# Scale-invariant Region-based Hierarchical Image Matching 

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## PROBLEM STATEMENT

## Problem Statement

## Given a set of images

# containing objects from an unknown category 

where the images are captured

## at varying distances from the objects

discover and segment all objects of the category

## Illustrative Example


input

output
Faces should be identified and segmented as the frequently occurring category in the input images

Note that this is not a face-detection talk

## Challenges: Scale Variations



- Geometric and photometric properties of objects change
- Details visible in the high zoom-in, disappear at coarse scales


## MOTIVATION

## Motivation: Image $=$ Segmentation Tree


multiscale segmentation
N. Ahuja 96, Tobb \& Ahuja 97, Arora\&Ahuja 06

## Motivation: Image $=$ Segmentation Tree


N. Ahuja 96, Tobb \& Ahuja 97, Arora\&Ahuja 06

## Motivation: Learning Objects = Tree Matching

## training



Objects $=$ Similar subtrees

## Effect of Scale Changes



Width and depth of object subtrees varies

PRIOR WORK

## Region Properties Associated with Each Node


$\psi \psi \psi$ vector of region properties:

- Contrast with surround
- Area
- Displacement of centroids
- Orientation of principal axes
- Perimeter
:

Todorovic \& Ahuja 06, Ahuja \& Todorovic 07

## Region Properties Associated with Each Node


$\psi \psi$ vector of region properties:

- Contrast with surround
- Area
- Displacement of centroids
- Orientation of principal axes
- Perimeter $:$

Area defined relative to parent's area


Invariance to small scale variations

Todorovic \& Ahuja 06, Ahuja \& Todorovic 07

Tree Matching = Subtree Isomorphism


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Match two regions

## Tree Matching = Subtree Isomorphism



Match two regions

- If their immediate properties are similar


## Tree Matching = Subtree Isomorphism



Match two regions

- If their immediate properties are similar
- AND the same holds for their subregions


## Addressing Instability of Image Segmentation



Many-to-many matching $=$ Augmenting trees with mergers

Todorovic \& Ahuja 06, Ahuja \& Todorovic 07

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Many-to-many matching = Augmenting trees with mergers

Todorovic \& Ahuja 06, Ahuja \& Todorovic 07

## Addressing Instability of Image Segmentation



Matching all descendants under a node $\Downarrow$
$\Rightarrow$ tree flattening
Matching transitive closures of trees
Torsello \& Hancock 03, Pelillo et al. 99, Glantz et al. 04

## OUR APPROACH

## Our Approach

- Represent images as segmentation trees
- Down-weight fine details closer to leaf nodes -Find weighted transitive closure of the trees
- Match by separating the scales of the objects and scene -Normalization of region properties


## Weighted Transitive Closures



Weights $\rho$ associated with all edges in the tree

$$
\rho(v, u)=\frac{\operatorname{area}(u)}{\operatorname{area}(v)}
$$

## Bottom-Up Matching



## Bottom-Up Matching



## Bottom-Up Matching



## Bottom-Up Matching



## Separation of Scene Scale from Object Scale

Example:

$\delta_{\text {area }}=\operatorname{area}(v) / \operatorname{area}\left(v^{\prime}\right) \rightarrow \widetilde{\operatorname{area}}\left(v^{\prime}\right)=\delta_{\text {area }} * \operatorname{area}\left(v^{\prime}\right)$

## Separation of Scene Scale from Object Scale

Example:

$\delta_{\text {area }}=\operatorname{area}(v) / \operatorname{area}\left(v^{\prime}\right) \rightarrow \widetilde{\operatorname{area}}\left(v^{\prime}\right)=\delta_{\text {area }} * \operatorname{area}\left(v^{\prime}\right)$
$\boldsymbol{\psi}(v), \boldsymbol{\psi}\left(v^{\prime}\right)=$ vectors of region properties
$\delta_{i}=\psi_{i}(v) \oslash \psi_{i}\left(v^{\prime}\right) \rightarrow \widetilde{\psi}_{i}\left(v^{\prime}\right)=\delta_{i} \otimes \psi_{i}\left(v^{\prime}\right)$
$\Rightarrow \boldsymbol{\Delta}=\left\{\delta_{1}, \ldots, \delta_{d}\right\}$ normalization factors

## Normalization



Use $\boldsymbol{\Delta}$ to normalize all descendants $\boldsymbol{u}^{\prime}$ of $\boldsymbol{v}^{\prime}$

$$
\widetilde{\boldsymbol{\psi}}_{i}\left(u^{\prime}\right)=\delta_{i} \otimes \boldsymbol{\psi}_{i}\left(u^{\prime}\right) \quad i=1, \ldots, d
$$

## Normalization



Use $\boldsymbol{\Delta}$ to normalize all descendants $u^{\prime}$ of $\boldsymbol{v}^{\prime}$

$$
\widetilde{\boldsymbol{\psi}}_{i}\left(u^{\prime}\right)=\delta_{i} \otimes \boldsymbol{\psi}_{i}\left(u^{\prime}\right) \quad i=1, \ldots, d
$$

$\Downarrow$
Properties of all nodes are normalized to those of root $v$
$\Downarrow$
Separation of the scale of the object from the scale of the scene

## Tree Matching: Formulation

Given two weighted trees: $t=(V, E, \psi, \rho)$ and $t^{\prime}=\left(V^{\prime}, E^{\prime}, \widetilde{\psi^{\prime}}, \rho^{\prime}\right)$

For each pair of nodes: $\left(v, v^{\prime}\right) \in V \times V^{\prime}$

Find bijection between the descendants of $v$ and $v^{\prime}$

$$
f=\left\{\left(u, u^{\prime}\right)\right\} \subset V \times V^{\prime}
$$

which minimizes the cost of matching:

$$
C_{v v^{\prime}}=\min _{f}\left[\sum_{\left(u, u^{\prime}\right) \in f} A_{v v^{\prime}}+\sum_{\left(w, w^{\prime}, u, u^{\prime}\right) \in f \times f} B_{v v^{\prime} u u^{\prime}}\right]
$$

where $A$ and $B$ are defined in terms of region properties and edge weights

## Tree Matching: Formulation

## Relaxation of the discrete problem

$$
C_{v v^{\prime}}=\min _{X}\left[A^{T} X+\frac{1}{2} X^{T} B X\right]
$$

$$
\stackrel{\text { s.t. }}{\boldsymbol{x}_{u u^{\prime}}} \in[0,1], \quad \sum_{u} x_{u u^{\prime}}=1 \sum_{u^{\prime}} x_{u u^{\prime}}=1
$$

Results: Discovery and Segmentation


## Results: Discovery and Segmentation



## prior work



## Results: Discovery and Segmentation



Caltech-101: Faces


## Results: Discovery and Segmentation



## matching the down-sampled textures

Texture


## Summary

- Scale-invariant object matching achieved by:


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- Down-weighting the effect of missing fine details at coarser scales


## Summary

- Scale-invariant object matching achieved by:
- Down-weighting the effect of missing fine details at coarser scales
- Separating the scale of the object from the scale of the scene

