CS450/550 — Written Homework 5 — Due February 27, 1996

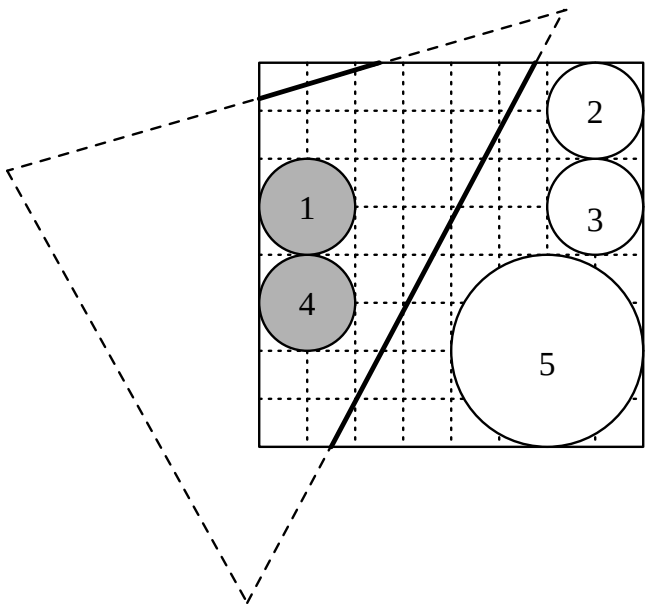
1. Problem 9 in Chapter 10 of the textbook.

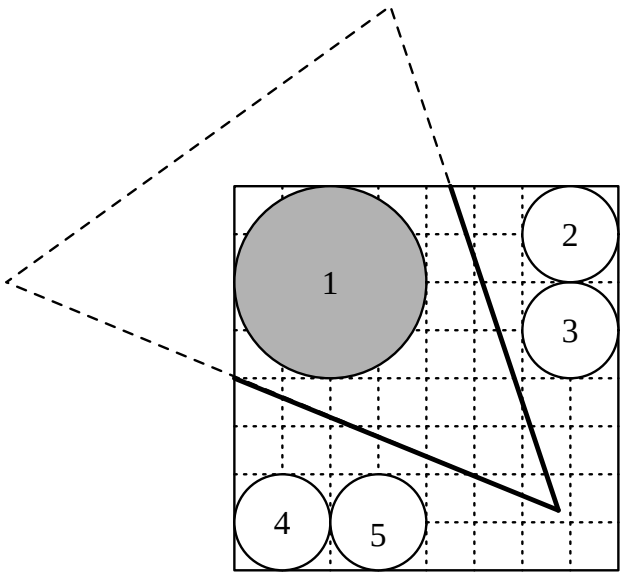
triangle	# behind	frontlist
1	2	NULL
2	0	3
3	1	1
4	0	1

2. Consider Figure 10.24 (the figure for Problem 9 in Chapter 10). There are six possible pairs of triangles that could be compared by the depth sorting algorithm. For each pair of triangles, tell which test (triangle bounding boxes; edge intersection; containment) determines whether the triangles are disjoint or overlapping. Explain your answer (e.g., by explaining why the earlier tests in the sequence do not succeed).

Triangle 1	Triangle 2	Test
1	2	Bounding rectangles fail Edge intersection tests show no intersections Bounding rectangle of 1 does not include center of 2
1	3	Bounding rectangles fail Edge intersection tests show intersection
1	4	Bounding rectangles fail Edge intersection tests show intersection
2	3	Bounding rectangles fail Edge intersection tests show intersection
2	4	Bounding rectangles succeed
3	4	Bounding rectangles fail Edge intersection tests show no intersections Bounding rectangle of 4 does not include center of 3

3 and 4. Work problems 2 and 3 from Chapter 11. Use the notation introduced in Figure 11.19.

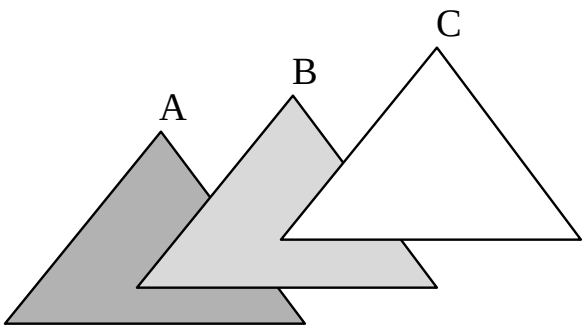




For Students Enrolled in CS550:

6. Let $T_1 < T_2$ be true if triangle T_1 is "in front of" triangle T_2 as computed by the depth sorting algorithm. True or false: This $<$ relation is transitive. Prove your answer.

False. Here is a counterexample. $C < B$ and $B < A$, but there is no depth relationship between C and A—they are disjoint.



7. Prove that the mathematics for edge intersection testing on page 381 is correct:

   a. Derive D and show that $D = 0$ if and only if the lines are parallel.

Proof: Let one edge run from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ and the other edge run from $P_3 = (x_3, y_3)$ to $P_4 = (x_4, y_4)$. The slope of the first edge is

$$\frac{y_2 - y_1}{x_2 - x_1}$$

and the slope of the second edge is

$$\frac{y_4 - y_3}{x_4 - x_3}.$$

These slopes will be equal iff

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_4 - y_3}{x_4 - x_3}.$$

Assuming $x_2 - x_1 \neq 0$ and $x_4 - x_3 \neq 0$, then this equation will hold iff

$$D = (y_2 - y_1)(x_4 - x_3) - (x_2 - x_1)(y_4 - y_3) = 0$$

This is the derivation of D. Note that this is equivalent to

$$D = (y_1 - y_2)(x_3 - x_4) - (x_1 - x_2)(y_3 - y_4).$$

What about the cases where one or both of $x_2 - x_1$ and $x_4 - x_3$ are zero?

Case 1: Suppose $x_2 - x_1 = x_4 - x_3 = 0$. Then the slopes are equal and infinite and $D = 0$, so the statement is trivially true.

Case 2: Suppose $x_2 - x_1 = 0$ but $x_4 - x_3 \neq 0$. If $D = 0$, then this implies that $y_2 - y_1 = 0$, so the first edge has no defined slope and consists of a single point. We are free to choose any slope for this line, so we can certainly say that it is parallel to the second line.

Case 3: Suppose $x_4 - x_3 = 0$ but $x_2 - x_1 \neq 0$. This is analogous to case 2.

This completes the proof.

b. Derive s and t and show that the edges intersect if and only if $0 < s, t < 1$.

Let us define two lines:

$$\begin{aligned} P &= P_1 + s(P_2 - P_1) \\ P &= P_3 + t(P_4 - P_3) \end{aligned}$$

If we set these equal to each other, we get the following two equations:

$$\begin{aligned} x_1 + s(x_2 - x_1) &= x_3 + t(x_4 - x_3) \\ y_1 + s(y_2 - y_1) &= y_3 + t(y_4 - y_3) \end{aligned}$$

Let us solve each of these for t:

$$\begin{aligned} t &= \frac{(x_1 - x_3) + s(x_2 - x_1)}{(x_4 - x_3)} \\ t &= \frac{(y_1 - y_3) + s(y_2 - y_1)}{(y_4 - y_3)} \end{aligned}$$

Set the two right-hand-sides equal to each other and solve for s.

$$\begin{aligned} \frac{(x_1 - x_3) + s(x_2 - x_1)}{(x_4 - x_3)} &= \frac{(y_1 - y_3) + s(y_2 - y_1)}{(y_4 - y_3)} \\ (x_1 - x_3)(y_4 - y_3) + s(x_2 - x_1)(y_4 - y_3) &= (y_1 - y_3)(x_4 - x_3) + s(y_2 - y_1)(x_4 - x_3) \\ (x_1 - x_3)(y_4 - y_3) - (y_1 - y_3)(x_4 - x_3) &= s[(y_2 - y_1)(x_4 - x_3) - (x_2 - x_1)(y_4 - y_3)] \\ s &= \frac{(x_1 - x_3)(y_4 - y_3) - (y_1 - y_3)(x_4 - x_3)}{(y_2 - y_1)(x_4 - x_3) - (x_2 - x_1)(y_4 - y_3)} \\ s &= \frac{(x_1 - x_3)(y_4 - y_3) - (y_1 - y_3)(x_4 - x_3)}{D} \end{aligned}$$

Similarly, we can eliminate s:

$$\begin{aligned} s &= \frac{(x_3 - x_1) + t(x_4 - x_3)}{(x_2 - x_1)} \\ s &= \frac{(y_3 - y_1) + t(y_4 - y_3)}{(y_2 - y_1)} \\ \frac{(x_3 - x_1) + t(x_4 - x_3)}{(x_2 - x_1)} &= \frac{(y_3 - y_1) + t(y_4 - y_3)}{(y_2 - y_1)} \end{aligned}$$

$$
\begin{aligned}
(x_3 - x_1)(y_2 - y_1) + t(x_4 - x_3)(y_2 - y_1) &= (y_3 - y_1)(x_2 - x_1) + t(y_4 - y_3)(x_2 - x_1) \\
(x_3 - x_1)(y_2 - y_1) - (y_3 - y_1)(x_2 - x_1) &= t[(y_4 - y_3)(x_2 - x_1) - (x_4 - x_3)(y_2 - y_1)] \\
t &= \frac{(x_3 - x_1)(y_2 - y_1) - (y_3 - y_1)(x_2 - x_1)}{(y_4 - y_3)(x_2 - x_1) - (x_4 - x_3)(y_2 - y_1)} \\
t &= \frac{(y_3 - y_1)(x_2 - x_1) - (x_3 - x_1)(y_2 - y_1)}{(x_4 - x_3)(y_2 - y_1) - (y_4 - y_3)(x_2 - x_1)} \\
t &= \frac{(y_3 - y_1)(x_2 - x_1) - (x_3 - x_1)(y_2 - y_1)}{D}
\end{aligned}
$$

If these equations give values for s and t between zero and one, then this means that the intersection point lies between points P_1 and P_2 on the first edge and between points P_3 and P_4 on the second edge. Otherwise, the intersection point lies beyond the end of one or both line segments.

This completes the proof.