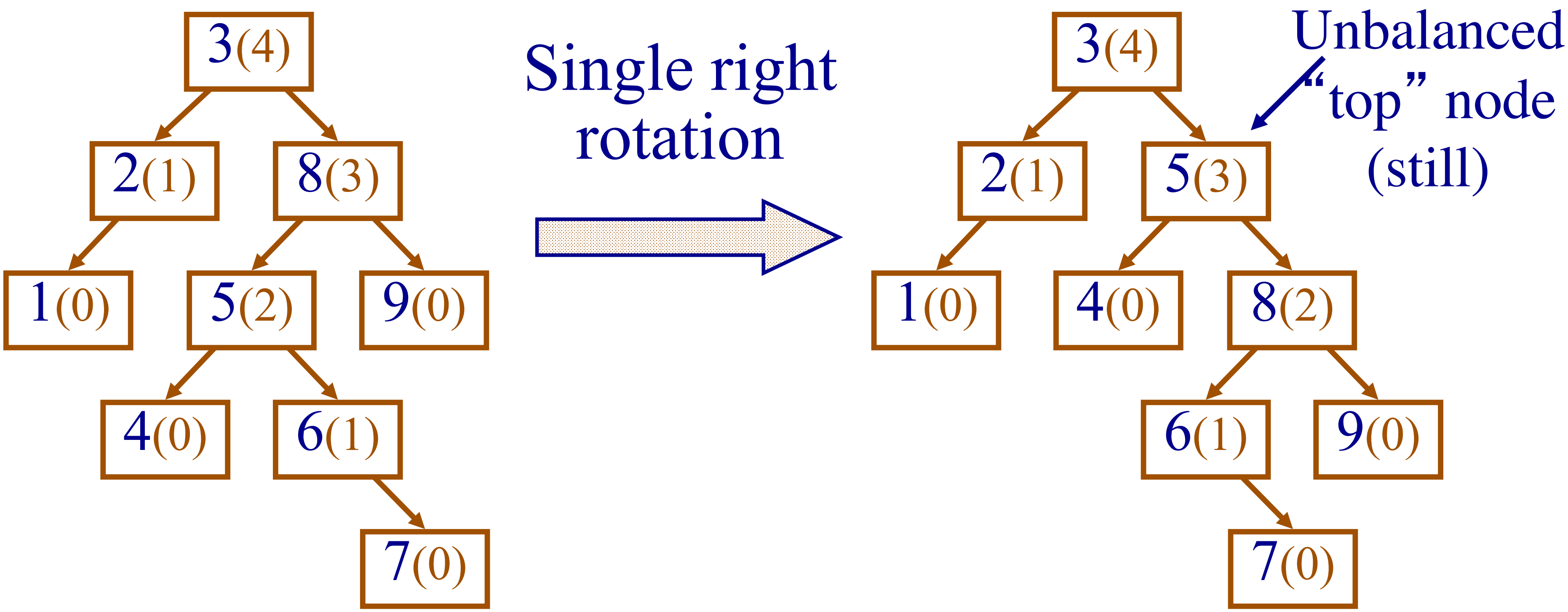
Example: Add 7 to the tree



Example – Suppose We Used Single Rotation

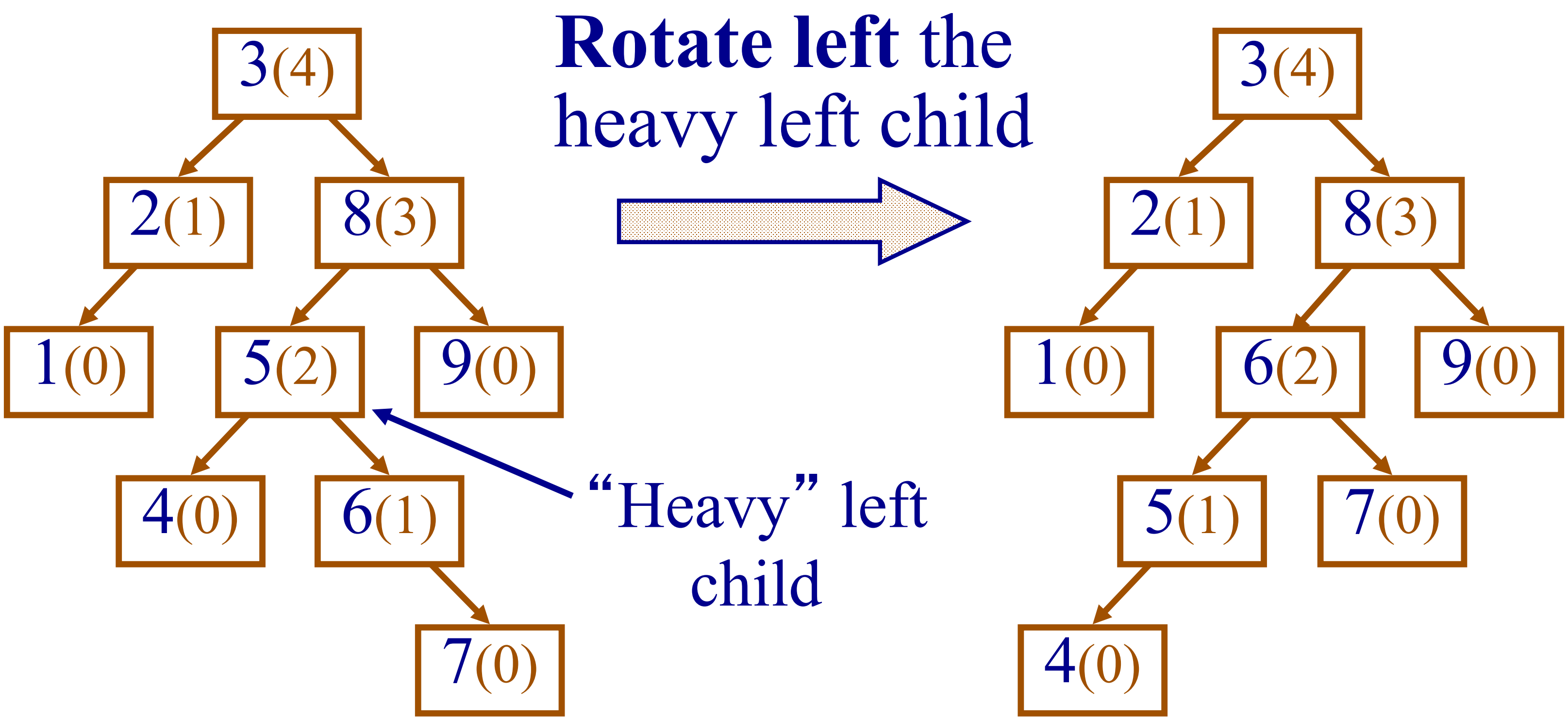
Unbalanced Tree

Tree Still Unbalanced



Example – Double Rotation Right

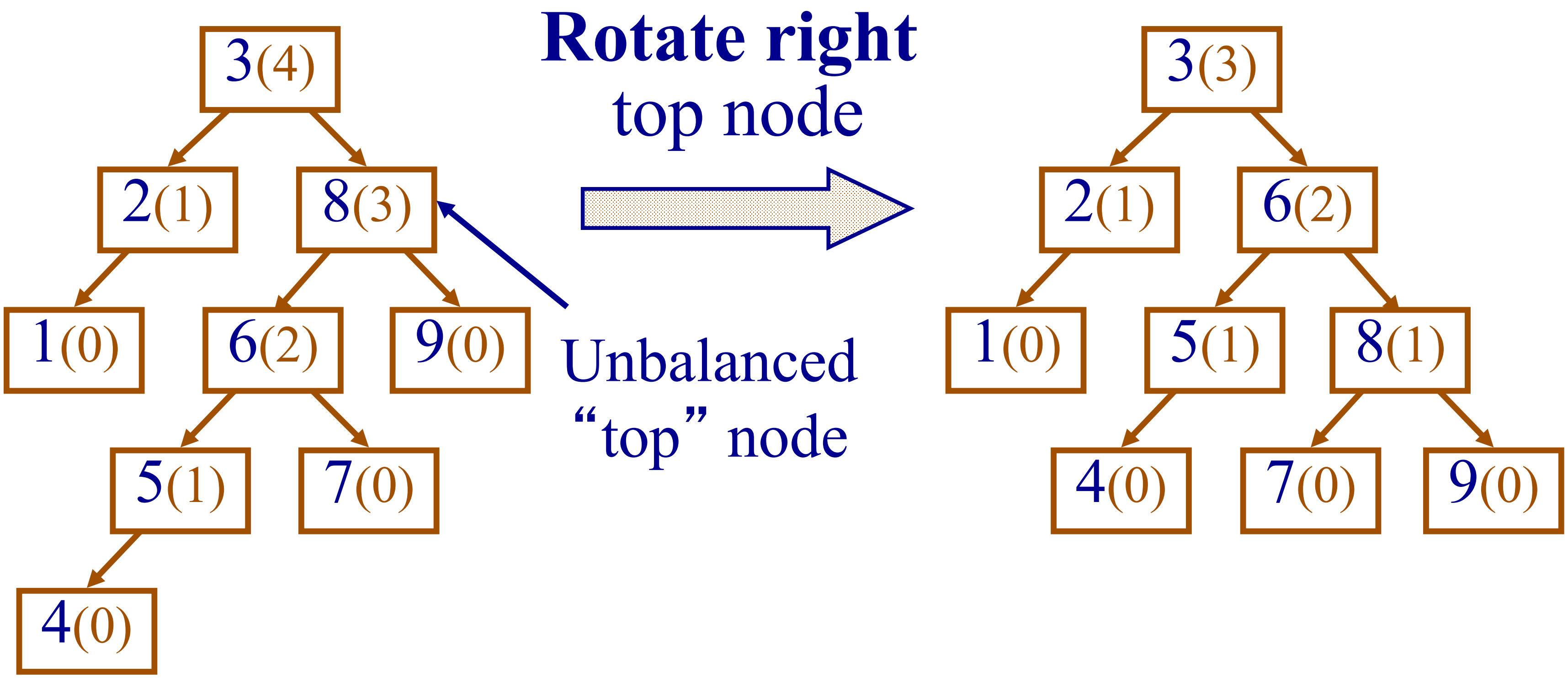
Unbalanced Tree | Tree Still Unbalanced, but …



Example – Double Rotation Right

Unbalanced Tree
(after 1st rotation)

Tree Now Balanced



Your Turn

- Any questions

- Worksheet:
  - Start by inserting values 1-7 into an empty AVL tree
  - Then write code for left and right rotations