Lecture #15

Introduction to multi-jointed control.

- More muscles & more complexity

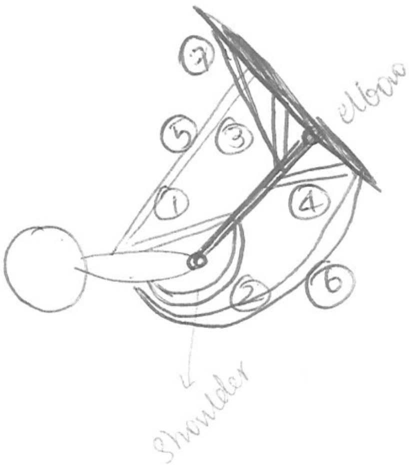
- In single joint, we know exactly what τ_j was for a given τ_{ext}

- In multijoint mvt, we need to consider the interaction betwn joints.
  - Force & motion vectors may not be collinear.
  - Impedance modulation @ the interaction pt (ie. the hand)

is important, but not @ each joint.

eg: compliant in one direction to accommodate an external kinematic constraint

-stiff in another direction to minimize disturbance.



In 2D top view

① Shoulder adductor

② " abductor

③ Elbow flexor

④ " extensor

⑤ Multi-joint muscle (polyarticular flexor)

⑥ Polyarticular extensor

⑦ Elbow flexor.

- Lots of redundancy – vary attachment pts. for different force vectors
  - helps modulate impedance

- Don't need ⑤ & ⑥ for motion control, but they modulate the interaction btwn joints (minimize chances of injury) by limiting velocities & preventing hitting joint limits)

- Neural feedback from ① & ② to ③ & ④ takes too long for rapid or high freq. interactions.

  ↳ so effectively not as much redundancy as it appears.

we know how individual muscles work.

So how can we transform this information to the joint level (or even end-pt.) information.

↳ the Transformation relationship is completely determined by the geometry of musculo-skeletal connections.

Assume for most cases that muscle lengths can be determined from the joint angles.

$$\bar{l} = \bar{l}(\bar{\theta}) \quad , \quad \bar{l} = [l_1, l_2, \cdots, l_m]^T$$

\# of muscles

$$\bar{\theta} = [\theta_1, \theta_2, \cdots \theta_r]^T$$

\# of joints.

$m >> r$

To allow sign matching,

(i.e. as $\bar{l}\downarrow$, force goes up)

$$\bar{h}(\bar{\theta}) = -\bar{l}(\bar{\theta})$$

$$d\bar{h} = \underbrace{\frac{\partial \bar{h}(\bar{\theta})}{\partial \bar{\theta}}}_{J(\bar{\theta})} \cdot d\bar{\theta} = -\frac{\partial \bar{l}(\bar{\theta})}{\partial \bar{\theta}} \cdot d\bar{\theta}$$

$J(\bar{\theta})$ Jacobian matrix

partial derivative of muscle length wrt joint angles

Because usually $m > r$, $J(\Theta)$ is not square.

- cannot be uniquely inverted.

Divide both side by dt.

$$\frac{d\bar{h}}{dt} = J(\bar{\Theta}) \frac{d\bar{\Theta}}{dt} \Rightarrow \underbrace{\bar{V}}_{\text{Muscle velocities}} = J(\bar{\Theta}) \underbrace{\bar{\omega}}_{\text{joint velocities}}$$

Energy is conserved

↳ work done by joints = work done by muscles.

$$\underbrace{dE}_{\text{potential field}} = \underbrace{\tau^T}_{\text{joint torque array}} d\bar{\Theta} = \underbrace{\bar{f}^t}_{\text{muscle force array}} d\bar{h}$$

By substitution,

$$\bar{f}^T d\bar{h} = \bar{f}^T J(\bar{\Theta}) d\bar{\Theta} = \tau^T d\Theta$$

$$\underbrace{\bar{\tau}}_{\text{joint torque}} = \underbrace{J(\bar{\Theta})^T}_{\text{moment arm}} \underbrace{\bar{f}}_{\text{muscle force}}$$

We can use the Jacobian to transform muscle impedance (k + b)

Viscosity

Muscle force $\bar{f}$ is dependent on muscle velocity $\bar{V}$

$$\bar{f} = \bar{f}(\bar{V})$$

$$\bar{\tau} = J^T(\bar{\theta})\,\bar{f}(\bar{V}) = J^T(\bar{\theta})\,\bar{f}(J(\bar{\theta})\bar{\omega}).$$

Muscle viscosity $b_m = \dfrac{\partial \bar{f}}{\partial \bar{V}}$ ← incremental change of force due to " " " velocity

Joint " $b_j = \dfrac{\partial \bar{\tau}}{\partial \bar{\omega}}$

$$b_j = \frac{\partial \bar{\tau}}{\partial \bar{\omega}} = \bar{J}^T(\theta)\frac{\partial \bar{f}(J(\bar{\theta})\bar{\omega})}{\partial \bar{\omega}} + \cancel{\frac{\partial J^T(\bar{\theta})}{\partial \bar{\omega}}}^{\,0} \circ \bar{f}(J(\bar{\theta})\bar{\omega})$$

$$= \bar{J}^T(\theta)\frac{\partial \bar{f}(\bar{V})}{\partial \bar{V}} \circ \frac{\partial \bar{V}}{\partial \bar{\omega}}$$

$$\underline{b_j = \bar{J}^T(\bar{\theta}) \circ b_m \circ J(\bar{\theta})}$$

↓

muscle damping applies directly to joints

Stiffness

Muscle force $\bar{f}$ depends on muscle position $(\bar{h})$

$$\bar{f} = \bar{f}(\bar{h}) = \bar{f}(\bar{h}(\bar{\theta}))$$

$$\bar{\tau} = J^T(\bar{\theta}) \cdot \bar{f}(\bar{h}(\bar{\theta}))$$

$$k_m = \frac{\partial \bar{f}}{\partial \bar{h}} \qquad k_j = \frac{\partial \bar{\tau}}{\partial \bar{\theta}}$$

$$k_j = \frac{\partial \bar{J}^T(\bar{\theta})}{\partial \bar{\theta}} \cdot \bar{f}(\bar{h}(\bar{\theta})) + J^T(\bar{\theta}) \cdot \frac{\partial \bar{f}(\bar{h}(\bar{\theta}))}{\partial \bar{\theta}}$$

$$= \frac{\partial \bar{J}^T(\bar{\theta})}{\partial \bar{\theta}} \cdot \bar{f}(\bar{h}(\bar{\theta})) + J^T(\bar{\theta}) \cdot \underbrace{\frac{\partial \bar{f}(\bar{h}(\bar{\theta}))}{\partial \bar{\theta}}}_{k_m} \cdot \underbrace{\frac{\partial \bar{h}}{\partial \bar{\theta}}}_{J(\bar{\theta})}$$

$\Gamma(\bar{\theta})$ array of 2nd partial derivatives
of muscle lengths wrt joint angles
"variation of moment arm"

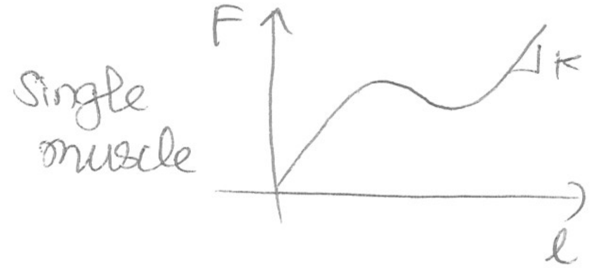
$$= \underbrace{\Gamma(\bar{\theta})\,\bar{f}}_{} + \underbrace{J^T(\bar{\theta}) \cdot K_m \cdot J(\bar{\theta})}_{\text{muscle stiffness transformed}}$$

"fictitious" joint stiffness

↳ zero if $\bar{f} = 0$

increases w/ increasing $\bar{f}$ (i.e. coactivation)

Now we know how to go from muscles to joints in velocity, "Spring-like" behavior was observed for single muscle or joint @ steady state



Single joint → multiple muscles

But there's always a unique torque/moment for every angle given no perturbation

Talked about equm. pt. hyp. based on the fact that single joint is spring-like

How about multi-jointed behavior?

Can we apply the Eq. pt. theory or Virtual traj. theory?

Now, there's no unique moment @ the joint becos it depends on the other joint angles too (moment arm changes based on configuration)

So it is more complex.

So how do we decide whether multijoint behavior is

Fundamental Req:

Potential fn analogous to elastic energy is definable.

$$E_p(\bar{\Theta}) = \underbrace{\int \bar{\tau}^T(\theta)\, d\bar{\theta}}_{\text{integral of torque wrt displacement is defineable}} \rightarrow \text{multijoint vector.}$$

$$\bar{\tau}(\theta) = \text{grad } E_p(\bar{\Theta})$$

$$= \frac{\partial E_p(\bar{\Theta})}{\partial \bar{\Theta}}$$

Basic fact from linear algebra → curl of a gradient = 0

$$\text{Curl}(\hat{\tau}(\theta)) = 0$$

$$\cancel{\text{Curl}}\ \text{Curl } \bar{\tau}(\bar{\Theta}) = \frac{\partial \tau_i}{\partial \theta_j} - \frac{\partial \tau_j}{\partial \theta_i} \quad \text{for } i, j = 1, \cdots, r \quad i \neq j$$

↓

Quantitatively these are the off-diag. terms in $\bar{K}_j$ $(\partial\bar{\tau} = \bar{K}\partial\bar{\theta})$

Therefore, spring-like behavior is observed when the stiffness matrix is symmetric

[Furthermore, if curl is zero for joint coordinates, it will be zero in all coordinates. So we can take any convenient coordinate frame + measure stiffness in that frame]