Lecture 18

Last time: Optimization theories

- min jerk : cartesian
  joint x2
- min torque change
- min variance in task space

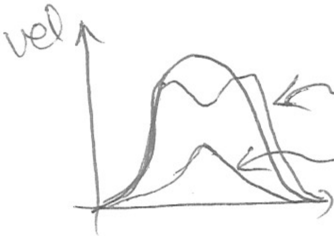
Regardless of what is being optimized, there are stereotypical mvt. characteristics people have found so far.

⟶ see p2, + p3 first

Curvature

$$C(t) = \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}} = \frac{1}{r(t)}$$ ← radius of curvature

Relating to the curves we have been drawing.



$v(t) =$ total velocity (speed)

$$\sqrt{\dot{x}^2 + \dot{y}^2}$$

The relationship betw'n V + C

$$V = \gamma C^{-1/3}$$

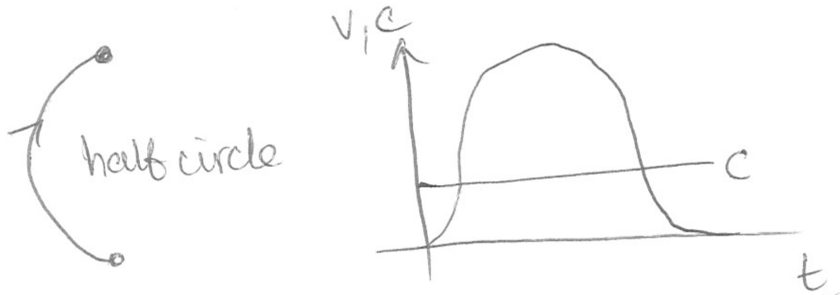
↑ const

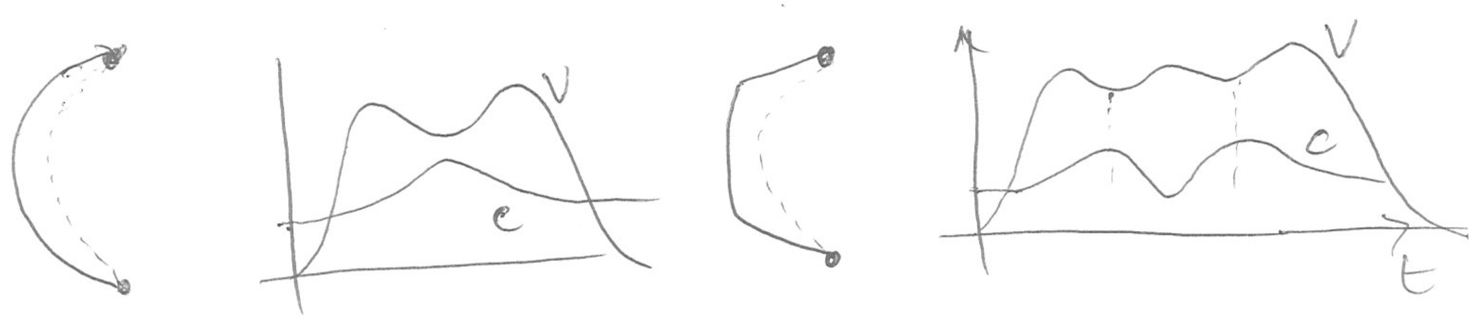
As V↑, C↓

Bell-shaped profile - Talked about this a lot.

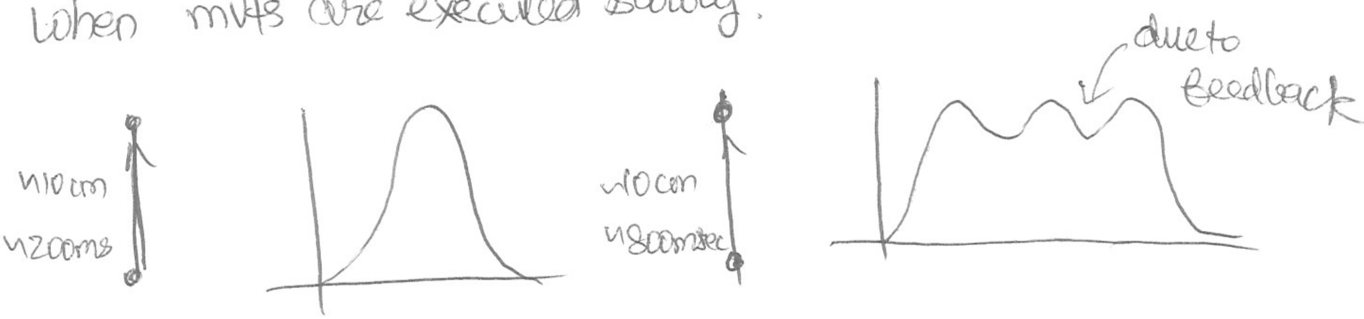
- In general, hard to violate, but there are two cases when it happens.

a. when mvts are curvy.





b. when mvts are executed slowly.



Generally, even if there is violation, profiles look like a combo of

Relate to thies

(2) $\frac{2}{3}$ power law. (Lacquaniti, 1983)

→ applies betwn velocity & mvt. curvature.

(continued on bottom of p1)

Definition of power law:

$$\log(v) \propto \log(C)$$

Relate "angular vel" w/ curvature

$$v = \omega r = \gamma C^{-1/3}$$

Note that $r = \frac{1}{C}$

$$\Rightarrow \omega = \gamma C^{2/3}$$

But we relate better to the tangential velocity

$$v = (\dot{x}^2 + \dot{y}^2)^{1/2} = \gamma C^{-1/3}$$

$$= \gamma \left(\frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \right)^{-1/3}$$

$$(\dot{x}^2 + \dot{y}^2)^{1/2} = \gamma \frac{(\dot{x}\ddot{y} - \dot{y}\ddot{x})^{-1/3}}{(\dot{x}^2 + \dot{y}^2)^{-1/2}}$$

$$\Rightarrow \gamma = (\dot{x}\ddot{y} - \dot{y}\ddot{x})^{+1/3}$$

$$\Rightarrow \dot{x}\ddot{y} - \dot{y}\ddot{x} = \gamma^3$$ ← still a constant.

Taking derivative $\dot{x}\dddot{y} + \cancel{\ddot{x}\ddot{y}} - \cancel{\ddot{y}\ddot{x}} - \dot{y}\dddot{x} = 0$

$\Rightarrow \dot{x}\dddot{y} = \dot{y}\dddot{x}$ or $\frac{\dot{x}}{\quad} = \frac{\dddot{x}}{\quad}$ ← Relationship betw'n velo.

what does this mean?

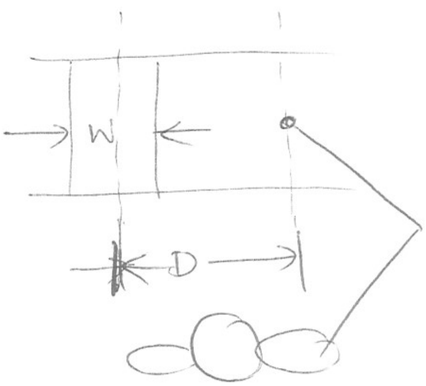
Vel & jerk parallel to each other.

Lots of papers on the 2/3 power law.

- By product of the smooth mvts ⇒ result in power law.

3) Fitts' Law (Fitts, 1954)

The more accuracy the movement needs to be, the slower it is expected



Moving time → constants

$$T = C_1 + C_2 \log_2(2D/W)$$

moving time

D = Distance to target center

W = Target width in dir'n of mvt.

T ↑ as D ↑ or W ↓ in log scale

Very frequently referred to still: impt. for work efficiency using mouse + clicking on buttons

Schmidt (1979) extended this to mvt. errors

$$W_e = K\left(\frac{D}{T}\right)$$

other impt characteristic that are known but do not have a name.

(4) Movement time does not scale w/ mvt. distance.

→ mvt. time approximately the same for a large range of motion

(of course, this can be changed w/ target size as in Fitt's law or just tell people to make fast mvts)

5) For sequential mvts, what stays constant is the mvt. time ratio even if the whole seq. is executed @ different speeds.

Finally, but very impt.

6) High variability in mvt. paths

Even for perfectly mastered mvts, pt. to pt. mvts are different every single time.

↳ this makes games like golf, tennis, darts fun even @ pro level.

↓

games that don't have many other external variability

P7

Where does this variability come from?
Why don't we correct this completely?
} Open question

Book → Good as a term paper idea.

Funnily, even w/ all the variability, we can still tell the handwritings, walking patterns that are signature/specific to the person.

So what are the domains ~~that~~ in which variabilities are tolerated w/i one person & " " " " ~~that~~" " ⇒ another person?