

**CS321**  
**Theory of Computation**  
**Quiz 5, Fall 2008**

Name:

1. For each of the following grammars, describe the corresponding language in set notation. In all cases,  $S, A, B$  denote variable symbols and  $a, b$  denote terminal symbols, and  $S$  is the start variable.

(a) [5pt]  $S \rightarrow aaSb \mid \lambda$

$$L(G) = \{a^{2i}b^i : i \geq 0\}$$

(b) [5pt]  $S \rightarrow AB$   
 $A \rightarrow aAb \mid \lambda$   
 $B \rightarrow bB \mid \lambda$

$$L(G) = \{a^i b^j : i \geq 0, j \geq i\}$$

(c) [5pt]  $S \rightarrow A \mid B$   
 $A \rightarrow aAb \mid \lambda$   
 $B \rightarrow bB \mid \lambda$

$$L(G) = \{a^i b^j : i = j, j \geq 0 \text{ or } i = 0, j \geq 0\}$$

2. For each of the languages below give a corresponding grammar.

(a) [5pt]  $L = \{w : w = uaaav, u \in \{a, b\}^*, v \in \{a, b\}^*\}$

$$S \rightarrow AaaaA$$
$$A \rightarrow aA \mid bA \mid \lambda$$

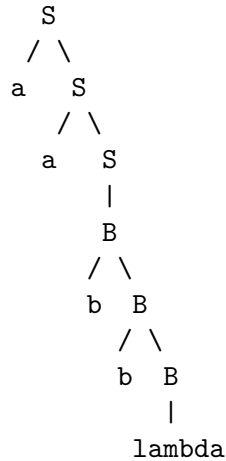
(b) [5pt]  $L = \{w_1 w_2 : w_1 = a^i b^i, w_2 = b^j a^j, i \geq 0, j \geq 0\}$

$$S \rightarrow AB$$
$$A \rightarrow aAb \mid \lambda$$
$$B \rightarrow bBa \mid \lambda$$

3. [5pt] For the follow grammar with variables  $V = \{S, A\}$ , terminals  $T = \{a, b\}$ , and start symbol  $S$  show a derivation tree for the string  $w = abb$ .

$$S \rightarrow aS \mid B$$

$$B \rightarrow bB \mid \lambda$$



4. [5pt] What does it mean for a context free grammar to be ambiguous?

A context free grammar  $G$  is ambiguous if there exists a string  $w \in L(G)$  such that there are two distinct derivation trees that both have a yield of  $w$ .

5. [10pt] Show that the following grammar is ambiguous.  $S$  is the start variable and  $a$  and  $b$  are terminals.

$$S \rightarrow SabS \mid SS \mid \lambda$$

Below we show two distinct derivation trees with a yield of  $\lambda$ , which proves that the grammar is ambiguous.

