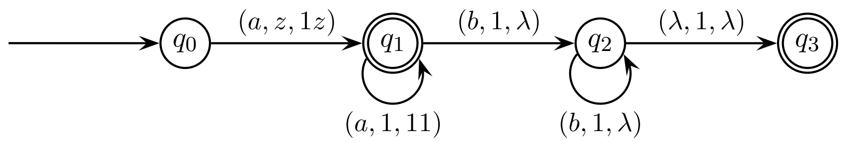
CS321
Theory of Computation
Quiz 6, Fall 2008

Name:

1. [5pt] What is a configuration of a non-deterministic pushdown automaton (NPDA)?

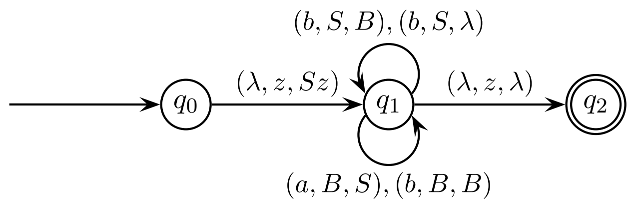
   A configuration of an NPDA is a triple (q, w, u) where q is a state, $w \in \Sigma^*$ is the remaining input to be read in, and $u \in \Gamma^*$ is the current stack.

2. [10pt] Draw an NPDA for the language $L = \{a^i b^k : i > k\}$.



3. [10pt] Draw an NPDA that is equivalent to the following grammar. Use the construction taught in class and in the book.

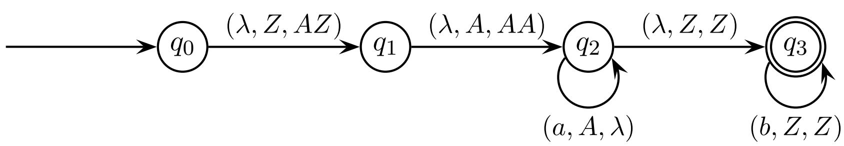
$$\begin{aligned} S &\rightarrow bB \mid b \\ B &\rightarrow aS \mid bB \end{aligned}$$



4. [10pt] Prove that context-free languages are closed under the concatenation operation. That is, show that if L_1 and L_2 are context-free languages, then $L_1 L_2$ is also context free.

   If L_1 and L_2 are context-free languages then there are context free grammars G_1 and G_2 such that $L_1 = L(G_1)$ and $L_2 = L(G_2)$. Without loss of generality assume that G_1 and G_2 do not share any variables (if this is not true, then we can just rename the variables). Let R_i be the set of rules for G_i and S_i be its start symbol. Now construct a new grammar G that has as rules the union of R_1 and R_2 with the addition of the rule $S \to S_1 S_2$, where S is the start symbol for G. From this definition it follows that any string in $L(G)$ must be the result of a string generated from the symbol S_1 concatenated with a string generated from S_2. This means that that $L(G) = L_1 L_2$. Since we have shown a grammar for $L_1 L_2$ we see that it is context free. Thus, context free languages are closed under concatenation.

5. Consider the following NPDA M,



   (a) [5pt] What is the sequence of NPDA configurations that M goes through when given string aa as input?

$$(q_0, aa, Z) \vdash (q_1, aa, AZ) \vdash (q_2, aa, AAZ) \vdash (q_2, a, AZ) \vdash (q_2, \lambda, Z) \vdash (q_3, \lambda, Z)$$

   (b) [5pt] What is $L(M)$ in set notation?

$$L(M) = \{aab^i : i \geq 0\}$$