JOINT RECEIVE ANTENNA SELECTION AND
USER SCHEDULING USING LOW-COMPLEXITY
MULTIUSER MIMO BEAMFORMING

by

Sherif Abdelwahab Ahmed Elgohari

Thesis submitted to the
Faculty of Engineering at Cairo University
in partial fulfillment of the
Requirements for the Degree of
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in
ELECTRONICS AND ELECTRICAL COMMUNICATIONS ENGINEERING

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
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GIZA, EGYPT
2010
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To Gaza
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<tr>
<td>3GPP</td>
<td>Third Generation Partnership Project</td>
</tr>
<tr>
<td>ACG</td>
<td>Amplitude-Craving Greedy</td>
</tr>
<tr>
<td>ADSL</td>
<td>Asymmetric Digital Subscriber Line</td>
</tr>
<tr>
<td>BABS</td>
<td>Bandwidth Assignment Based on SNR</td>
</tr>
<tr>
<td>BB</td>
<td>Branch and Bound</td>
</tr>
<tr>
<td>BD</td>
<td>Block Diagonalization</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BF</td>
<td>Beamforming</td>
</tr>
<tr>
<td>BTS</td>
<td>Base Transceiver Station</td>
</tr>
<tr>
<td>BWA</td>
<td>Broadband Wireless Access</td>
</tr>
<tr>
<td>CCI</td>
<td>Co-Channel Interference</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>CSIR</td>
<td>Channel State Information at the Receiver</td>
</tr>
<tr>
<td>CSIT</td>
<td>Channel State Information at the Transmitter</td>
</tr>
<tr>
<td>DAB</td>
<td>Digital Audio Broadcast</td>
</tr>
<tr>
<td>DMT</td>
<td>Discrete Multi Tone</td>
</tr>
<tr>
<td>DVB</td>
<td>Digital Video Broadcast</td>
</tr>
<tr>
<td>E-UTRA</td>
<td>Evolved UTRAN</td>
</tr>
<tr>
<td>FDD</td>
<td>Frequency Division Duplex</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FSA</td>
<td>Fair Scheduling Algorithm</td>
</tr>
<tr>
<td>GRA</td>
<td>Greedy Releasing Algorithm.</td>
</tr>
<tr>
<td>GRF</td>
<td>Gain Reduction Factor</td>
</tr>
<tr>
<td>GS</td>
<td>Greedy Search</td>
</tr>
<tr>
<td>HSPDA</td>
<td>High Speed Downlink Packet Access</td>
</tr>
<tr>
<td>ICI</td>
<td>Interchannel Interference</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>FFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IP</td>
<td>Internet Protocol</td>
</tr>
<tr>
<td>IQ</td>
<td>Quadrature-Phase</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol Interference</td>
</tr>
<tr>
<td>LA</td>
<td>Link Adaptation</td>
</tr>
<tr>
<td>LMS</td>
<td>Least Mean-Square</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>LR</td>
<td>Lagrangian Relaxation</td>
</tr>
<tr>
<td>LS</td>
<td>Least Square</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>-------------</td>
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</tr>
<tr>
<td>MA</td>
<td>Margin Adaptive</td>
</tr>
<tr>
<td>MAC</td>
<td>Media Access Control</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi Input Multi Output</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean-Square</td>
</tr>
<tr>
<td>MUD</td>
<td>Multiuser Diversity</td>
</tr>
<tr>
<td>Mu-MIMO</td>
<td>Multiuser MIMO</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non-Line-of-Sight</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>OSI</td>
<td>Open System Interconnecting</td>
</tr>
<tr>
<td>PHY</td>
<td>Physical</td>
</tr>
<tr>
<td>PMP</td>
<td>Point-to-Multipoint</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>PSK</td>
<td>Phase Shift Key</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Modulation</td>
</tr>
<tr>
<td>RA</td>
<td>Rate Adaptive</td>
</tr>
<tr>
<td>RCG</td>
<td>Rate-Craving Greedy</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SDMA</td>
<td>Space Division Multiple Access</td>
</tr>
<tr>
<td>SDMA</td>
<td>Multi Input Single Output</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Noise-And-Interference Ratio</td>
</tr>
<tr>
<td>SLNR</td>
<td>Signal-to-Leakage-and-Noise Ratio</td>
</tr>
<tr>
<td>SLR</td>
<td>Signal-to-Leakage Ratio</td>
</tr>
<tr>
<td>SLRBF</td>
<td>Signal-to-Leakage-and-Noise-Ratio based beamformer</td>
</tr>
<tr>
<td>SMMSE</td>
<td>Successive Minimum Mean-Square</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SS</td>
<td>Sub Station</td>
</tr>
<tr>
<td>SSLRBF</td>
<td>Simplified Signal-to-Leakage-and-Noise-Ratio based beamformer</td>
</tr>
<tr>
<td>SUI</td>
<td>Stanford University Interim</td>
</tr>
<tr>
<td>TDD</td>
<td>Time Division Duplex</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>ZF</td>
<td>Zero Forcing</td>
</tr>
</tbody>
</table>
MATHEMATICAL NOTATIONS

\( \lambda_i(.) \) \( i^{th} \) eigenvalue of a matrix

\( w_i \) Beamforming Vector \( \in \mathbb{C}^{N \times 1} \)

\( \Lambda_i(.) \) Eigenvector corresponding to the \( i^{th} \) eigenvalue of a matrix

\( \Lambda_{\text{max}}(.) \) Eigenvector corresponding to the dominant eigenvalue of a matrix

\( I_M \) Identity matrix of size \( M \times M \)

\( d(.) \) Matrix column corresponds to dominant diagonal element

\( \mathcal{H} \) Matrix Hermitian conjugate

\( \sigma_i \) Noise Power at user \( i \)

\( M_i \) Total number of receive antennas at user \( i \)

\( T_i \) Number of active receive antennas at user \( i \)

\( N \) Number of Transmit antennas at Base station

\( \psi \) Scaling factor satisfying Neumann Series condition \( \rho(A) > 1 \)

\( S \) Set of scheduled users at same time-frequency instance such that \( |S| = L \)

\( \text{SINR}_i \) Signal-To-Interference-And-Noise-Ratio at user \( i \)

\( \text{SLNR}_i \) Signal-To-Leakage-And-Noise-Ratio at user \( i \)

\( H_i \) Spatial Channel Gain at user \( i \in \mathbb{C}^{M_i \times N} \)

\( \rho(.) \) Spectral Radius of a matrix defined as \( := \lambda_{\text{max}}(.) \)

\( Tr(.) \) Trace of a matrix

\( \sigma_c \) Variance of Estimated MIMO Channel

\( \tilde{H}_i \) Extended Matrix of spacial channels of co-scheduled users with user \( i \)
Praise and thanks to ALLAH the mighty who provided me with patience and provision to complete this work. First and foremost, I would like to express my deepest gratitude to my advisor Dr. Mohamed Khairy for his continual guidance, supervision and support throughout this thesis. Without his technical insight, and on-going motivation this thesis would have never been possible. I can never forget several discussions over the phone late at night to refine this work and develop our journal paper. It has been a real pleasure to have Dr. Mohamed as an advisor and mentor. A special and warm thanks to Dr. Amin Nassar for hosting this work in Computer Electronics Group and to the Examining Committee for their time evaluating this thesis.

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being a very good mentor and sincere friend. His Moral support was very important
factor to get this work done.
ABSTRACT

Multiuser Multiple-Input-Multiple-Output (MIMO) is a key physical layer technique in current and future wireless technologies which require high throughput, efficient resource usage, and effective multiple access. In this thesis, an efficient, low complexity design of a transmit beamformer is considered. In addition, a joint user scheduler and receive antenna selector is proposed. Neumann Series and Jacobi-Davidson algorithms are used to drive a simplified form of the Signal-to-Noise-and-Leakage (SLNR) based beamformer and to study its convergence for different Signal To Noise Ratios. Using this simplified form, the joint problem of receive antenna selector and user scheduler is formulated as a Mixed Binary Quadratic Programming problem. Branch and Bound method is used to derive a near-optimal, low complexity algorithm. The performance of the proposed beamformer, scheduler and antenna selector is assessed by simulations using WINNER MIMO channel model [1] in addition to taking into consideration the effect of imperfect channel estimation. The proposed beamformer is shown to have a linear complexity compared to the SLNR scheme [2] which requires polynomial computation time. The proposed joint receive antenna selection and user scheduling algorithm results in sum rate performance near to that achieved by Exhaustive Search, and significantly outperforms Greedy Search with both better sum rate and lower computational requirements.
Chapter 1

Introduction and Review of Literature

Application of MIMO technology exploits the spatial-domain as another new dimension for wireless transmission. This in turn increases spectral efficiency. Theoretically, spectral efficiency scales linearly with the minimum of the number of transmit and receive antennas employed. Several features can be realized based on theory of multiple antennas, but not all of them easily deliver their theoretical promises when it comes to implementation in practical systems. MIMO can be used in different ways:

- **Diversity gain.** Use of the space-diversity provided by the multiple antennas to improve the robustness of the transmission against multipath fading.

- **Array gain.** Concentration of energy in one or more given directions via precoding or beamforming. This also allows multiple users located in different directions to be served simultaneously (so-called multi-user MIMO).

- **Spatial multiplexing gain.** Transmission of multiple signal streams to user on multiple spatial layers created by combinations of the available antennas.

Multiuser MIMO (Mu-MIMO) is one of the most advanced physical layer techniques that provide high throughput, efficient resource usage, and effective multiple access. The problem of designing efficient and low-complexity Mu-MIMO Radio Resource Management (RRM) algorithms is addressed. We focus on Mu-MIMO transmit beamforming (beamforming), and multiuser scheduling with receive antenna selection for downlink MIMO broadcast channel with Space Division Multiple Access (SDMA).
A real challenge meet practical implementation of Mu-MIMO RRM algorithms is its computational requirements. These RRM procedures are designed in this thesis while exploiting cross-layer information and reducing computational requirements.

1.1 Contributions and Thesis Outline

In this thesis, we propose an efficient and low complexity multiuser MIMO beamformer based on Signal-to-Leakage-and-Noise Ratio beamforming. It is shown that the proposed scheme has BER and SINR cumulative distribution function close to the SLNR beamformer with significant complexity reduction. A near optimal and computationally efficient joint user scheduling and receive antenna selection algorithm using Branch and Bound Method is then proposed. The sum-capacity of the proposed algorithm is significantly higher than that achieved using Greedy search and requires lower computational power. We also show that the performance of the proposed algorithm performs much closer to the optimal exhaustive search algorithm than the greedy search.

The rest of this thesis is organized as follows. In this chapter we introduce the topic of MIMO technology with focus on Physical Layer MIMO techniques. We develop our System model used throughout the thesis in Chapter 2 and elaborate about WINNER channel model. Transmit beamforming techniques including the Signal to Leakage and Noise based beamformer are discussed in Chapter 3. Then in Chapter 4 we propose a simplified near optimal beamformer that is applicable in practical systems and discuss beamformer complexity issues. We then use the results from Chapter 4 in Chapter 5 to reformulate the problem of user scheduling and receive antenna selection and propose a novel algorithm to solve it. We conclude the thesis in Chapter 6.

1.2 MIMO in the Standards

Multiuser MIMO finds its place in modern cellular and wireless standardization efforts [3]. 3GPP Release 6 and Release 7 both support smart antenna schemes with
Figure 1.1: Application of MIMO in different standardization efforts with expected theoretical DL spectral efficiency. Source: EDGE, HSPA & LTE, Mobile Broadband Innovation, Rysavy Research September 2008

Figure 1.2: Application of MIMO in different standardization efforts with expected theoretical UL spectral efficiency. Source: EDGE, HSPA & LTE, Mobile Broadband Innovation, Rysavy Research September 2008
single antenna receiver and multiple antennas receiver respectively. Peak data rate requirements of Evolved-UTRAN [4] cannot be met without employment of MIMO techniques. A large part of the LTE Study Item phase was therefore dedicated to the selection and design of the various multiple antenna features to be included in LTE. The final system includes several complementary options which allow for adaptability according to the deployment and the propagation conditions of the different users.

Currently IEEE 802.16e includes intensive usage of MIMO-OFDM solutions for two/four transmit antennas. IEEE 802.11n proposes MIMO with possible combination of beamforming and space-division multiplexing. This motivates researchers and practitioners to develop advanced MIMO Radio Resource Management (RRM) procedures. Beamforming, multiuser scheduling, and antenna selection are crucial RRM techniques, that promise high data rate and exploitation of multiuser diversity (MUD) in Mu-MIMO based systems. Complexity considerations in these RRM algorithms is an important factor for practical implementations and success of Multiuser MIMO in typical wireless systems. Figure 1.1 and Figure 1.2 show theoretical spectral efficiency in different standardization effort for downlink and uplink respectively.

1.3 Multiple Antenna Techniques

In this section we provide the reader with theoretical background that is necessary for good understanding of MIMO techniques in general. We first tackle single user MIMO techniques, then extend our introduction to Mu-MIMO with comprehensive literature review on transmit beamforming, receive antenna selection and multiuser scheduling.

MIMO exploits spatial dimension of wireless channel utilizing space-time processing methods with the aim of improving the links performance in terms of one or more possible metrics, such as the error rate, communication data rate, coverage area and spectral efficiency. Both Single user and Multiuser MIMO build on key fundamental principles as stated before:

- Diversity gain.
• Array gain.
• Spatial multiplexing gain.

Diversity mitigates multipath fading, by transmitting or receiving over multiple antennas that have decorrelated channels. This in turn improves statistics of instantaneous SNR in the channel. Array gain concentrates energy in one particular spatial dimension, which is analogous to a matched filter gain in time-domain receivers. Finally multiplexing gain is related to transmitting parallel data streams and the ability to separate them based on what is so called spatial signature. This later gain has no cost on bandwidth, but rather on application of several antennas and on computational requirements of advanced signal processing techniques.

\[ y = Hs + z \]  

(1.1)

where \( z \) is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian noise for user \( i \) with distribution \( \mathcal{C}\mathcal{N}(0, \sigma_i^2) \) and is spatially white. \( H \) is \( M \times N \) channel matrix and represented as

\[
\begin{bmatrix}
h_{11} & \cdots & h_{1N} \\
\vdots & \ddots & \vdots \\
h_{M1} & \cdots & h_{MN}
\end{bmatrix}
\]

where \( h_{ij} \) is the channel coefficient from the \( j \)-th transmit antenna at the base station to the \( i \)-th receive antenna at the user \( i \). Basically, these coefficients are modeled as complex Gaussian random variables with zero mean and unit variance. Mapping trans-
mission symbols to the transmitted signal determines which MIMO scheme (diversity, array gain, spatial multiplexing, or a combination of three) is adopted.

**Capacity Of MIMO Channel**

We explore the information theoretical limits introduced by single user MIMO systems. Consider a MIMO channel with $N$ transmit antennas and $M$ receive antennas. Assume a frequency flat quasi-static channel of unitary bandwidth. In the discussion hereafter we assume that the only source of interference is interference between the input stream.

The following discusses capacity bound of different antenna configuration $(N \times M)$. For a memory less $1 \times 1$ SISO system the capacity is given by [5]

$$C_{siso} = \log_2 \left(1 + \rho |h|^2\right) \text{ bps/Hz} \quad (1.2)$$

where $\rho$ is the Signal to Noise Ratio at Rx Antenna, and $h$ is the channel gain. By increasing the number of receive antennas the system turns into $1 \times M$ SIMO with capacity given as [5]

$$C_{simo} = \log_2 \left(1 + \rho \sum_{i=1}^{M} |h_i|^2\right) \text{ bps/Hz} \quad (1.3)$$

where $h_i$ is the channel gain for receive antenna $i$. From (1.3) we note that increasing the number of receive antennas ($M > 1$) results only in logarithmic increase in Ergodic capacity. Similarly by introducing transmit diversity, we increase the number of transmit antennas ($N > 1$). We get to a Multiple-Input-Single-Output (MISO) system and the capacity is given by [5]

$$C_{miso} = \log_2 \left(1 + \frac{\rho}{N} \sum_{i=1}^{N} |h_i|^2\right) \text{ bps/Hz} \quad (1.4)$$

Here, $h_i$ is the channel gain for transmit antenna $i$. The above formula assumes Equal Power (EP) allocation to $N$ uncorrelated sources. We note that in (1.4), we have no array gain compared to (1.3). Similar to (1.3), the capacity has logarithmic relationship with $N$. Now we come to employing diversity at both the transmitter and
the receiver forming a MIMO system. With the (EP) case as in (1.4) the capacity of a $N \times M$ MIMO channel with gain $H$ is given by [5], [6]

$$C_{EP} = \sum_{i=1}^{r} \log_2 \left( 1 + \frac{\rho}{N} \lambda_i \right) \text{bps/Hz}$$

(1.5)

where $r = \min(N, M)$, and $\lambda_1, \lambda_2, \ldots, \lambda_r$ are the nonzero eigenvalues of $HH^H$ in case $N \leq M$ and of $HH^H$ in case $N > M$. The capacity in 1.5 increases with $r = \min(N, M)$. We note the effect of capacity multiplication in case of MIMO channel, as the capacity becomes sum-capacity of the individual channel eigenmode.

Figure 1.4 and Figure 1.5 show the mean capacity and 10% outage capacity $C_{0.1}$. The 10% outage capacity is the capacity that is supported 90% of the time and contains information about system reliability. This is shown for different antenna configurations with Equal Power (EP) allocation (i.e. Channel Unknown to transmitter).

![Ergodic Capacity of Different Antenna Configuration](image_url)

Figure 1.4: Ergodic Capacity with Different Antenna Configuration.

The ergodic capacity increases with increasing the SNR, $N$ and $M$. The ergodic capacity of a SIMO channel will be greater than that of a MISO channel when the channel is unknown at the transmitter (Equal Power allocation). This is expected refer-
Figure 1.5: 10% Outage Capacity with Different Antenna Configuration.

ring to later discussion about capacities in (1.3) and (1.4). The above results hold for i.i.d Rayleigh fading channel, and neglecting effects of antenna correlation, and Line of Sight (LOS) component.

Further we review capacity behavior when channel is known at the transmitter (Via feedback or through the reciprocity in duplex systems). When the channel is known at the transmitter, an optimal capacity is found by allocating power on different spatial sub-channels using Water Pouring (WP) algorithm. The capacity is then given by

\[ C_{WP} = \sum_{i=1}^{r} \log_2 \left( 1 + \frac{\rho}{N} \gamma_{i}^{opt} \lambda_{i} \right) \text{ bps/Hz} \]  

(1.6)

where \( \gamma_{i}^{opt} \) reflects the transmit energy in the \( i \)th spatial sub-channel and is found iteratively using the water pouring algorithm such that

\[ \gamma_{i}^{opt} = \left( \mu - \frac{N}{\rho \lambda_{i}} \right) , \ i = 1, \ldots, r, \]  

(1.7)

\[ \sum_{i=1}^{r} \gamma_{i}^{opt} = N, \]  

(1.8)
Figure 1.6: Ergodic Capacity when Channel is Known and Unknown by the transmitter.

where $\mu$ is a constant reflecting total energy available for a spatial sub-channel and $x_+^+$ implies

$$x_+^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$  \hspace{1cm} (1.9)

Figure 1.6 and Figure 1.7 show the mean capacity and 10% Outage capacity comparing the cases when the channel is known and unknown at the transmitter. Unlike EP, WP optimizes power allocation among the diagonal of the covariance matrix of the transmitted signal vector. WP gain over EP is significant at low SNR but converges to zero as SNR increases. This result can be intuited as follows: The transmit channel knowledge provides transmit array gain, that is not possible when channel is not known at the transmitter. Diversity gain and Multiplexing gain do not require such channel knowledge and are blindly achievable. Since the relative importance of transmit array gain in boosting average SNR vanishes in high SNR, the benefit of feedback also reduces. In the next chapter we show higher importance of channel knowledge at the transmitter at all SNR regions, when we discuss the multiuser MIMO case.

The multiplexing gain corresponds to the multiplicative factor by which the spec-
Figure 1.7: 10% Outage Capacity when Channel is Known and Unknown by the transmitter.

Central efficiency is increased by a given scheme. Perhaps the single most important requirement for MIMO multiplexing gain to be achieved is for the various transmit and receive antennas to experience a sufficiently different channel response. This translates into the condition that the spatial signatures of the various transmitters (the $h_i$s) (or receivers) be sufficiently decorrelated and linearly independent to allow for the channel matrix $H$ to be invertible (or more generally, well-conditioned). An immediate consequence of this condition is the limitation to $\min(M,N)$ of the number of independent streams which may be multiplexed into the MIMO channel, or more generally to $\text{rank}(H)$ streams. As an example, single-user MIMO communication between a four-antenna base station and a dual antenna UE can, at best, support multiplexing of two data streams, and thus a doubling of the UEs data rate compared with a single stream.

A fundamental aspect of the benefits of MIMO lies in the fact that any given multiple antenna configuration has a limited number of degrees of freedom. Thus there exists a compromise between reaching full beamforming gain in the detection of a desired stream of data and the perfect canceling of undesired, interfering streams. Sim-
ilarly, there exists a trade-off between the number of streams that may be multiplexed across the MIMO channel and the amount of diversity that each one of them will enjoy. Such a trade-off can be formulated from an information-theoretic point of view. In the particular case of spatial multiplexing of $N$ streams over a $N$ to $M$ antenna channel, with $M \geq N$, and using a linear detector, it can be shown that each stream will enjoy a diversity order of $M - N + 1$. To some extent, increasing the spatial load of MIMO systems (i.e. the number of spatially-multiplexed streams) is akin to increasing the user load in CDMA systems. This correspondence extends to the fact that an optimal load level exists for a given target error rate in both systems.

1.4 Literature Review

1.4.1 MIMO Wireless Linear Precoding

Multiple-input multiple-output (MIMO) systems enable a growth in transmission rate linear in the minimum number of antennas at transmitter or receiver. MIMO techniques also enhance link reliability and improve coverage [7].

Beamforming is a processing technique that exploits CSIT by operating on the signal before transmission. For many common forms of partial CSIT, a linear beamformer is optimal from an information theoretic view point [2], [8], [9],[10]. A linear beamformer essentially functions as a multimode beamformer, optimally matching the input signal on one side to the channel on the other side. It does so by splitting the transmit signal into orthogonal spatial eigenbeams and assigns higher power along the beams where the channel is strong but lower or no power along the weak. Precoding design varies depending on the types of CSIT and the performance criterion.

MIMO Transmit beamforming allows several receivers to receive downlink data simultaneously on same time-slot and frequency resources, by maximizing signal power at each receiver using appropriate beamforming matrix. Optimal Capacity achieving coding is Dirty Paper Code (DPC) [5],which requires considerable computational complexity. Simpler linear beamforming schemes are designed by exploiting Channel State
Information at the Transmitter (CSIT) with different goals in mind like minimizing Bit Error Rate (BER), maximizing Signal to Interference and Noise Ratio (SINR), and Channel Ergodic Capacity [7].

Zero Forcing (ZF) Transmit Beamforming [11] cancels interference among users. However, closely spaced antennas at user terminals, introduce inter-stream interference. Canceling inter-stream interference will result in noise enhancement that will cause performance loss. The authors in [12] designed a Successive Minimum Mean Square Error (SMMSE) beamformer to mitigate the interference among users taking into account the effect of closely spaced receive antennas. In [10] and [2], the authors proposed a new design metric named Signal to Leakage and Noise Ratio (SLNR) and designed an SLNR maximizing beamformer. The SLNR based beamformer was shown to outperform Zero Forcing choice in general MIMO configuration, and is shown in our work to outperform SMMSE as well. However, the requirements of computing complex matrix inversion and calculating the generalized dominant eigenvector is very high. In this thesis we propose methods of developing an efficient form of SLNR based beamformer with a much lower complexity.

1.4.2 Scheduling Algorithms for MIMO Based Wireless Systems

When the number of users becomes greater than the number of transmit antennas, multiuser interference increases and causes severe performance loss that cannot be mitigated by transmit beamforming alone. It is then necessary to select a subset of the data streams intended for specific users for transmission to effectively reduce multiuser interference. Scheduling transmission of selected users exploits multi-user diversity gain in MIMO Systems. This can be done by applying cross layer design scheduling algorithm that tracks channel conditions for each user and selects the best users that achieve particular system design goals [13]-[14]. There are several approaches for scheduling algorithms in multiuser MIMO systems such as opportunistic transmission which selects the user with maximum channel gain, proportional fairness in which fairness among the users is considered, and QoS aware scheduling that considers QoS require-
ments of different applications [13]. In this thesis, we consider the sum SINR maximizing approach, which can be extended to account for fairness and QoS requirements among users [15],[13].

1.4.3 Receive Antenna Selection

Employing multiple receive antennas in multiuser MIMO systems, increases CCI among users. To mitigate this CCI by user scheduling only, the scheduler packs a lower number of users in the same scheduling instance resulting in noticeable throughput loss. Receive antenna selection overcomes this problem by selecting only partial receive antennas at each user such that the total SINR is maximized. Two approaches are mainly employed to perform receive antenna selection. The first [16] is to shutdown some of the receive antennas of certain users not satisfying an SINR threshold. A more general, and proven to be a better approach [17], is by selecting receive antennas that maximizes the SINR for each user.

1.5 Conclusion

Multiple antennas can be employed in wireless systems in several ways.

- **Diversity gain.** Use of the space-diversity provided by the multiple antennas to improve the robustness of the transmission against multipath fading.
- **Array gain.** Concentration of energy in one or more given directions via precoding or beamforming. This also allows multiple users located in different directions to be served simultaneously (so-called multi-user MIMO).
- **Spatial multiplexing gain.** Transmission of multiple signal streams to a single user on multiple spatial layers created by combinations of the available antennas.

The single most important requirement for MIMO multiplexing gain to be achieved is for the various transmit and receive antennas to experience a sufficiently different channel response. Also, there exists a trade off between the number of streams that may be multiplexed across the MIMO channel and the amount of diversity that each one of
them will enjoy.

MIMO has a great potential in modern wireless standardization efforts. A real challenge that meet practical implementation of Mu-MIMO RRM algorithms is its computational requirements. These RRM procedures are designed in this thesis while exploiting cross-layer information and reducing computational requirements.
Chapter 2

System Modeling and Simulation

Framework

2.1 Mu-MIMO System Model

We consider a Multiuser MIMO (Mu-MIMO) system consisting of a single base station communicating with $K$ users. The base station has $N$ transmitting antennas, such that $K > N$. The system employs Space-Division-Multiple-Access (SDMA), by spatially multiplexing a subset of users on same frequency $f$ and same timeslot $n$. In this setup we assume that each subscriber station $i$ is equipped with $M_i$ receive antennas. For simplicity of presentation equal number of receive antennas $M$ at each subscriber station is assumed. Figure 2.1 shows a block diagram of the system. The link adaptation and Resource assignment block is responsible for coordinating cross layer interactions between PHY and MAC layers. The proposed joint user and receive antenna selection should reside in this block to enable control on scheduling, beamforming and antenna summation blocks.

The task of user selection (scheduler) is to select a user group $S$ (where $|S| = L$) out of the $K$ users in the system for transmission. $L$ should be less than or equal to $N$ at any time $n$, to be able to have efficient beamforming (BF) vectors that in turn optimizes a performance goal metric such as BER or SINR. Different beamforming schemes with
different performance goals are elaborated in the next section. We define $T_i$ as the number of active receive antennas per user $i$. The scheduler is assumed to select a subset, $T_i$, of the receive antennas for each user $i$ jointly with user scheduling. Each of the selected data streams $s_i$ is multiplied by the beamforming vector $w_i \in \mathbb{C}^{N \times 1}$, then passed to the antenna summation block for summation and transmission. Receive antenna selection is performed jointly within the scheduler block. Antenna summation block adds the input data streams and transmits the resultant signal on the transmit antennas. The transmitted vector $x \in \mathbb{C}^{N \times 1}$ at time $n$ is given by

$$x(n) = \sum_{\ell \in S} w_{\ell} s_{\ell}(n)$$

(2.1)

where $|s_{\ell}|^2 = 1$,

and $\|w_{\ell}\|^2 = 1, \forall \ell \in S$

Throughout the thesis, the time index $n$ is omitted for notational simplicity. The two conditions guarantee normalized data streams as well as normalized beamforming coefficients. Equal power allocation among the spatial dimension is assumed (i.e. no water pouring is used). The vector $x$ is then transmitted and is affected by a different
spatial channel $H_i \in \mathbb{C}^{T_i \times N}$ for each user. Hence the received vector $y_i \in \mathbb{C}^{T_i \times 1}$ at the $i^{th}$ user can be written as

$$y_i = H_i \sum_{\ell \in S} w_\ell s_\ell + z_i$$

where $z_i$ is independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian noise for user $i$ with distribution $\mathbb{C}\mathcal{N}(0, \sigma_i^2)$ and is spatially white. The average Signal-to-Noise Ratio (SNR) at each receive antenna of user $i$, is given as $\frac{1}{\sigma_i^2}$, independent of the number of transmitting antennas. $H_i$ is the $T_i \times N$ channel matrix and is represented as

$$
\begin{bmatrix}
    h_i^{(1,1)} & \ldots & h_i^{(1,N)} \\
    \vdots & \ddots & \vdots \\
    h_i^{(T_i,1)} & \ldots & h_i^{(T_i,N)}
\end{bmatrix}
$$

where $h_i^{(r,t)}$ is the channel coefficient from the $t^{th}$ transmit antenna at the base station to the $r^{th}$ receive antenna at user $i$. These coefficients are modeled as complex Gaussian random variables with zero mean and unit variance. The wireless channel is assumed to be constant over a packet length and changes independently on a packet by packet basis. Realistic channel models based on the work done in WINNER project [1] are considered in the numerical results. We assume that $H_i$ is available either through efficient feedback mechanism in Frequency-Division-Duplex (FDD) systems, or through reverse channel estimation in Time-Division-Duplex (TDD) systems. Each spatial channel $H_i$ is assumed to be known at the corresponding user, but not necessarily known or shared by other users. It is assumed that user $i$ is estimating received symbols $\hat{s}_i$ from $y_i$ using classical maximum likelihood detection $\hat{s}_i \equiv \frac{w_i^H H_i^H y_i}{\| H_i w_i \|^2}$. The output SINR for user $i$ would be given by [2]}

$$SINR_i = \frac{\| H_i w_i \|^2}{\sigma_i^2 + \frac{\sum_{k=1, k \neq i}^{K} \| w_i^H H_i^H H_i w_k \|^2}{\| H_i w_i \|^2}}$$

(2.3)
2.2 WINNER Channel Model

The Wireless World Initiative New Radio (WINNER) project [1] is a collaborative research initiative, dedicated to addressing the challenge of 4G air interface design and specification. Within its simulation framework, WINNER project integrated spatial channel model for MIMO simulations. The covered propagation scenarios are indoor office, large indoor hall, indoor-to-outdoor, urban micro-cell, bad urban micro-cell, outdoor-to-indoor, stationary feeder, suburban macro-cell, urban macro-cell, rural macro-cell, and rural moving networks.

The channel model is a geometry-based stochastic model. The model allows simulation of different antenna configuration. The channel parameters are determined stochastically, based on statistical distributions extracted from channel measurement [18]. The distributions are defined for, e.g., delay spread, delay values, angle spread, shadow fading, and cross-polarization ratio. For each channel snapshot the channel parameters are calculated from the distributions. Channel realizations are generated by summing contributions of rays with specific channel parameters like delay, power, angle-of-arrival and angle-of departure. Different scenarios are modeled by using the same approach, but different parameters. The parameter tables for each scenario are included in this deliverable. The model supports LOS and NLOS propagation conditions.

As a ray based channel model, the channel coefficient between each transmit and receive antenna pair is the summation of all rays at each tap and at each time instant, according to the antenna configuration, gain pattern, and the amplitude, AoA, AoD of each ray. The temporal channel variation depends on the traveling speed and direction relative to the AoA/AoD of each ray.

Through the thesis we promote a Typical Urban Micro Cell environment as our baseline simulation. This model is defined in [18] as ‘B1’ with NLOS propagation. The height of both the antenna at the BS and at the MS is assumed to be well below the tops of surrounding buildings. Both antennas are assumed to be outdoors in an area where streets are laid out in a Manhattan-like grid. This scenario is defined for both the LOS and the NLOS cases.
2.2.1 Path-Loss Model

Based on measurement WINNER developed a set of path-loss models for each simulation scenario. We discuss here path-loss model applied in case of typical urban environment. The shadow fading standard deviation used in this scenario are as \( \sigma = 3 \) for LOS and \( \sigma = 4 \) for NLOS. In case of LOS operation the following path-loss model is applied [1]

\[
PL_{LOS} = 22.7 \log_{10}(d_1) + 41 + 20 \log_{10}\left(\frac{f_c}{5}\right), \quad 10m < d_1 < d_{BP} \tag{2.4}
\]

\[
\text{and} \quad 40 \log_{10}(d_1) + 9.45 - 17.3 \log_{10}(h_{BS}') - 17.3 \log_{10}(h_{MS}') + 2.7 \log_{10}\left(\frac{f_c}{5}\right), d_{BP} < d_1 < 5km \text{ and } h_{BS} = 10m, \ h_{MS} = 1.5m \tag{2.5}
\]

where \( d_{BP} = 4h_{BS}h_{MS} \frac{f_c}{c} \), where \( f_c \) is the center frequency in Hz, \( c \) is the propagation velocity in free space, and \( h_{BS} \) and \( h_{MS} \) are the effective antenna heights at the BS and the MS, respectively. In case of NLOS operation [1]

\[
PL_{NLOS} = \min(PL(d_1, d_2), PL(d_2, d_1)) \tag{2.6}
\]

where

\[
PL(d_k, d_l) = PL_{LOS}(d_k) + 20 - 12.5n_j + 10n_j \log_{10}(d_1) + 3 \log_{10}\left(\frac{f_c}{5}\right), \quad k \in \{1, 2\}
\]

\[
\text{and} \quad n_j = \max(2.8 - 0.0024d_k, 1.84)
\]

\( PL_{LOS} \) is applicable for \( d_1 \) and \( d_2 \) in the ranges

\[
10m < d_1 < 5km, \quad \frac{w}{2} < d_2 < 2km
\]

\[
w = 20m(\text{streetwidth})
\]

\[
h_{BS} = 10m, \ h_{MS} = 1.5m
\]

When \( 0 < d_2 < \frac{w}{2} \), the LOS model is applied. These \( d_1 \) and \( d_2 \) distances are defined with respect to a rectangular street grid, as illustrated in Figure 2.2, where the MS is shown moving along a street perpendicular to the street on which the BS is located.
(the LOS street). $d_1$ is the distance from the BS to the center of the perpendicular street, and $d_2$ is the distance of the MS along the perpendicular street, measured from the center of the LOS street.

![Geometry for $d_1$ and $d_2$ path-loss model](image)

**Figure 2.2: Geometry for $d_1$ and $d_2$ path-loss model**

### 2.2.2 Channel Segments, and Drops

Channel segment represents a period of quasi-stationarity during which probability distributions of low level parameters are not changed noticeably. During this period all large-scale parameters, as well as velocity and direction-of-travel for mobile station (MS), are practically constant.

Allowing the channel segment length go to zero, we specify a drop: In a drop all parameters are fixed, except the phases of the rays. Motion within a drop is only virtual and causes fast fading and the Doppler effect by superposition of rotating phasors, rays. It can be said, that a drop is an abstract representation of a channel segment, where the inaccuracies caused by the change of the terminal location have been removed.
2.2.3 Cluster Delay Line model

Clustered delay line (CDL) model is used by WINNER in which the fading process for each tap is modeled in terms of a sum of sinusoids. The CDL model describes the propagation channel as being composed of a number of separate clusters with different delays. Each cluster, in turn, is composed of a number of multi-path components (rays) that have the same delay values but differ in angle-of-departure and angle-of-arrival. The angular spread within each cluster can be different at the BS and the MS. The offset angles represent the Laplacian Power Angle Spectrum (PAS) of each cluster. The average power, mean AoA, mean AoD of clusters, angle-spread at BS and angle-spread at MS of each cluster in the CDL represent expected output of the stochastic model with parameters listed in Table 2.1. Parameter tables for the CDL models are given in Table 2.2 and Table 2.3 for the cases of LOS and NLOS respectively.

<table>
<thead>
<tr>
<th>Scenario</th>
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<tr>
<td>NLOS</td>
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<table>
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<tr>
<th>DS (ns)</th>
<th>AS at BS (°)</th>
<th>AS at MS (°)</th>
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</thead>
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<td>3</td>
<td>25</td>
</tr>
<tr>
<td>76</td>
<td>15</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 2.2: LOS Cluster Delay Line model parameters Cluster ASD = 3°, Cluster ASA = 18°, XPR = 9 dB

<table>
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<td>-31.4</td>
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</tbody>
</table>

A Rician fading channel is used to model dominant ray component in LOS case. In the LOS model Ricean K-factor (defined as the ratio of signal power in dominant
Table 2.3: NLOS Cluster Delay Line model parameters Cluster ASD = 10°, Cluster ASA = 22°, XPR = 8 dB

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</tr>
<tr>
<td>16</td>
<td>615</td>
<td>-29.9</td>
<td>46</td>
<td>-107</td>
<td>-42.9</td>
</tr>
</tbody>
</table>
component over the scattered power) is 3.3 dB. The simulation results shown in rest of the thesis were carried out applying simulation parameters in Table 2.4

Table 2.4: WINNER [1] Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>[5 8 4 3 10]</td>
</tr>
<tr>
<td>M</td>
<td>[3 2 1 1 2]</td>
</tr>
<tr>
<td>Scenario</td>
<td>'B1' (Typical Urban Micro Cell environment)</td>
</tr>
<tr>
<td>PropagCondition</td>
<td>'NLOS' (Non Line Of Sight)</td>
</tr>
<tr>
<td>NumSubPathsPerPath</td>
<td>10</td>
</tr>
<tr>
<td>BsGainPattern</td>
<td>1 (Directive)</td>
</tr>
<tr>
<td>BsElementPosition</td>
<td>0.5 (Ratio of wavelength)</td>
</tr>
<tr>
<td>MsGainPattern</td>
<td>1 (Omi)</td>
</tr>
<tr>
<td>MsElementPosition</td>
<td>0.5 (Ratio of wavelength)</td>
</tr>
<tr>
<td>K</td>
<td>{10, 20, 30, 40, 50, 60}</td>
</tr>
<tr>
<td>SNR</td>
<td>-10 to 50 dB</td>
</tr>
</tbody>
</table>

2.3 Channel Estimation Error Model

We consider the case of channel estimation error, particularly found in TDD systems. This model applies as well for FDD systems with lossless channel feedback. The error model introduces uncertainty to the exact channel gain as indicated in (2.7).

\[ \hat{H}_i = H_i + \Theta_i \]  

(2.7)

Here \( \Theta_i \) is i.i.d. circularly symmetric complex Gaussian estimation error of user \( i \) with distribution \( \mathbb{C}^{M \times N} \mathcal{N}(0, \sigma_e^2) \). Estimation quality is assumed to be equal for all users for simplicity and expressed in dB as \( \frac{1}{\sigma_e^2} \).

2.4 Conclusion

In this chapter we have developed multiuser MIMO system model that is to be employed throughout this work. The model consists of a single base station commu-
communicating with several users. The scheduler selects optimal set of users that optimize certain objective function. In this thesis, we are mainly interested about maximizing system throughput. The link adaptation and Resource assignment block is responsible for coordinating cross layer interactions between PHY and MAC layers. Antenna summation block effectively transmits data streams according to receive antenna selection decision by link adaption and resource assignment block.

WINNER Channel model promises accurate and efficient spatial channel models. It covers different propagation scenarios including indoor office, large indoor hall, indoor-to-outdoor, urban micro-cell, bad urban micro-cell, outdoor-to-indoor, stationary feeder, suburban macro-cell, urban macro-cell, rural macro-cell, and rural moving networks. Channel model in WINNER is a geometry-based stochastic model. The channel parameters are determined stochastically, based on statistical distributions extracted from channel measurement.

Typical wireless systems suffer from imperfections in practical implementations. One major imperfection that greatly affected flow of this work is channel estimation error. We adopted a simplified channel estimation error model that typically influences TDD systems and FDD systems with lossless feedback channels.
Chapter 3

Mu-MIMO Transmit Beamforming

3.1 Introduction

In this chapter, conventional single user beamforming, multiuser Zero Forcing beamforming (ZF), multiuser Successive MMSE beamforming (SMMSE), and multiuser Signal-to-Leakage-and-Noise-Ratio based beamforming (SLRBF) are defined and presented. We discuss the well known Zero forcing beamforming, and the present the concept of leakage [2]. The Signal-To-Leakage-And-Noise ratio based beamformer is then discussed in details. A comparison between different beamforming techniques is then shown. We further discuss the effect of increasing the scheduled number of user on performance of the transmit beamforming. We end our demonstration by current method of realizing beamforming in LTE system.

3.2 Conventional single user beamforming

Conventional single user beamforming [5] maximizes SINR for the user ignoring co-channel interference (CCI) from other users in the system. The beamforming vector for single user beamforming is computed as in (3.1) and is mentioned here for comparison with our proposed scheme and completeness of presentation.
\begin{equation}
\mathbf{w}^{(SU)}_i = \Lambda_{\text{max}} (\mathbf{H}_i^H \mathbf{H}_i)
\end{equation}

\section{3.3 Zero Forcing Transmit Beamforming}

The zero forcing beamformer cancels Co-Channel Interference (CCI) completely by using pseudo-inversion to force $\mathbf{H}_i \mathbf{w}_k$ to zero for all users except user $i$. For simplicity, we present the case of single receive antenna where the beamforming vector is computed as follows \cite{11}:

Let $\mathbf{H}(\mathcal{S})$ be the concatenated channel vectors for the set of scheduled users $[h_i h_j ... h_S]^H$. Where $h_i \in \mathbb{C}^{1 \times N}$ is the channel matrix vector of user $i$.

If users employ multiple receive antennas, we can apply the formula in (3.2) by treating each receive antenna as a separate receiver. A beamforming vector results for each receive antenna, where the same user data is multiplied by all receive antenna beamforming vectors. The antenna summation block in Figure 2.1, then adds resulting signals. We can also apply Block Diagonalization (BD) beamforming explained in \cite{11}.

ZF beamforming requires the following condition on the number of transmit antennas

\begin{equation}
N \geq \sum_{\ell \in \mathcal{S}} T_\ell
\end{equation}

\section{3.4 Successive MMSE Transmit Beamforming}

Successive MMSE deals with the problem of interference between close antennas \cite{12}. Interference lessening is achieved by successively computing the beamforming matrix $\mathbf{F}_i$ for each receive antenna separately. The beamforming vector for SMMSE is defined as
\[
\mathbf{w}_i^{(\text{SMMSE})} = \Lambda_{\text{max}} (\mathbf{H}_i \mathbf{F}_i) \tag{3.4}
\]

where

\[
\mathbf{F}_i = \left[ \mathbf{F}_i^{(1)} \mathbf{F}_i^{(2)} \ldots \mathbf{F}_i^{(j)} \right] \tag{3.5}
\]

\(\mathbf{F}_i^{(j)}\) is the beamforming matrix corresponds to the \(j\)-th antenna of user \(i\), such that

\[
\mathbf{F}_i^{(j)} = \left( \mathbf{H}_i^{(j)} \mathbf{H}_i^{(j)} + \sigma_i^2 \mathbf{I} \right)^{-1} \mathbf{H}_i^{(j)\mathsf{H}} \tag{3.6}
\]

and

\[
\mathbf{H}_i^{(j)} = \begin{bmatrix}
\mathbf{h}_{ij}^\mathsf{T} \\
\mathbf{H}_1 \\
\vdots \\
\mathbf{H}_i \\
\mathbf{H}_{i+1} \\
\vdots \\
\mathbf{H}_S
\end{bmatrix} \tag{3.7}
\]

where \(\mathbf{h}_{ij} \in \mathbb{C}^{1 \times N}\) is the channel matrix vector of user \(i\) and receive antenna \(j\). The formulation in (3.4) extracts maximum diversity and array gain of the MIMO system by transmitting users’ data on the dominant eigenmode of the corresponding user, while successively canceling interstream interference between closely spaced receive antennas [12]. The simulations showed that the performance of this scheme is worse than that promised by SLRBF, at similar complexity. This scheme can also be reformulated to maximize system capacity by using waterfilling on the eigenmodes of all users instead of transmitting on the dominant eigenmode.
3.5 Signal-To-Noise-And-Leakage based Transmit Beamforming

The Signal-to-Leakage-and-Noise based beamformer (SLRBF) is based on the concept of signal leakage [10]. While CCI refers to the interference at a desired user that is caused by all the other users, leakage refers to the interference caused by the signal intended for a desired user on the remaining users. That is, leakage is a measure of how much signal power leaks into the other users. In this scheme, the beamforming vector is chosen such that it maximize the Signal-to-Leakage-and-Noise-ratio (SLNR) for all users simultaneously. SLNR for user $i$ is given by [2]

$$SLNR_i = \frac{\|H_i w_i\|^2}{\sigma_i^2 + \sum_{k=1, k \neq i}^{K} \|H_k w_i\|^2}$$

(3.8)

Which can be written in reduced form as

$$SLNR_i = \frac{\|H_i w_i\|^2}{\sigma_i^2 + \|H_k w_i\|^2}$$

(3.9)

where

$$\tilde{H}_i = [H_i^H ... H_{i-1}^H H_{i+1}^H ... H_L^H]^H$$

(3.10)

Using Rayleigh-Ritz quotient, the beamforming vector that maximizes (3.9), is given by the generalized eigenvector corresponding to the dominant eigenvalue of the matrix pair $\sigma_i^2 I + \tilde{H}_i^H \tilde{H}_i$ and $H_i^H H_i$ [10]. Since $\sigma_i^2 I + \tilde{H}_i^H \tilde{H}_i$ is invertible, the beamforming vector is reduced to

$$w_i^{(slr)} = \Lambda_{max} \left( \left( \sigma_i^2 I + \tilde{H}_i^H \tilde{H}_i \right)^{-1} H_i^H H_i \right)$$

(3.11)

Taking the estimation error model given in (2.7) into consideration, the SLNR beamformer is modified as shown in (3.12) [2]. Where $\tilde{H}_i$ is the extended matrix, formulated using the estimated channel gain. SLRBF maximizes the SLNR and has substantial gain over the ZF beamformer and the conventional beamformer. Numerical
results also show that SLRBF outperforms SMMSE with similar computational complexity.

\[ w^{(slr)}_i = \Lambda_{\text{max}} \left( \sigma^2 \mathbf{I} + \tilde{\mathbf{H}}_i \tilde{\mathbf{H}}_i^H + \left( \sum_{k=1, k \neq i}^{K} T_k - T_i \right) \sigma^2 \mathbf{I} \right)^{-1} \left( \tilde{\mathbf{H}}_i \tilde{\mathbf{H}}_i^H + T_i \sigma^2 \mathbf{I} \right) \]

\[ (3.12) \]

### 3.6 Performance Comparison of Different Beamforming Techniques

![Figure 3.1: SER Comparison of ZF, SMMSE, and SLR Based beamformers](image)

A typical multi-user MIMO system is simulated to compare the performance of different beamforming schemes. While the single-user beamforming fails in a multi-user environment in terms of BER, the SLNR based beamformer and SMMSE provide an acceptable BER in the presence of 2 and 3 active users. In Figure 3.1 BER performance is shown. It’s noticed that SLNR outperforms ZF at all SINR range, and SMMSE for moderate and high SINR.
Figure 3.2: SINR Distribution Comparison ZF, SMMSE, and SLR Based beamformer

SINR distributions at $SNR = 8$ dB of different beamforming schemes are shown in Figure 3.2. The SLNR beamformer shows at least 2 dB gain of system capacity compared to SMMSE. Both SMMSE and SLNR require the same computational power due to channel inversion and eigenvector calculations.

### 3.6.1 Case of Large Number of Users

Increasing number of simultaneous users enhances total system capacity at the expanse of average user throughput. System capacity and average user throughput are shown in Figure 3.3 and Figure 3.4 respectively. Enhancement of total system capacity is only valid for low SNR regime. This is explained as multiuser interference is not dominant as noise in low SNR and there is a considerable gain from multiuser user diversity.

Degradation of average user throughput and system capacity at high SNR motivates developing of user scheduling so that the system efficiently exploits multi-user diversity and maximizes total system capacity. Design of efficient, low complexity user
scheduling is addressed later in the thesis.

Figure 3.3: Sum-Capacity ZF, SMMSE, and SLR Based comparing $K < N$ and $K > N$ scenarios

3.7 MIMO techniques in LTE

LTE adopted various MIMO technologies including transmit diversity, single user (SU)-MIMO, multiuser (MU)-MIMO, closedloop rank-1 precoding, and dedicated beamforming [4],[19].

The SU-MIMO scheme is specified for the configuration with two or four transmit antennas in the downlink, which supports transmission of multiple spatial layers with up to four layers to a given User Equipment (UE). The transmit diversity scheme is specified for the configuration with two or four transmit antennas in the downlink, and with two transmit antennas in the uplink. The MU-MIMO scheme allows allocation of different spatial layers to different users in the same time-frequency resource, and is supported in both uplink and downlink. The closed-loop rank-1 precoding scheme is used to improve data coverage utilizing SU-MIMO technology based on the cell-
Figure 3.4: Average User Rate ZF, SMMSE, and SLR Based comparing $K < N$ and $K > N$ scenarios

specific common reference signal while introducing a control signal message that has lower overhead. The dedicated beamforming scheme is used for data coverage extension when the data demodulation based on dedicated reference signal is supported by the UE [20].

In the closed-loop rank-1 precoding mode, the eNodeB operates the closed-loop SU-MIMO scheme based on the cell-specific common reference signal with the limitation of selecting a rank-1 precoding matrix for transmission to a UE among predefined Table for two transmit antennas and predefined Table for four transmit antennas to improve data coverage without relying on the UE-specific reference signal. Since the transmission rank is fixed to one in this mode, the related downlink control signaling overhead is smaller than the case of operating the closed-loop SU-MIMO scheme, of which control signaling allows full freedom of selecting the transmission rank among all possible rank values.

MU-MIMO scheme is supported in both the uplink and downlink of the LTE standard. In the uplink, the eNodeB can always schedule more than one UEs to transmit
in the same time-frequency resource, which forms a MU-MIMO transmission configuration. However, in order for the eNodeB to be able to correctly differentiate and demodulate these UEs signals, eNodeB needs to assign orthogonal reference signals for these UEs scheduled for the MU-MIMO transmission. For a given slot and subframe in each cell, a Zadoff-Chu sequence [21] is defined as the base sequence for uplink reference signals. The cyclically shifted versions of a given Zadoff-Chu sequence form an orthogonal set of sequences. Each UE scheduled for MU-MIMO transmission is assigned a distinctive cyclic shift value, and the UE combines this cyclic shift value with the knowledge of the base Zadoff-Chu sequence to form a reference signal sequence that is orthogonal to other UEs reference signal sequences. It is noted that the cyclic shift value is always contained in the control signaling, which the UE has to receive for data transmission on uplink, regardless whether the MU-MIMO is operated or not.

In the downlink, if a UE is configured to be in the MU-MIMO transmission mode, only rank-1 transmission can be scheduled to the UE. The eNodeB can schedule multiple UEs, which are configured to be in the MU-MIMO transmission mode, in the same time-frequency resource using different rank-1 precoding matrices from predefined tables. Note that the UE receives only the information about its own precoding matrix. The scheduled UE then decodes the information data utilizing the common reference signal together with the precoding information obtained from the control signaling. The UE generates the CQI feedback without any knowledge about other simultaneously scheduled UEs. Hence, there could be mismatch between the UEs CQI report and the actual CQI experienced due to lack of knowledge of interference caused by another UEs scheduled simultaneously. In LTE, for support of receiving higher-order modulation signal such as 16QAM and 64QAM without causing too much complexity in the UE, the transmit power level for each UE is configured in a long-term manner. The per-UE preconfigured power level is hard to maintain in MU-MIMO transmission mode, since eNodeB power amplifier has to support multiple UEs scheduled on the same time-frequency resource. A 1-bit signaling is therefore introduced to indicate whether there is 3 dB power reduction with respect to the per-UE configured power level if a UE is configured in the MU-MIMO transmission mode.
3.8 Support of Beamforming in IEEE 802.16 Standard

IEEE 802.16e supports both open-loop and closedloop MIMO. Open-loop MIMO techniques include spatial multiplexing (SM) and space-time coding (STC) [22]. 802.16e includes support for up to four spatial streams and therefore a maximum of $4 \times 4$ MIMO configuration. STC is based on the Alamouti scheme (also STBC) and is also called space-time transmit diversity (STTD). It is an optional feature and may be used to provide higher order transmit diversity on the downlink [22].

In closed-loop MIMO, full or partial CSI is available at the transmitter through feedback. Eigenvector steering is employed to approach full capacity of the MIMO channel and water filling can be used to maximize throughput by allocating power in an optimal manner. IEEE 802.16e supports closed-loop MIMO precoding for SM and also closed-loop STC. However, closed-loop MIMO is not yet supported in the latest WiMAX Forum Wave 2 profiles. Another MIMO mode called collaborative spatial multiplexing is also specified where two subscriber stations (SS), each having a single antenna, use the same subchannel for uplink transmission in order to increase the throughput.

The adaptive antenna systems (AAS) supported in 802.16e also include closed-loop adaptive beamforming, which uses feedback from the SS to the base station (BS) to optimize the downlink transmission.

3.9 Conclusion

In this chapter we developed a ground base of different multiuser MIMO transmit beamforming found in literature. We reviewed the beamformer based on what is called Signal-To-Leakage based beamformer. SLNR based beamformer is minimizeing signal leakage from one user to all other users in the system. The well known Zero forcing beamforming is compared to SMMSE and SLNR based beamformer. It’s noticed that SLNR based outperforms ZF at all SINR range in terms of Symbol Error Rate. It also outperforms SMMSE for moderate and high SINR. SLNR based is also outperforming
both SMMSE and ZF in terms of capacity as we demonstrated by SINR distribution.

In case of large number of users, multiuser diversity provides a gain on system capacity at low SNR. This diversity gain vanishes at moderate and high SNR. On the other hand average user throughput degrades as a result of bad resource sharing. Schedulers are employed to utilize multiuser diversity at all SNR range by effectively maximizing SNR. This comes at new computational requirements as we will discuss later in this thesis.
Chapter 4

Proposed Low Complexity Mu-MIMO Beamforming

4.1 Introduction

A fundamental problem with SLRBF is its computational complexity. SLRBF requires computing the inverse of a matrix and the generalized eigenvector corresponding to the maximum eigenvalue for each user in the system. The complexity of these computations increases polynomially with increasing the number of users and the number of transmit antennas. Matrix inversion has $O(N^3)$ computational complexity, and calculating the eigenvectors of a matrix has $O(N^3)$ or $O(N^2)$ in case of parallel implementation [23]. Combined together, these complexities make SLRBF implementations impractical and unfeasible for large scale systems.

In this thesis, we propose a simplified beamforming technique derived from the SLNR based beamformer. Our approach utilizes numerical algorithms to evaluate matrix inversion and eigenvectors. We employ Neumann series to compute the matrix inverse, and study the properties of the matrix $\sigma_i^2 I + \tilde{H}_i^H \tilde{H}_i$ that lead to convergence in an adequate number of iterations. Jacobi-Davidson (JD) algorithm [24] is then used to find an approximate value for the dominant eigenvector. Convergence of JD algorithm with different starting vectors is studied. Based on the two numerical techniques, we
propose a beamformer equation that significantly reduces the computational complexity with minimal performance loss, specially when channel estimation error is encountered.

### 4.2 Mathematical Background

In this section we introduce mathematical tools employed in the derivation of the proposed low complexity transmit beamforming. We start by numerical method of matrix inversion calculation, then eigenvector computation.

#### 4.2.1 Matrix Inversion of Sums

A common matrix sum that often requires inversion is \( I - A \). The inverse \((I - A)^{-1}\) exist if the elements of \( A \) are sufficiently small so that \( \lim_{n \to \infty} A^n = 0 \).

**Theorem 1.** (Neumann Series [24]) *If \( \lim_{n \to \infty} = 0 \) then \( I - A \) is nonsingular and.*

\[
(I - A)^{-1} = I + A + A^2 + \ldots = \sum_{r=0}^{\infty} A^r. \tag{4.1}
\]

It provides approximations of \((I - A)^{-1}\) when \( A \) has entries of small magnitude. To insure the convergence condition we write:

\[
\rho (A) < 1 \tag{4.2}
\]

#### 4.2.2 Jacobi-Davidson Algorithm

Eigenpair calculation is necessary in transmit beamforming and applied in several beamforming methodologies. The Jacobi-Davidson Algorithm finds an approximated value of the eigenpair of a matrix \( A \) corresponding to the largest eigenvalue of the matrix. The basic for of the algorithm is listed in Table.4.1.

The details of the algorithm are described as follows

**INIT:** The search subspace is expanded in each iteration by a vector \( t \), and we start this process with a given vector \( t = v_0 \). Ideally, this vector should have a significant component in the direction of the wanted eigenvector.
Table 4.1: Jacobi-Davidson (JD) Algorithm

Require: \( t = v_0 \) \{Starting guess\}

Ensure: \( (\tilde{\lambda}, \tilde{x}) \) approximated eigenpair for the eigenpair of the matrix \( A \)

1: for \( m = 1, \ldots \) do
2:    for \( i = 1, \ldots, m - 1 \) do
3:        \( t = t - (v_i^*t) v_i \)
4:    end for
5:    \( v_m = t \| t \|_2, v_m^A = Av_m \)
6:    for \( i = 1, \ldots, m - 1 \) do
7:        \( M_{i,m} = v_i^*v_m^A \)
8:    end for
9:    Compute the largest eigenpair \((\theta, s)\) of \( M (\| s \|_2 = 1) \)
10:   \( u = Vs \)
11:   \( u^A = V^A s \)
12:   \( r = u^A - \theta u \)
13:   if \( \| r \|_2 \leq \epsilon \) then
14:       \( \tilde{\lambda} = \theta, \tilde{x} = u \)
15:    Break
16:    end if
17:   Solve (approximately) a \( t \perp u \) from \((I - uu^*) (A - \theta I) (I - uu^*) t = -r\)
18: end for

Unless one has some idea of the wanted eigenvector, it may be a good idea to start with a random vector. This gives some confidence that the wanted eigenvector has a nonzero component in the starting vector, which is necessary for detection of the eigenvector.

(2)-(4): This represents the modified Gram-Schmidt process for the orthogonalization of the new vector \( t \) with respect to the set \( v_1, \ldots, v_{m-1} \). If \( m = 1 \), this is an empty loop.

(6)-(8): In this phase, computation of the \( m \)th column of the upper triangular part of the matrix \( M \equiv V^*AV \) occurs. The matrix \( V \) denotes the \( n \) by \( m \) matrix with columns \( v_j, V^A \) likewise

(9): Computing the largest eigenpair of the \( m \times m \) Hermitian matrix \( M \), with elements \( M_{i,j} \) in its upper triangular part.
The vector $u^A$ may either be updated as described here or recomputed as $u^A = Au$, depending on which is cheapest. The choice is between an $m$-fold update and another multiplication with $A$; if $A$ has fewer than $m$ nonzero elements on average per row, the computation via $Au$ is preferable. If $u^A$ is computed as $Au$, it is not necessary to store the vectors $v_j^A$.

The algorithm is terminated if $\|Au - \theta u\|_2 \leq \epsilon$. In that case $A$ has an eigenvalue $\lambda$ for which $|\lambda - \theta| \leq \epsilon$.

The approximate solution for the expansion vector $t$ can be computed with a Krylov solver.

### 4.3 Low Complexity Transmit Beamforming

The SLRBF in (3.11) can be written as

$$w_i^{(slr)} = \Lambda_{\text{max}} \left( (\psi E_i)^{-1} \tilde{H}_i^H \tilde{H}_i \psi \right)$$  \hspace{1cm} (4.3)

where

$$E_i = \sigma^2 I + \tilde{H}_i^H \tilde{H}_i$$  \hspace{1cm} (4.4)

$\psi$ is a scaling factor. By applying Neumann series

$$(\psi E_i)^{-1} = \sum_{r=0}^{\infty} (I - \psi E_i)^r$$  \hspace{1cm} (4.5)

Provided that [24]

$$|1 - \rho(\psi E_i)| < 1$$  \hspace{1cm} (4.6)

where $\rho(\cdot)$ is defined as the maximum eigenvalue of a matrix. This implies that

$$0 < \rho(\psi E_i) < 2$$  \hspace{1cm} (4.7)

Since $E_i$ is a positive definite matrix [2], then for convergence

$$0 < \psi < \frac{2}{\rho(E_i)}$$  \hspace{1cm} (4.8)
Since
\[ Tr(E_i) > \rho(E_i) \]  

(4.9)

We choose a more strict, and computationally simpler bound as
\[ 0 < \psi < \frac{2}{Tr(E_i)} \Rightarrow 0 < \psi < \frac{2}{\rho(E_i)} \]  

(4.10)

\[ w_i = \Lambda_{\text{max}} \left( \sum_{r=0}^{R} (I - \psi E_i)^r H_i^H H_i \right) \]  

(4.11)

Let
\[ A = \sum_{r=0}^{R} (I - \psi E_i)^r H_i^H H_i \]  

(4.12)

We then apply Jacobi-Davidson algorithm to compute the dominant eigenvector of matrix \( A \). The starting vector in JD should be chosen such that it does not belong to the null space of the matrix to ensure convergence. The starting vector is usually chosen randomly [24]. However, for the algorithm to converge in an adequate number of iterations, the starting vector should be chosen such that it has a significant component in the direction of the dominant eigenvector [25]. We propose to use the dominant column of \( A \) (i.e. the column corresponding to the maximum absolute diagonal element of \( A \)) as a starting vector. In the special case of a diagonal matrix the dominant eigenvector of the matrix is exactly the dominant column of the matrix, so we propose to choose the dominant column vector as a starting vector in the JD algorithm. Using the dominant column results in faster converge than if a random vector is used as a starting vector in JD algorithm. As will be shown later, 3 iterations of the JD algorithm are the maximum number of iteration needed. To further simplify the computation of the beamformer, we propose the following beamformer vector that directly uses the dominant column instead of the dominant eigenvector without applying the JD algorithm.

\[ w_i^{(SSLR)} = d \left( \sum_{r=0}^{R} (I - \psi E_i)^r H_i^H H_i \right) \]  

(4.13)

The proposed beamformer given by (4.13) transforms the computational complexity of the problem to be linear in \( N \) and \( R \) (\( \in O(RN) \)) instead of exponential com-
plexity. The effect of using the dominant column is evaluated in Section 4.5. Typically, when $R$ is in the range of 20 to 70, $w_i^{(SSLR)}$ becomes very close to $w_i^{(SLR)}$. Determination of the value of $R$ depends on the operating range of the SNR. For low SNR, the matrix $E$ in (4.4) tends to be less random and Neumann series requires few iterations to converge. When the SNR increases, the matrix becomes more random and $w_i^{(SSLR)}$ requires more iterations to converge to the SLNR beamformer vector. In simulations, we take $\psi$ equals $\frac{2}{Tr(E_i)}$, which ensures fastest convergence of Neumann series for low and medium range of the SNR.

Symbol Error Rate and SINR CDF of the proposed beamformer are very close to that of the SLNR based beamformer when $N \geq \sum_{\ell \in S} T_\ell$. When $N < \sum_{\ell \in S} T_\ell$, SINR degrades as shown later in Section 4.5. The constraint ($N \geq \sum_{\ell \in S} T_\ell$) is analogous to the constraint for successful operation of zero-forcing beamformer. In the next section, we propose a simplified user and receive antenna selection algorithm that ensures satisfaction of this constraint while maximizing the system sum rate.

### 4.3.1 Proposed Beamformer with Channel Estimation Error

Following similar approach as we had in (3.12), the proposed beamformer in case of estimation error is modified to be

\[
\begin{align*}
    w_i^{(SSLR)} &= d \left( \sum_{r=0}^{R} \left( I - \hat{\psi} \hat{E}_i \right)^r \left( \hat{H}_i^H \hat{H}_i + T_i \sigma^2 \sigma^2 I \right) \right) \\
    &= \psi \left( \sum_{r=0}^{R} \left( I - \hat{\psi} \hat{E}_i \right)^r \left( \hat{H}_i^H \hat{H}_i + T_i \sigma^2 \sigma^2 I \right) \right) \\
    &= \psi \left( \sum_{r=0}^{R} \left( I - \hat{\psi} \hat{E}_i \right)^r \left( \hat{H}_i^H \hat{H}_i + T_i \sigma^2 \sigma^2 I \right) \right)
\end{align*}
\]

where

\[
\hat{E}_i = \sigma_i^2 I + \tilde{\sigma}_i^2 \tilde{H}_i^H \tilde{H}_i + \left( \sum_{k=1, k \neq i}^{K} T_k - T_i \right) \sigma_c^2 I
\]

and

\[
0 < \hat{\psi} < \frac{2}{Tr(E_i)}
\]
4.4 Discussion on Convergence

To study the effectiveness of Neumann series in finding an approximate value of \((\psi E_i)^{-1}\), we show in Table 4.2 the number of iterations required (i.e. series order) to achieve a \(\pm 1\%\) error between the exact and the approximated inverse. It should be emphasized that the randomness of the matrix \(E_i\) affects convergence of Neumann series. For low SNRs, Neumann series converges faster than for higher SNRs as the scaling factor \(\psi\) is near unity in case of low SNR.

Table 4.2: Convergence of Neumann Series \(K = 3\), equal \(T_i\) for all users.

| SNR | \(N = 9\)  \
<table>
<thead>
<tr>
<th></th>
<th></th>
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<td>67</td>
<td>70</td>
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<td>58</td>
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Table 4.3 shows the number of iterations required by Jacobi-Davidson method such that a \(\pm 1\%\) error between the calculated approximate eigenvector and the exact one is achieved. The dominant column choice converges in less than one iteration with 70% probability in cases of \(N = 5\). This is at least seven times faster than using a random starting vector as the initial vector.

Table 4.3: Convergence of Jacobi-Davidson Algorithm \(K = 3\), comparing Dominant Column (DC) and Random Vector (RV) as starting vectors and equal \(T_i\) for all users

| SNR | \(N = 9\)  \
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<td>9</td>
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</table>
4.5 Numerical Results

The performance of the beamformer proposed in (4.13) referred to hereafter as SSLRBF is demonstrated in comparison to the SLNR-based beamformer given in (3.11), ZF beamformer given in (3.2), SMMSE beamformer given in (3.4) and Conventional single user beamformer given in (3.1) and referred to in the figure as MAXEIG. Performance is expressed in BER curves for different beamformers and Cumulative Density Function of SINR at the receiver and we take $\psi$ equals $\frac{2}{Tr(E_i)}$. For clarity of presentation, we assume in this section that the scheduler selects exactly $K$ users (i.e. $L = K$) and no receive antenna selection is performed (i.e. $T_i$ is equal for all users and $T_i = M$).

We first demonstrate the effect of satisfying the condition $N \geq \sum_{i \in S} T_i$ on the SINR distribution of SSLRBF compared to the SLNR beamformer. It’s the task of the scheduler to satisfy this condition all the time by solving the formulated quadratic problem in (5.14). Figure 4.1 shows the CDF of the SINR of the proposed beamformer with $K = 3$, Neumann series order $R = 10$, and $SNR = 10dB$. The performance

![Figure 4.1: CDF of SINR of the proposed beamformer with $R = 10$ compared to SLNR beamformer at $SNR = 10dB$, $\frac{1}{\sigma^2_c} = 10dB$, and $L = 3$. Shown two cases with $N \geq \sum_{i \in S} T_i$ ($N = 8$ and $T_i = 2$) and $N < \sum_{i \in S} T_i$ ($N = 5$ and $T_i = 3$)
with estimation error $\frac{1}{\sigma_z^2} = 10$ dB is also shown. With $N = 5$ and $T_i = 3$, SSLRBF performs 2.8 dB worse than the SLNR beamformer in case of estimation error. In case the $N \geq \sum_{\ell \in S} T_\ell$ condition is fulfilled (e.g. shown with $N = 8$ and $T_i = 2$), the proposed scheme performance is only 0.8 dB worse than the SLNR beamformer (3.11). This small degradation could be tolerable given the significant complexity reduction of the proposed scheme compared to the polynomial complexity of the SLNR beamformer.

Figure 4.2 and Figure 4.3 show the BER and SINR distribution behavior of the SSLRBF compared to the SLNR beamformer. In this simulation, it is assumed that the exact-CSIT is available at the transmitter with $N = 5$ and $T_i = 3$. The proposed scheme also outperform SMMSE in terms of BER, with significant reduction in computational complexity. Neumann series order $R$, has a great effect on the performance of the proposed scheme. A significant gain is observed when using $R = 20$ instead of $R = 10$. Generally, the gain obtained by increasing the value of $R$ depends on the operating SNR. As shown in Figure 4.2, increasing $R$ from 10 to 70 has negligible effect on BER performance at low SNR, while this gain increases as SNR increases.

The effect of Dominant Column approximation is assessed in Figure 4.2. When applying Jacobi-Davidson approximation to the dominant eigenvector problem while using exact matrix inversion, the performance is not much affected. On the other hand we have significant complexity reduction, by directly using dominant column instead of dominant eigenvector.

When estimation error is present, which is the case in practical systems, the proposed scheme demonstrates less BER and SINR loss for the same SNR values compared to the SLNR beamformer. Figure 4.4 and Figure 4.5 show BER and SINR distribution, respectively, with $N = 10$, $T_i = 2$ and estimation error $\frac{1}{\sigma_z^2} = 10$ dB. It’s shown that for both SSLRBF and SMMSE to have the same BER as SLNR beamformer, better radio conditions (i.e. higher SNR) are required than the case of Full-CSIT. Zero Forcing and conventional beamforming both fail dramatically for this configuration.
Figure 4.2: Uncoded BER for proposed beamformer with $N = 5$, $T_i = 3$ and $L = 3$. Exact-CSIT at the transmitter is assumed.

Figure 4.3: CDF of SINR at the receiver for proposed beamformer with $N = 5$, $T_i = 3$ and $L = 3$. Snapshot at $SNR = 4dB$ and Exact-CSIT at the transmitter is assumed.
Figure 4.4: Uncoded BER for proposed beamformer with $N = 10$, $T_i = 2$ and $L = 3$ and Estimation error with $\frac{1}{\sigma_c^2} = 10dB$ at the transmitter is assumed.

Figure 4.5: CDF of SINR at the receiver for proposed beamformer with $N = 10$, $T_i = 2$ and $L = 3$. Snapshot at $SNR = 10dB$ and Estimation error with $\frac{1}{\sigma_c^2} = 10dB$ at the transmitter is assumed.
4.6 Conclusion

Multiuser transmit beamforming is necessary in realizing multiuser MIMO and SDMA. This comes at hard computational requirements that rise mainly from need to compute Matrix inverse and Generalized Eigenvector. In this chapter we investigated methods of reducing computational requirements of Mu-MIMO transmit beamforming while maintaining satisfactory system performance.

We proposed a simplified efficient Mu-MIMO transmit beamforming in (4.13), that achieves near optimal performance compared to the SLNR beamformer at linear complexity. This is done by employing numerical algorithms to calculate matrix inversion and eigenvector. Matrix inversion is approximated in linear complexity using Neumann Series. We also relaxed the requirement to calculate eigenvector. This is achieved by replacing the dominant eigenvector by the dominant column. This is in direct result of Jacobi-Davidson algorithm.

Following our derivation of the new scheduling formulation we found the need to maintain the condition that the number of transmit antennas should remain larger than or equal to the total number of the receive antennas of all users. This is achieved later by the scheduler. The performance of the proposed beamforming method was shown to perform as close to the optimal SLNR beamformer considering BER curves and SINR distribution. The proposed beamformer can be directly modified to take into account channel estimation error (4.14).
Chapter 5

Proposed Joint Receive Antenna Selection and User Scheduling

5.1 Introduction

Multiuser MIMO scheduling is a complex combinatorial problem in general. This problem can only be optimally solved using extensive search, requiring exponentially growing complexity with the number of scheduled users as described in [26]-[27]. When we combine multiuser scheduling with transmit beamforming, the complexity increases due to the requirement of computing the beamforming matrix at each scheduling interval. This problem has been addressed recently in the literature. A MIMO/OFDM system was considered in [27], and a reduced complexity scheduling algorithm based on channel correlation for users on the same carrier was proposed. This algorithm requires estimation of the angle of arrival at the transmitter. The authors in [26] and [9] studied the problem both when channel gains and phases are fully and partially available at the transmitter. The authors in [28] used channel correlation and formulated the SDMA algorithm as a convex quadratic problem, which can be efficiently solved using convex optimization. This results in sub-optimal but efficient solution to the user scheduling problem. Threshold based on inter-user correlation was adopted in [26]-[27], so that users with channel correlation below particular threshold are only scheduled.
Using the beamformer proposed in Section 4, joint design of user scheduling and receive antenna selection is proposed. The algorithm’s objective is to satisfy the condition \( N \geq \sum_{\ell \in S} T_\ell \) while maximizing the total system’s SINR. This is achieved by scheduling users’ data such that users with better channel conditions are served during a certain scheduling interval using antennas that maximize the overall system’s SINR. We propose a novel joint user and receive antenna selection algorithm. The algorithm treats every receive antenna as an individual user. The problem is then formulated as follows:

\[
S = \arg \max_S \left( \sum_{\ell \in S} SINR_\ell \right)
\]

Subject to:

\[
\sum_{\ell \in S} P_\ell \leq 1,
\]

and \( N \geq \sum_{\ell \in S} T_\ell \),

The size of solution space set then becomes \( 2^{(K \times M)} \). The problem with this formulation (5.1) is that the complexity increases due to the increasing solution space of the algorithm. Unlike other approaches where antenna selection is carried out separately from user scheduling, the solution space in the formulation in (5.1) scales linearly with the number of receive antennas. The problem is NP-hard and the optimal solution is directly an exhaustive search across the whole solution space with number of combinations \( 2^{(K \times M)} \), making it unfeasible for large scale systems. Heuristic methods like greedy search can be used to solve the problem in (5.1), however its performance is usually far from optimal as shown later.

### 5.2 Greedy Search

We outline in the following the greedy user scheduling algorithm for comparison with our proposed Branch and Bound algorithm. Receive antenna selection is also performed jointly by treating every receive antenna as separate user. The algorithm is outlined below. In this algorithm the users’ receive antenna with the highest Sum SINR
is first selected, and in next steps only receive antennas that increase the Sum SINR function are added to the selected users’ receive antennas set $S$. Note in the special case where single receive antenna is used for each user, $S$ becomes the set of scheduled users.

**Require:** $H \lor K \{\text{Channel Estimated for each user}\}$

**Ensure:** $S$ is the users’ receive antennas group maximizing SINR

1: $S \leftarrow \emptyset$
2: $R \leftarrow 0$
3: $Rk \leftarrow 1, .., KM \{\text{Set of non-active receive antennas}\}$
4: for $i = 1$ to $N$ do
5:  $\text{Anew} = \arg \max_{k \in S} \text{SumSINR}(S \cup \{k\})$
6:  if $\text{SumSINR}(S \cup \{\text{Anew}\}) > R$ then
7:  $S = S \cup \text{Anew}$
8:  $Rk = Rk \setminus \text{Anew}$
9:  $R = \text{SumSINR}(S \cup \{\text{Anew}\})$
10:  else
11:    Break
12:  end if
13: end for

where $\text{SumSINR}(S) = \sum_{\ell \in S} \text{SINR}_{\ell}$ and $\text{Anew}$ is the best selected receive antenna to be active in one algorithm iteration. The resultant $S$ is then processed to determine the users $1, .., K$ to be scheduled and their respective receive antennas to be active.

### 5.3 Proposed Problem Reformulation as Quadratic Programming

To overcome the complexity problem we use the beamformer obtained in (4.13) to derive a quadratic formulation of the problem in (5.1). This new formulation enables us to employ the Branch and Bound (BB) algorithm to find a close to optimal solution.
of the problem and at a complexity less than the heuristic algorithm. We start by a novel derivation of a simplified form of the joint receive antenna selection and user scheduling problem, then we propose an efficient solution.

When employing the SLNR based beamformer, the Signal-to-Leakage ratio in (3.9) can be written as [16]

\[ SLNR_i = \lambda_{\text{max}} \left( \left( \sigma_i^2 I + \bar{H}_i^H \bar{H}_i \right)^{-1} H_i^H H_i \right) \]  

\[ \propto \text{Tr} \left( \left( \sigma_i^2 I + \bar{H}_i^H \bar{H}_i \right)^{-1} H_i^H H_i \right) \]  

\[ \propto \text{Tr} \left( \sum_{r=0}^{R} (I - \psi E_i)^r H_i^H H_i \psi \right) \]

where in the scheduler and receive antenna selection context, the spatial channel is defined as \( H_i \in \mathbb{C}^{M \times N} \). As we are only interested in proportionality between users, we can take \( R = 1 \)

\[ SLNR_i \propto \text{Tr} \left( (I - \psi E_i) H_i^H H_i \psi \right) \]  

Maximizing the total SINR is equivalent to maximizing the total SLNR, hence the problem in (5.1) can be reformulated directly to

\[ S = \arg \max_S \left( \sum_{\ell \in S} \text{Tr} \left( (I - \psi E_\ell) H_\ell^H H_\ell \psi \right) \right) \]  

s.t. \( \sum_{\ell \in S} P_\ell \leq 1 \),

and \( N \geq \sum_{\ell \in S} T_\ell \),

Let

\[ v = [v_0 \; v_1 \; \ldots \; v_{K \times M}] , \; v_i \in \{0, 1\} \]

be a binary vector of size \( K \times M \), where \( v_i \) element takes a value of zero if the corresponding receive antenna at a particular user is meant to be shutdown and one if it should be active. Define \( A, A_s, O, \) and \( Q \) as follows:

\[ A = [H_1^H H_2^H \ldots H_K^H]^H \]

51
\[
A_s = \begin{bmatrix}
H_1 & 0 & \ldots & 0 \\
0 & H_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & H_K
\end{bmatrix}
\] (5.9)

\[
O = (2 - \psi \sigma^2) \psi I + \psi^2 (A_s A_s^H - A A^H)
\] (5.10)

\[
Q = \begin{bmatrix}
Q_{11} & Q_{12} & \ldots & Q_{1K} & Q_{1M} \\
\vdots & Q_{22} & \ldots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
Q_{K \times M} & \ldots & \ldots & Q_{K \times M} & Q_{K \times M}
\end{bmatrix}
\] (5.11)

such that

\[
Q_{ij} = 2 \text{Tr} (a_i^H O_{ij} a_j)
\] (5.12)

As elements in \( O \) are relative quantities of all users, \( \psi \) in (5.10) can take any arbitrary value calculated for any user \( i \) as \( \frac{2}{\text{Tr}(E_i)} \). \( a_i \) and \( a_j \) are row vectors \( i \) and \( j \) in matrix \( A \), respectively. Then the total system SLNR becomes

\[
f(v) = \frac{1}{2} v^T Q v
\] (5.13)

And the problem is formulated in Mixed Binary Quadratic Programming (MBQP) form as

\[
v^\text{opt} = \arg \min_v \left( -\frac{1}{2} v^T Q v \right)
\] (5.14)

s.t. \( v_i \in \{0, 1\} \),

and \( [1 \ 1 \ \ldots \ 1]^T v \leq N \)

The matrix \( Q \) can be modified in a straightforward manner to reflect the total throughput of the system instead of the total SINR, by redefining (5.12) to reflect capacity formulation. This can be done by modifying (5.12) to become

\[
Q_{ij} = 2 \log (1 + \text{Tr} (a_i^H O_{ij} a_j))
\] (5.15)

### 5.4 Proposed Branch And Bound Algorithm

**Require:** \( Q \) {Matrix concluded in (5.12)}
Ensure: $\mathbf{v}$ is the optimal solution of the problem formulated in (5.14)

1: $\bar{f} \leftarrow +\infty$
2: $\mathbf{v} \leftarrow \text{void}$
3: $STACK \leftarrow \text{push} \left( \mathbb{P} \right)$
4: while not empty($STACK$) do
5: $\mathbb{P}_i \leftarrow \text{pop} \left( STACK \right)$
6: $\mathbb{P}_i^R, S_i^R \leftarrow \text{relax} \left( \mathbb{P}_i \right)$
7: $f^i, v^i \leftarrow \text{quadprog} \left( \mathbb{P}_i^R \right)$
8: if $S_i^R = \phi$ then
9: $\mathbb{P}_i$ has no feasible solution.
10: else if $f^i \geq \bar{f}$ then
11: No feasible solution for $\mathbb{P}_i$ better than $\mathbf{v}$.
12: else if $v^i \in S_i$ then
13: $v^i$ is feasible and optimal in $\mathbb{P}_i$
14: $\bar{f} \leftarrow f^i$
15: $\mathbf{v} \leftarrow v^i$
16: else
17: $S_{i0}, S_{i1} \leftarrow \text{split} \left( S_i \right)$
18: $STACK \leftarrow \text{push} \left( \mathbb{P}_{i0}, \mathbb{P}_{i1} \right)$
19: end if
20: end while

Based on the above analysis, the multiuser scheduling problem is equivalent to MBQP problem. MBQP is proved to be NP-hard [29]. Branch and Bound (BB) algorithm is one of the well known, low complexity methods [29] that can be used to solve the MBQP problem given in (5.14) using integer programming relaxation. BB algorithm is based on the observation that the enumeration of integer solutions has a tree structure. Each node of this tree represents a subproblem which can also be split into two subproblems by adding two mutually exclusive and exhaustive constraints. BB algorithm do not search the complete tree. It prunes some subtrees by the bounding
strategy so as to reduce the searching space.

**Branching**

The objective function of (5.14) is quadratic in \( v \) and subjects to box constraints.

Branching is used to split the feasible solution into several smaller feasible subregions. Let \( S = \{ [v_0, v_1, \ldots, v_{KM}] \} \), \( v_i \in \{0, 1\} \) denotes the solution space set of the MBQP optimization problem. The solution is divided into several subsets using the branching method so that \( S = \bigsqcup_{i} S_i \). Then the optimal solution over \( S \) is computed over the smaller sets \( S_i \), separately formulated in a Quadratic programming subproblem \( P_i \). This naturally forms a tree structure known as search tree or branch and bound tree as shown in the example shown in Figure 5.1.

Associated with this problem is the quadratic relaxation created by relaxing the integer constraints \( v_i \in \{0, 1\} \) into interval constraints \( v_i \in [0, 1] \). In each node of Figure 5.1, the corresponding feasible set \( S_i \) is shown. The symbol * is used to denote that this variable is between 0 and 1. The rows of nodes in the tree are called levels. The top node is called the root node. All the nodes in the tree, except the nodes in the bottom of the tree, have two nodes connected to the lower side of the node. These two nodes are called the children of the node; any node above a child is called a parent node. The solution of a subproblem is done using the method in [30], and implemented by `quadprog` function in MATLAB.
Bounding

Bounding is used to compute lower bound and upper bounds for the optimal objective function value for the subproblem in the nodes. We denote \( \overline{f} \) as the global upper bound of the objective function, \( (f^*) \) is the optimal objective function over \( S \), \( (f^i) \) is a local bound of the objective function value over \( P_i \), and \( (f^{i*}) \) is the optimal objective function over the subproblem \( P_i \).

Branch and Bound tree structure enables complexity reduction by the ability to discard entire subtrees. Subtree discarding is performed if no feasible solution \( S_i \) exits, an optimal value is found, or at dominance condition \( f^{i} \geq \overline{f} \). This step in the algorithm is called pruning. Two important parameters that affect the algorithm are node selection and the branch variable.

In this thesis, we adopt Depth-First Search Node Selection with backtracking. In depth-first search, if the current node is not pruned, the next node is considered one of its children. With backtracking, when a node is pruned, we go back on the path from this node until we find the first node. We also use a method that selects a branch variable which has the lowest function value determined by the quadratic function \( 5.14 \).

The algorithm above outlines the branch and bound algorithm. The split, push, and pop subroutines are implemented to reflect the depth-first strategy. On exit, \( \overline{f} \) contains the optimal value of the global problem \( (f^*) \) and \( \nabla \) is the optimal solution.

5.5 Numerical Results

The sum rate achieved by the proposed scheduling and receive antenna selection algorithm based on Branch and Bound method is compared to the exhaustive search. We also compare the proposed algorithm to Greedy user selection which is employed in several papers such [9] and [26]. The branch and bound algorithm is computed using two branching iterations. The Greedy algorithm is modified to include receive antenna selection by treating each receive antenna as separate user. We elaborate the significant complexity reduction introduced by the proposed algorithm. In simulations, \( K \) is set
such that $K > N$, the scheduler then selects $L$ users out of $K$ such that $N \geq \sum_{\ell \in S} T_\ell$.

Figure 5.2 shows the system sum rate with $N = 4$, $M = 2$ and $K = 6$. Exhaustive search is introduced only for this configuration due to the simulation complexity for larger MIMO configuration and larger number of users. Sum Rate is computed based on the SLNR beamformer vector in (3.11) as:

$$\text{SUMRATE} = \sum_{\ell \in S} \log (1 + \text{SINR}_\ell)$$  \hspace{1cm} (5.16)

The proposed algorithm has sum rate very close to that obtained by the optimal exhaustive search. It also significantly outperforms Greedy search, specially at moderate and high SNRs. For example, in this configuration, the Greedy search requires 30dB higher SNR to achieve 20 bits/s/Hz than the proposed algorithm.

Figure 5.3 shows the system’s sum rate for $N = 5$, $M = 3$ and $K = 40$. In this case, the proposed Branch and Bound algorithm not only outperforms the Greedy search, but also extracts antenna and multiuser diversity gain. The gain of the proposed algorithm over Greedy search varies according to the operating SNR and is shown to
Figure 5.3: SumRate of Proposed Algorithm Vs SNR in comparison with Greedy for multiple receive antenna and $N = 5$, $M = 3$, and $K = 40$
have twice the sum rate as the Greedy Search for SNRs above 30dB and 1.5 times for moderate SNRs.

5.5.1 Complexity Analysis

A key advantage of the proposed scheduling algorithm is its reduced computational complexity. We numerically evaluate the complexity as the number of objective function evaluations, in (5.14), required by the Exhaustive Search (ES), Greedy Search (GS), and Branch and Bound (BB) for different number of users and MIMO configurations. The proposed algorithm (BB) is shown in Table 5.1 to require less objective function evaluations compared to the GS.

Table 5.1: Complexity comparison for different scheduling and Receive antenna selection Techniques

| K | N = 4, M = 2 |  |  |  |
|---|---|---|---|
| | ES | GS | BB | BB/GS(%) |
| 10 | 1.05E+06 | 58 | 33 | 56.90% |
| 20 | 1.10E+12 | 118 | 73 | 61.86% |
| 30 | 1.15E+18 | 178 | 113 | 63.48% |
| 40 | 1.21E+24 | 238 | 153 | 64.28% |
| 50 | 1.27E+30 | 298 | 193 | 64.77% |

| K | N = 5, M = 3 |  |  |  |
|---|---|---|---|
| | ES | GS | BB | BB/GS(%) |
| 10 | 1.07E+09 | 88 | 51 | 57.95% |
| 20 | 1.15E+18 | 178 | 113 | 63.48% |
| 30 | 1.24E+27 | 268 | 171 | 63.81% |
| 40 | 1.33E+36 | 358 | 231 | 64.53% |
| 50 | 1.43E+45 | 448 | 291 | 64.96% |
5.6 Conclusion

In this chapter we reformulated the problem of joint scheduling and receive antenna selection as Mixed Binary Quadratic Programming, and proposed a novel joint optimization algorithm based on Branch and Bound. The proposed algorithm efficiently maximizes total system SINR and is near optimal to Exhaustive Search. Design Criterion of the formulated problem can be straightforwardly adapted to sum-capacity maximization and users Quality of Service consideration. The proposed algorithm was shown to significantly outperform Greedy Search with less complexity.
Chapter 6

Discussions and Conclusions

In this thesis, we have studied the performance of different beamforming schemes for multiuser MIMO systems. Both the case of perfect CSIT and channel estimation errors have been studied. The performance has been evaluated with Monte-Carlo Simulations. We have utilized WINNER channel model for relaistic spatial channel model throughout the simulations. The result showed that Signal-To-Noise-And-Leakage based beamformer is better than both SMMSE and Zero Forcing beamfoming scheme.

SMMSE and SLNR based schemes relies on matrix inversion and eigenvector computation for their implementation. Motivated by this we investigated methods of simplifying computational requirement of the transmit beamforming vectors. We have proposed a simplified version of the optimal SLNR based beamformer by employing Neumann Series to calculate matrix inversion and Jacobi-Davison algorithm to compute dominant eigenvector. The proposed method was shown to have linear complexity compared to polynomial complexity of the optimal SLNR beamformer. This new method was shown as well to have near optimal performance.

Based on the results of the proposed simplified beamformer, we reformulated the problem of joint scheduling and receive antenna selection as Mixed Binary Quadratic Programming, and proposed a joint optimization algorithm based on Branch and Bound. The proposed algorithm efficiently maximizes total system SINR and is near optimal
to Exhaustive Search. Design Criterion of the formulated problem can be straightforwardly adapted to sum-capacity maximization and users Quality of Service consideration. The proposed algorithm was shown to significantly outperform Greedy Search at lesser complexity.

This work can be extended in several ways. Study of the system in multicell environment is highly encouraged. In this regime interference from other neighbor basestations can be taken into account when designing the beamformer. Interference lessing can be done in non-cooperative and cooperative schemes between different basestations. The later requires designing efficient protocols for coordination between basestations. Also modification of the proposed algorithms in unlicensed frequency band can be studied. In such scenario cognitive features like spectrum sensing can be integrated with the beamformer and scheduler to achieve optimal performance in unlicensed bands. MIMO Beamforming schemes can be utilized to avoid interference of secondary users on primary users.
Appendix A

Quadratic Programming Overview

A linearly constrained optimization problem with a quadratic objective function is called a quadratic program (QP) [29]. It forms the basis of several general nonlinear programming algorithms. In this appendix we introduce a general QP problem and examine Karush-Kuhn-Tucker (KKT) condition of the problem.

A.1 Quadratic Programming Problem

The general quadratic program can be written as

$$x^{opt} = \arg \min_x \left( \frac{1}{2} x^T Q x + c x \right)$$  \hspace{1cm} (A.1)

\[s.t.] \quad A x \leq b, \quad \text{and} \quad x \geq 0 \]

where \(c\) is an \(n\)-dimensional row vector describing the coefficients of the linear terms in the objective function, and \(Q\) is an \((n \times n)\) symmetric matrix describing the coefficients of the quadratic terms. If a constant term exists it is dropped from the model. As in linear programming, the decision variables are denoted by the \(n\)-dimensional column vector \(x\), and the constraints are defined by an \((m \times n)\) \(A\) matrix and an \(m\)-dimensional column vector \(b\) of right-hand-side coefficients. We assume that a feasible solution exists and that the constraint region is bounded. When the objective function is strictly convex for all feasible points the problem has a unique local minimum which is also the
global minimum. A sufficient condition to guarantee strictly convexity is for $Q$ to be positive definite.

### A.1.1 Karush-Kuhn-Tucker Condition

The Karush Kuhn Tucker conditions (also known as the Kuhn-Tucker or KKT conditions) are necessary for a solution in nonlinear programming to be optimal, provided some regularity conditions are satisfied. Excluding the non-negativity conditions, the Lagrangian function for the quadratic program is

$$L(x, \mu) = \frac{1}{2}x^TQx + cx + \mu (Ax - b) \quad (A.2)$$

where $\mu$ is an $m$-dimensional row vector. The Karush-Kuhn-Tucker conditions for a local minimum are given as follows.

$$\frac{\partial L}{\partial x_j} \geq 0, j = 1, \ldots, n \quad \rightarrow \quad x^TQ + c + \mu A \geq 0 \quad (A.3)$$

$$\frac{\partial L}{\partial \mu_i} \geq 0, i = 1, \ldots, m \quad \rightarrow \quad Ax - b \leq 0 \quad (A.4)$$

$$x_j \frac{\partial L}{\partial x_j} \geq 0, j = 1, \ldots, n \quad \rightarrow \quad x^T(c^T + Qx + A^T\mu) = 0 \quad (A.5)$$

$$\mu_i g_i(x) = 0, i = 1, \ldots, m \quad \rightarrow \quad \mu (Ax - b) = 0 \quad (A.6)$$

$$x_j \geq 0, j = 1, \ldots, n \quad \rightarrow \quad x \geq 0 \quad (A.7)$$

$$\mu_i \geq 0, i = 1, \ldots, m \quad \rightarrow \quad \mu \geq 0 \quad (A.8)$$

To put the first and last equation into a more manageable form we introduce nonnegative surplus variables $y \in \mathbb{R}^n$ to the first condition and nonnegative slack variables $v \in \mathbb{R}^m$ to the inequalities in second condition to obtain the equations

$$Qx + c^T + A^T\mu - y = 0 \quad and \quad Ax - b + v = 0 \quad (A.9)$$
The KKT conditions can then be written with the constraints moved to the right-hand side

\[ Qx + A^T \mu - y = -c^T \]  \hspace{1cm} (A.10)
\[ Ax + v = b \]  \hspace{1cm} (A.11)
\[ x \geq 0, \mu \geq 0, y \geq 0, v \geq 0 \]  \hspace{1cm} (A.12)
\[ y^T x = 0, \mu v = 0 \]  \hspace{1cm} (A.13)

A.1.2 The Simplex Algorithm

The simplex algorithm can be used to solve (A.10) (A.13) by treating the complementary slackness conditions (A.13) implicitly with a restricted basis entry rule. The procedure for setting up the linear programming model follows.

- Let the structural constraints be Eqs. (A.10) and (A.11) defined by the KKT conditions.
- If any of the right-hand-side values are negative, multiply the corresponding equation by 1.
- Add an artificial variable to each equation.
- Let the objective function be the sum of the artificial variables.
- Put the resultant problem into simplex form.

The goal is to find the solution to the linear program that minimizes the sum of the artificial variables with the additional requirement that the complementarity slackness conditions be satisfied at each iteration. If the sum is zero, the solution will satisfy (A.10) (A.13). To accommodate (A.13), the rule for selecting the entering variable must be modified with the following relationships in mind.

- \( x_j \) and \( y_j \) are complementary for \( j = 1, \ldots, n \)
- \( m_i \) and \( v_i \) are complementary for \( i = 1, \ldots, m \)

The entering variable will be the one whose reduced cost is most negative provided that its complementary variable is not in the basis or would leave the basis on the same
iteration. At the conclusion of the algorithm, the vector $x$ defines the optimal solution and the vector $\mu$ defines the optimal dual variables.

This approach has been shown to work well when the objective function is positive definite, and requires computational effort comparable to a linear programming problem with $m + n$ constraints, where $m$ is the number of constraints and $n$ is the number of variables in the QP. Positive semi-definite forms of the objective function, though, can present computational difficulties. The simplest practical approach to overcome any difficulties caused by semi-definiteness is to add a small constant to each of the diagonal elements of $Q$ in such a way that the modified $Q$ matrix becomes positive definite. Although the resultant solution will not be exact, the difference will be insignificant if the alterations are kept small.

### A.1.3 Mixed Integer Quadratic Programming

Mixed Integer Quadratic programming (MIQP) problem looks similar to the ordinary QP problem. However, the optimization variables are not only allowed to be real valued, but also integer valued. This turns the easily solved QP problem, into an NP-hard problem. A common special case of MIQP is when the integer variables are constrained to be 0 or 1. This problem is called a Mixed Binary Quadratic Programming (MBQP) problem. The mathematical definition of an MBQP problem is

$$
\mathbf{x}^{opt} = \arg \min_{\mathbf{x}} \left( \frac{1}{2} \mathbf{x}^T \mathbf{Q} \mathbf{x} + c \mathbf{x} \right)
$$

$$
s.t. \quad \mathbf{A} \mathbf{x} \leq \mathbf{b},
$$

$$
and \quad \mathbf{x} \in \{0, 1\}
$$

There exist several methods for solving MIQP problems. The four most commonly used methods for these kind of problems are:

- Cutting plane methods
- Decomposition methods

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• Logic-based methods
• Branch and bound methods

Branch and Bound is found to be the best method for mixed integer programs. Branch and Bound is the superior method for solving MIQP problems. With a few exceptions, branch and bound is an order of magnitude faster than any of the other methods. An important explanation to why branch and bound is so fast is that the QP subproblems are very cheap to solve. There exist several software for solving MIQP problems. For MATLAB, free software like YALMIP or miqp.m can be used. A commonly used commercial software is CPLEX.
Bibliography


