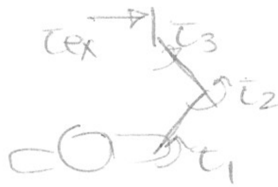
Lecture 16

Last time: talked about multi-jointed sys.

Not like the single jointed sys: no ~~~~ unit joint torque τ_i

for a given τ_{ex}.



- Different combinations ok

- Moment arms changes depending on other joint angles.

By having multiple joints – clearly an advantage.

Imagine having sticks (one joint limb) instead of legs & arms.

~~~~ – You have limited workspace

– " " " force capability

– " " " stiffness modulation capability

We have talked about the transformation from

muscle → joints

position → geometry

velocity → Jacobian

force → J^T

viscosity → direct transformation

" " [illegible] transformation
~~~~

Then we talked about springiness of multi-jointed sys. If (P2)

Spring-like, we can talk in terms of a virtual trajectory.

↳ specification of neural signals

Convenient theory to use to talk about neural commands + actual movements.

We showed potential function is defineable if $\text{Curl}\,\bar{\tau}^T(\theta)d\theta = 0$

$(\int \bar{\tau}(\theta)d\bar{\theta}$

- which is to have $\bar{K}_j$ to be symmetric

$$K_j = \begin{bmatrix} \frac{\partial \tau_1}{\partial \theta_1} & \frac{\partial \tau_1}{\partial \theta_2} & \cdots & \frac{\partial \tau_1}{\partial \theta_r} \\ \vdots & & & \vdots \\ \frac{\partial \tau_r}{\partial \theta_1} & & \cdots & \frac{\partial \tau_r}{\partial \theta_r} \end{bmatrix}$$

If $\bar{K}_j$ is symmetric, then multi-joint sys. is spring like.

Then we can use EP + VT in multi-joint sys.

But! multi-joint sys. are still complex to talk about if we includes all the DOFs

- large matrices
- hard to visualize
- difficult to specify goals

can we talk about things wrt the hand coordinates?

- simple $[x, y, z]$
- easy to visualize
- relate to the goal/external world

Energy conservation

$$\int \bar{f}^T d\bar{h} = \int \bar{\tau}^T d\bar{\theta} = \int \bar{F}^T d\bar{x} \quad !!$$

muscles joints hand

If $\int \bar{\tau}^T d\bar{\theta}$ is defineable (as we saw), $\int \bar{F}^T d\bar{x}$ is defineable ($\mathrm{curl}\ \bar{F}^T = 0$)

that is $\mathrm{Curl}\ \bar{F} = \frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x}$ for 2D

where $\bar{X} = \begin{bmatrix} x \\ y \end{bmatrix}$, $F = \begin{bmatrix} F_x(x, y) \\ F_y(x, y) \end{bmatrix}$

No matter how many joints are involved (shoulder, elbow, wrist, fingers), as long as mvts. are 2D, this eqn is good enough.

Assuming the sys. is linear (was not an assumption until now) for small displacement (dx, dy) w/some force F_x, F_y

$\bar{F} = -K_h d\bar{x}$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = - \begin{bmatrix} K_{xx} & K_{xy} \\ K_{xy} & K_{yx} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix},$$

where $-K_{xx} = \frac{\partial F_x}{\partial x}$ $\quad -K_{xy} = \frac{\partial F_x}{\partial y}$

$-K_{yx} = \frac{\partial F_y}{\partial x}$ $\quad -K_{yy} = \frac{\partial F_y}{\partial y}$

Thus if $K_{xy} = K_{yx}$, then K_h is symmetric

What is $\frac{\partial F_y}{\partial x}$ physically?

- incremental force produced in y dir'n due to incremental displ. in x dir'n

– perpendicular force

Behaviorally, it makes the hand rotate or curl.

(and thus its name)

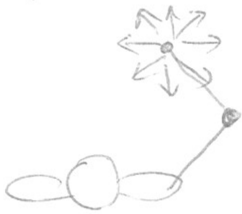
$\frac{\partial F_x}{\partial y}$ is also a perpendicular force.

If $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$, they cancel each other

curl $= 0$

We can measure $\bar{K}_h$

Experiment by Mussa-Ivaldi



@ an E.P.

Small ΔF was applied in 8 different directions

Then measured incremental displacement in

X+Y.

That allows us to solve for $\begin{bmatrix} -K_{xx} & -K_{xy} \\ -K_{xy} & -K_{yy} \end{bmatrix}$

K_{xy} & K_{yx} were similar but not exactly zero

Then he divided $\bar{K}_h$ into symmetric + asymmetric component

$$\bar{K}_h = \bar{K}_{sym} + \bar{K}_{asym}$$

We can do this by assuming

$$K_{sym} = \frac{\bar{K}_h + \bar{K}_h^T}{2}, \quad K_{asym} = \frac{\bar{K}_h - \bar{K}_h^T}{2}$$

Thus,

$$K_{sym} = \begin{bmatrix} -K_{xx} & -(K_{xy}+K_{yx})/2 \\ -(K_{xy}+K_{yx})/2 & -K_{yy} \end{bmatrix}$$

$$K_{asym} = \begin{bmatrix} 0 & -(K_{xy}-K_{yx})/2 \\ -(K_{yx}-K_{xy}) & 0 \end{bmatrix}$$

Then they accounted for symmetric force + asymmetric force

$$\bar{F} = \underbrace{\bar{K}_{sym} d\bar{x}}_{\text{Symm force}} + \underbrace{\bar{K}_{asym} d\bar{x}}_{\text{asymmetric force}}$$

It was found that the symmetric force dominated, + the asymm. force was statistically insignificant.

⇒ Multijoint behavior is effectively spring-like

⇒ we can apply E.P. + V.T. theories to multijoint analysis

- Much more convenient than to do this for each joint individually.

Now how can we talk about mvt. optimization in terms of the end pt.?

Yes we can!

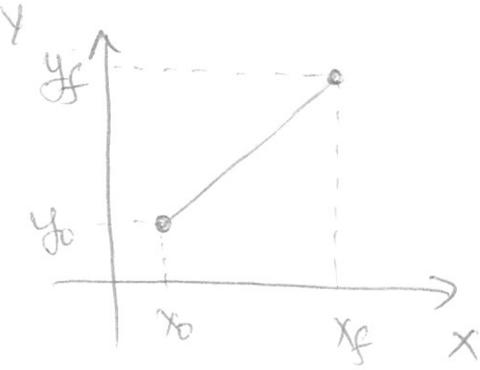
For a single joint, the objective function to minimize was $C = \frac{1}{2}\int_0^d \left(\frac{d^3\theta}{dt^3}\right)^2 dt$

For 2D handspace, the objective function is

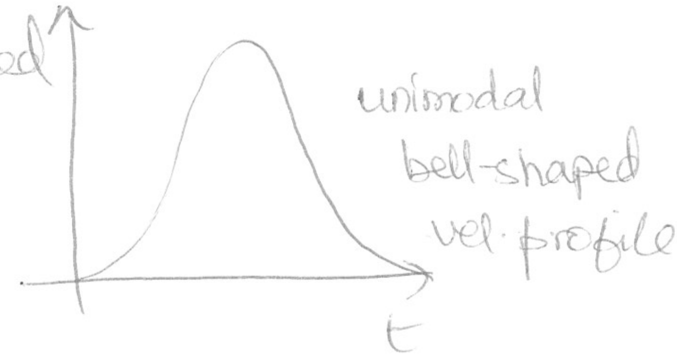
$$C = \frac{1}{2}\int_0^d \left[\left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2\right] dt$$

we want to minimize jerk in all directions.

[won't tell you here what the minimizing functions turn out to be, ~~that~~ becos you will solve for it in PS4, but you can use the Euler Poisson eq'n to get 5th order polynomial for position function]



Straight



These shapes are invariant under changes in mov. amplitude, direction of mov, change or init or final posns

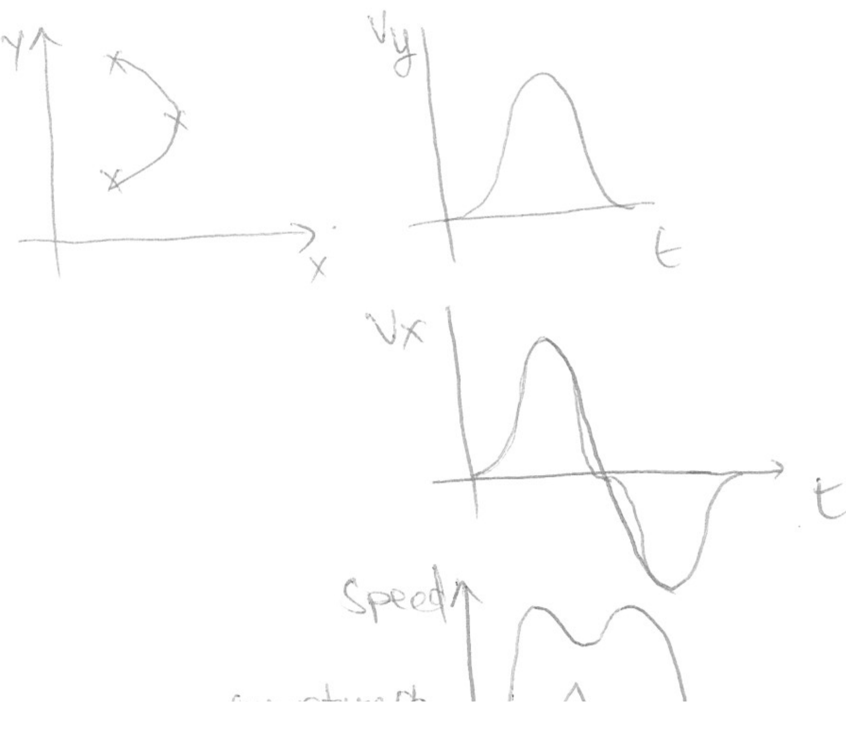
~~But~~ wait! there's more!

The same optimization can predict curved mvts.

For curved mvts, we assume there is a third pt. to pass through that is not on the straight line betwn the initial & final pts.

If we give three pts w/ $v = a = 0$ @ these points, we get

We put constraints in the optimization so that vel. & acc are continuous (details are in Flash & Hogan, 1985)

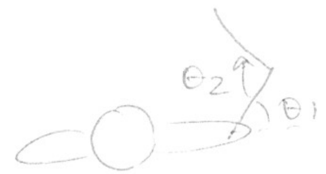


Are they really like real mvts?

Can they be optimized in other coordinate frames like joint angles?

In 2DOF planar mvt again

$$C = \frac{1}{2}\int_0^d \left[\left(\frac{d^3\theta_1}{dt^3}\right)^2 + \left(\frac{d^3\theta_2}{dt^3}\right)^2\right] dt$$



How about other joint coordinates?

$$C = \frac{1}{2}\int_0^d \left[\left(\frac{d^3\rho_1}{dt^3}\right)^2 + \left(\frac{d^3\rho_2}{dt^3}\right)^2\right] dt$$

You will try these different optimization strategies in PS4!

Think about what you find out & compare w/ what you think you do.

x, y appears to be goal oriented $\rightarrow$ good

θs related more w/ muscles $\rightarrow$ good