CS450/550 — Written Homework 4 — Solutions

1. Consider the CIE chromaticity diagram. Suppose we have three primary colors located at positions Red = (.6, .3), Green = (.1, .3), and Blue = (.1, .1). For each of the following colors indicate whether it can be produced by mixing these primaries. If it can be produced, what intensities of these three primary colors are needed to produce it?

(a) The color at (.5,.4)?

No. It lies outside the gamut.

(b) The color at (.3,.3)?

Yes. We need a 2/5:3/5:0 ratio of R:G:B.

(c) The color at (.1, .2)?

yes. We need a 0:1/2:1/2 ratio of R:G:B.

(d) The color at (.2, .2)? Hint: the amounts of primaries that you mix should sum to 1. Hence, you should be able to formulate a system of 3 equations in 3 unknowns.

To develop the system of equations, we first write down the requirement that the linear combination of the primaries must equal the desired color:

$$r(.6, .3) + g(.1, .3) + b(.1, .1) = (.2, .2)$$

Then we add the constraint

$$r + g + b = 1.$$

This gives us the 3 equations

$$\begin{array}{rcrcrcl} 0.6r & + & 0.1g & + & 0.1b & = & 0.2 \\ 0.3r & + & 0.3g & + & 0.1b & = & 0.2 \\ r & + & g & + & b & = & 1.0 \end{array}$$

The solution is 2/10:3/10:5/10 for R:G:B.

2. Convert each of the following RGB colors into their HSV equivalents and give a verbal description of the resulting color. (Be sure you can work these problems without computer assistance, although you may use `xv` or other programs to check your answers). Express hue as an angle in degrees, saturation and value on a scale from 0 to 1.

(a) (1.0 1.0 1.0)

(any, 0.0, 1.00); pure white

(b) (0.5 0.5 0.5)

(any, 0.0, 0.50); medium grey

(c) (0.0 1.0 0.0)

(120, 1.0, 1.0); saturated green

(d) (0.0 0.5 0.5)

(180, 1.0 0.50); dark grey-green

3. Suppose we have a triangle with vertexes at (0, 0, 0), (0, 3, 0), and (2, 0, 0) and a point light source at (2, 0, -5). The RGB color of the point light source is (1.0, 0.8, 0.5). The viewpoint is at (0, 0, -10). Suppose the surface material has diffuse reflectivity of (0.6, 1.0, 0.2) and no specular reflectivity. Assume no ambient light source, and include the $1/d$ decay of intensity with distance in your calculations. Show your work.

(a) What is the apparent RGB color of the point (0, 0, 0) on the triangle?

$$\begin{aligned}
d &= 5.38516 \\
N &= (0, 0, -1) \\
L &= (0.37139, 0.0, -0.92848) \\
\cos\phi &= L \cdot N = 0.92848 \\
I_r &= 0.6 \times 1.0 \times 0.92848/5.38516 = 0.1035 \\
I_g &= 1.0 \times 0.8 \times 0.92848/5.38516 = 0.1379 \\
I_b &= 0.2 \times 0.5 \times 0.92848/5.38516 = 0.0172
\end{aligned}$$

(b) What is the apparent RGB color of the point (1, 1, 0) on the triangle?

$$\begin{aligned}
d &= 5.19615 \\
N &= (0, 0, -1) \\
L &= (0.19245, -0.19245, -0.96225) \\
\cos\phi &= L \cdot N = 0.96225 \\
I_r &= 0.6 \times 1.0 \times 0.96225/5.19615 = 0.11111 \\
I_g &= 1.0 \times 0.8 \times 0.96225/5.19615 = 0.14815 \\
I_b &= 0.2 \times 0.5 \times 0.96225/5.19615 = 0.01852
\end{aligned}$$

4. Repeat problem 3, but now assume that the triangle has no diffuse reflectivity but instead has specular reflectivity of (0.5, 0.5, 0.5) and a Phong's rule value for n of 10. Show your work.

(a).

$$\begin{aligned}
d &= 5.38516 \\
N &= (0, 0, -1) \\
L &= (0.37139, 0.0, -0.92848) \\
V &= (0, 0, -1) \\
R &= 2(L \cdot N)N - L = (-0.37139, 0.0, -0.92848) \\
(R \cdot V)^n &= 0.47611 \\
I_r &= 1.0 \times 0.5 \times 0.47611/5.38516 = 0.04479 \\
I_g &= 0.8 \times 0.5 \times 0.47611/5.38516 = 0.03583 \\
I_b &= 0.5 \times 0.5 \times 0.47611/5.38516 = 0.02210
\end{aligned}$$

(b).

$$\begin{aligned}
d &= 5.19615 \\
N &= (0, 0, -1) \\
L &= (0.19245, -0.19245, -0.96225) \\
V &= (-0.09901, -0.09901, -0.99015) \\
R &= 2(L \cdot N)N - L = (-0.38113, 0.00377, -0.92452) \\
(R \cdot V)^n &= 0.61643 \\
I_r &= 1.0 \times 0.5 \times 0.61643/5.19615 = 0.05932 \\
I_g &= 0.8 \times 0.5 \times 0.61643/5.19615 = 0.04745 \\
I_b &= 0.5 \times 0.5 \times 0.61643/5.19615 = 0.02966
\end{aligned}$$