

Lecture Notes 6

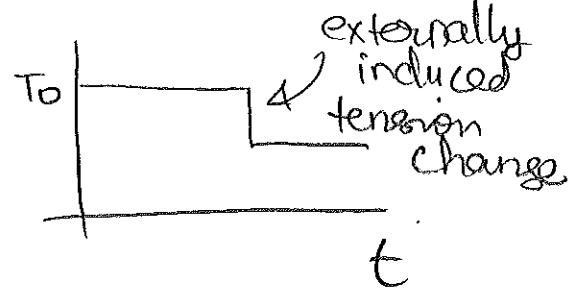
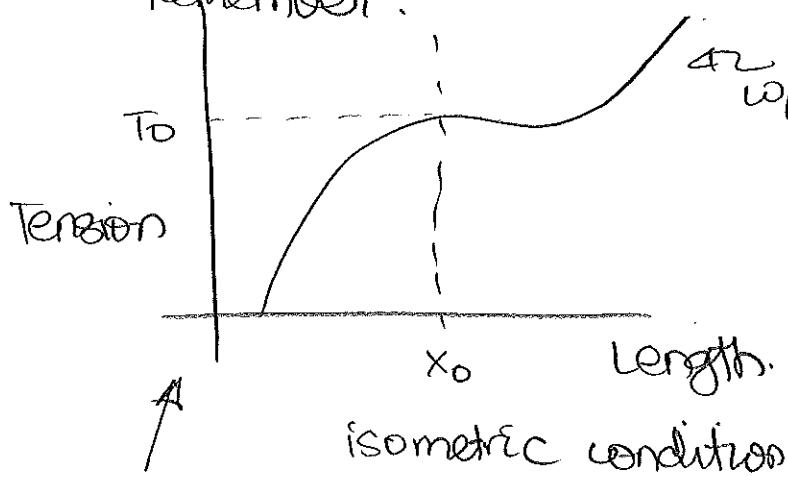
(P1)

More muscle mechanics \rightarrow models.

In an attempt to model muscle,

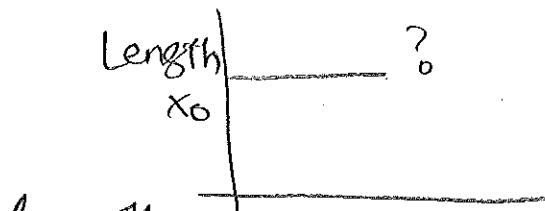
Quick Release expt.

Remember.

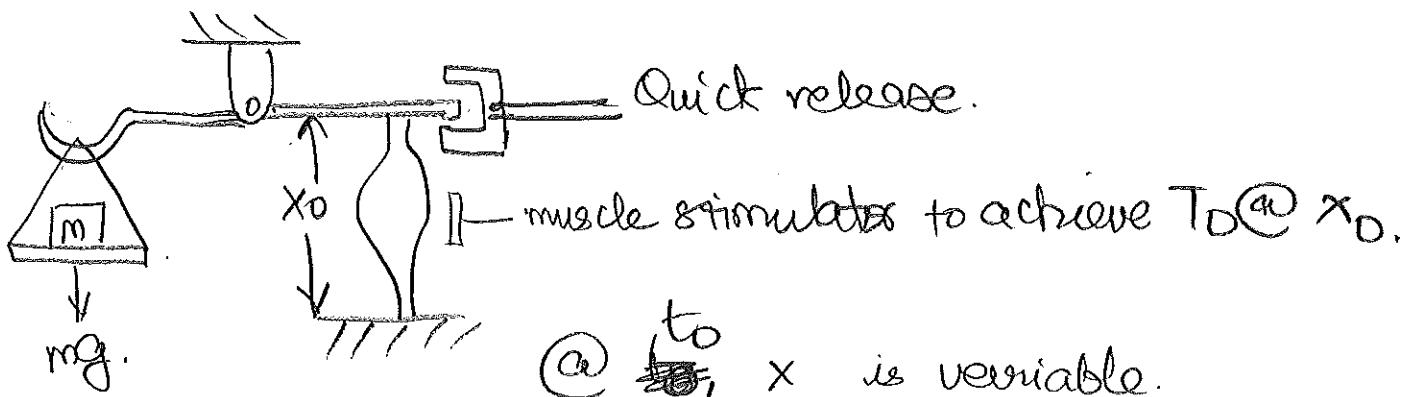


contractile

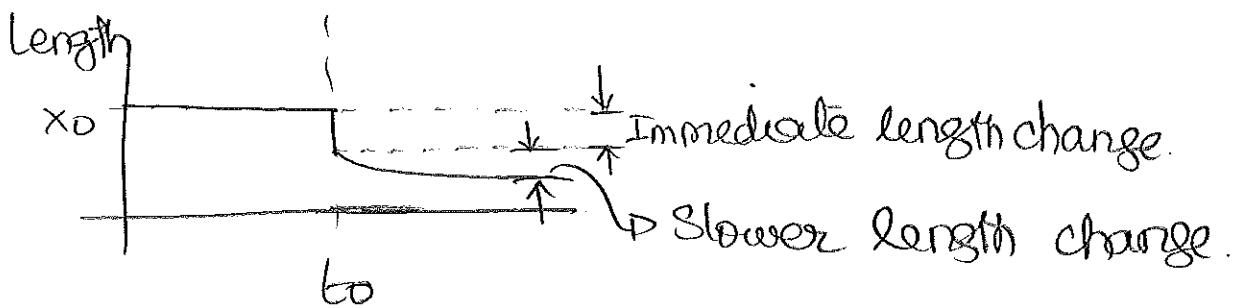
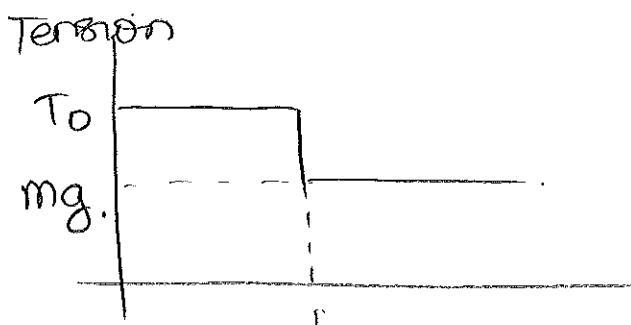
component alone.



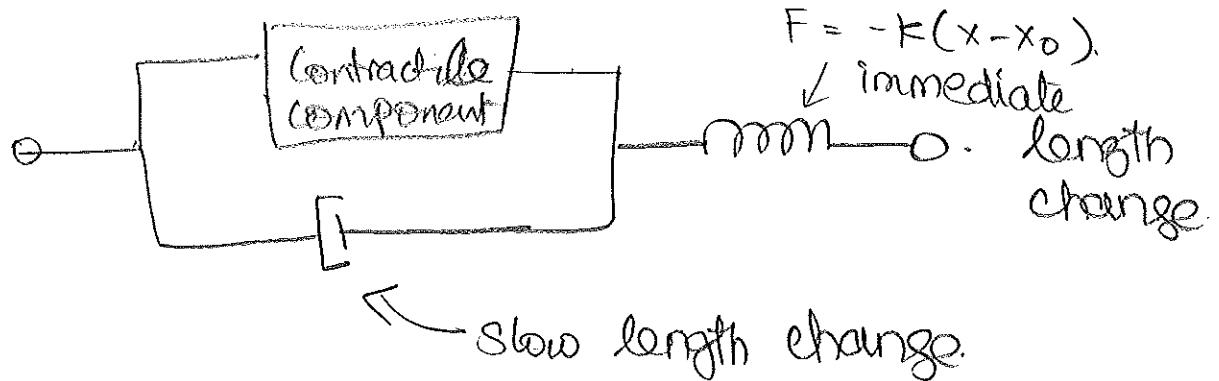
What would happen to length of muscle?



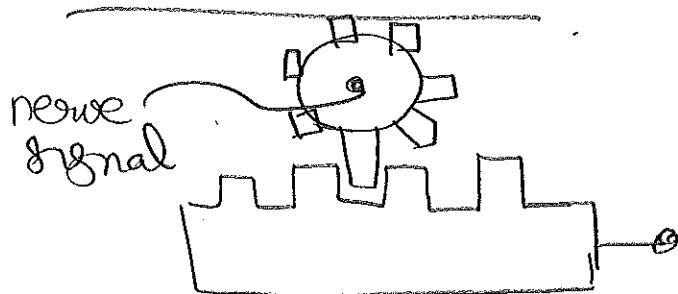
$$T = mg.$$



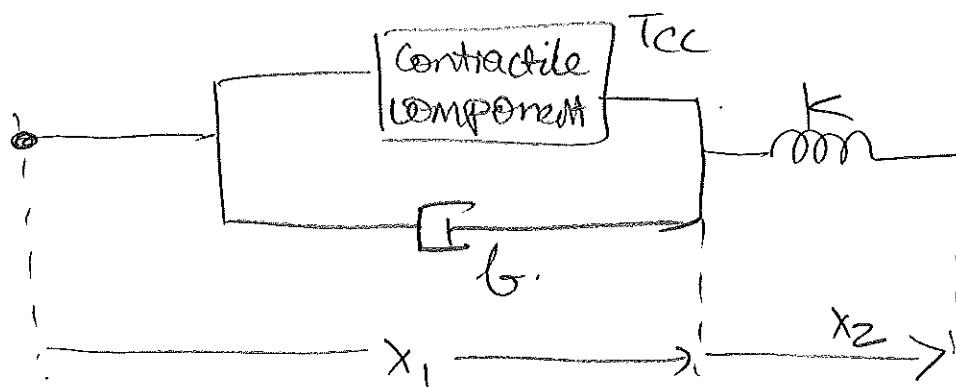
Model



Contractile component:

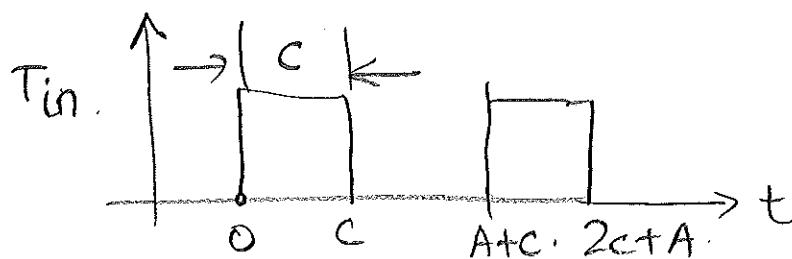


nerve receives signal & contracts.



$$\text{Isometric} \Rightarrow x_1 + x_2 = \text{constant}$$

Contractile component receives pulse input



$$\text{Tension } (t=0) = 0$$

$$\text{Tension } (0 < t < c) = T_{in}$$

What is the tension for the whole sys. under an isometric condition as a function of time?

Use simple physics + differential eqn.

If whole muscle experiences tension T,

$$\text{Spring} \quad " \quad T = K \Delta x_2 \quad (1)$$

So as CC + dashpot

$$T = b \ddot{x}_1 + T_{cc} \quad (2)$$

Remember Isometric $x_1 + x_2 = \text{const}$ ③ (P4).

$$\downarrow \\ \overset{\circ}{x_1} + \overset{\circ}{x_2} = 0.$$

Differentiate ① $\overset{\circ}{\dot{T}} = k \overset{\circ}{\dot{x}_2}$

$$\Rightarrow \overset{\circ}{\dot{x}_1} = -\overset{\circ}{\dot{x}_2} = -\overset{\circ}{\dot{T}}/k$$

$$\Rightarrow T = b(-\overset{\circ}{\dot{T}}/k) + T_{cc}.$$

$$\Rightarrow T + \frac{b}{k} \overset{\circ}{\dot{T}} = T_{cc}.$$

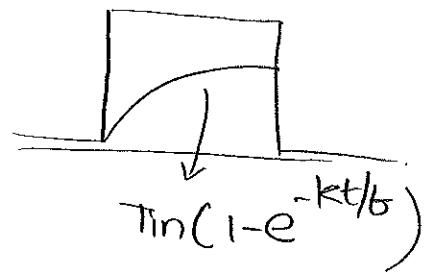
For $t < 0$, $T = 0$.

$$0 \leq t \leq c, T_{cc} = T_{in}, T(0) = 0$$

$$T(t) = T_{in}(1 - e^{-kt/b})$$

@ $t=c$

$$T(c) = T_{in}(1 - e^{-kc/b}).$$

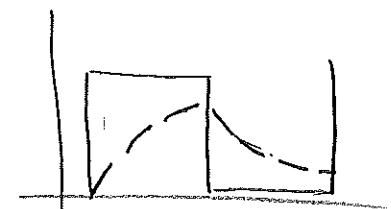


For $c \leq t \leq c+A$, $T_{cc} = 0$, ~~$T(c)$~~ $\Rightarrow T(c)$. ✓

$$T(t) = T(c) \left(e^{-k(t-c)/b} \right).$$

@ $t = c+A$

$$T(c+a) = T(c) e^{-ka/b}.$$

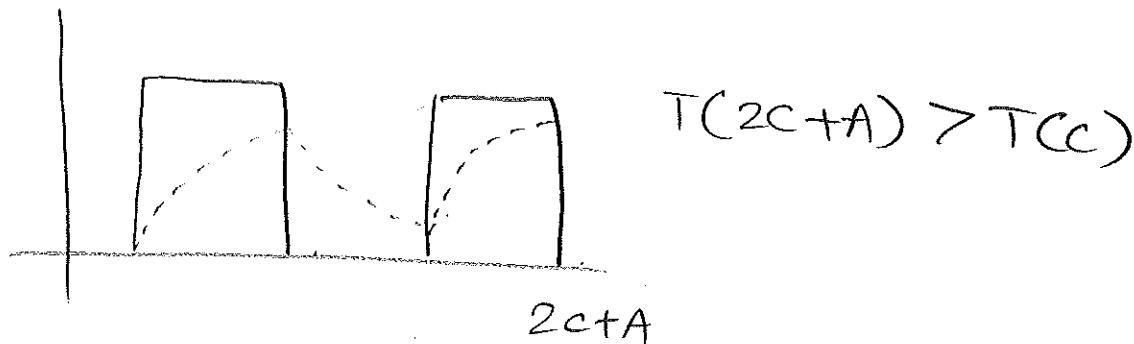


Now @ $c+A \leq t \leq 2c+A$.

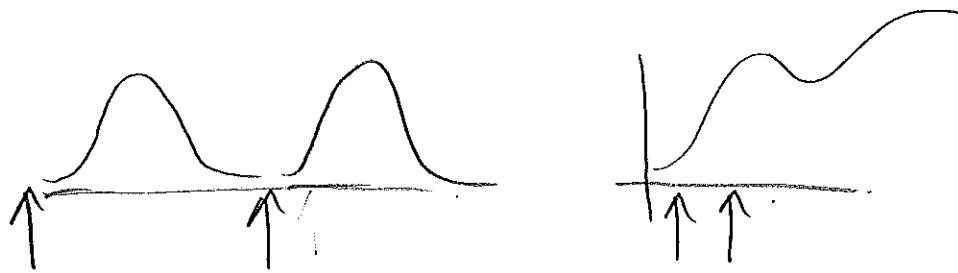
$T_{cc} = T_{in}$ again but @ $t = c+A$.

So it is a superposition of 2 responses.

$$T(t) = \underbrace{T_{cc} e^{-k(t-c)/6}}_{\text{Still decaying.}} + \underbrace{T_{in} (1 - e^{-k(t-c-A)/6})}_{\text{new response}}$$



Explains two-trifl response expt.



The tension @ $2c+A \rightarrow T_{max}$

How do they change w/ $A \rightarrow 0^-$?
 $A \rightarrow \infty$.

$$\begin{aligned} T(2c+A) &= T_{cc} e^{-k(c+A)/6} + T_{in} (1 - e^{-kc/6}) \\ &= T_{in} (1 - e^{-kc/6}) (1 + e^{-k(A+c)/6}) \end{aligned}$$

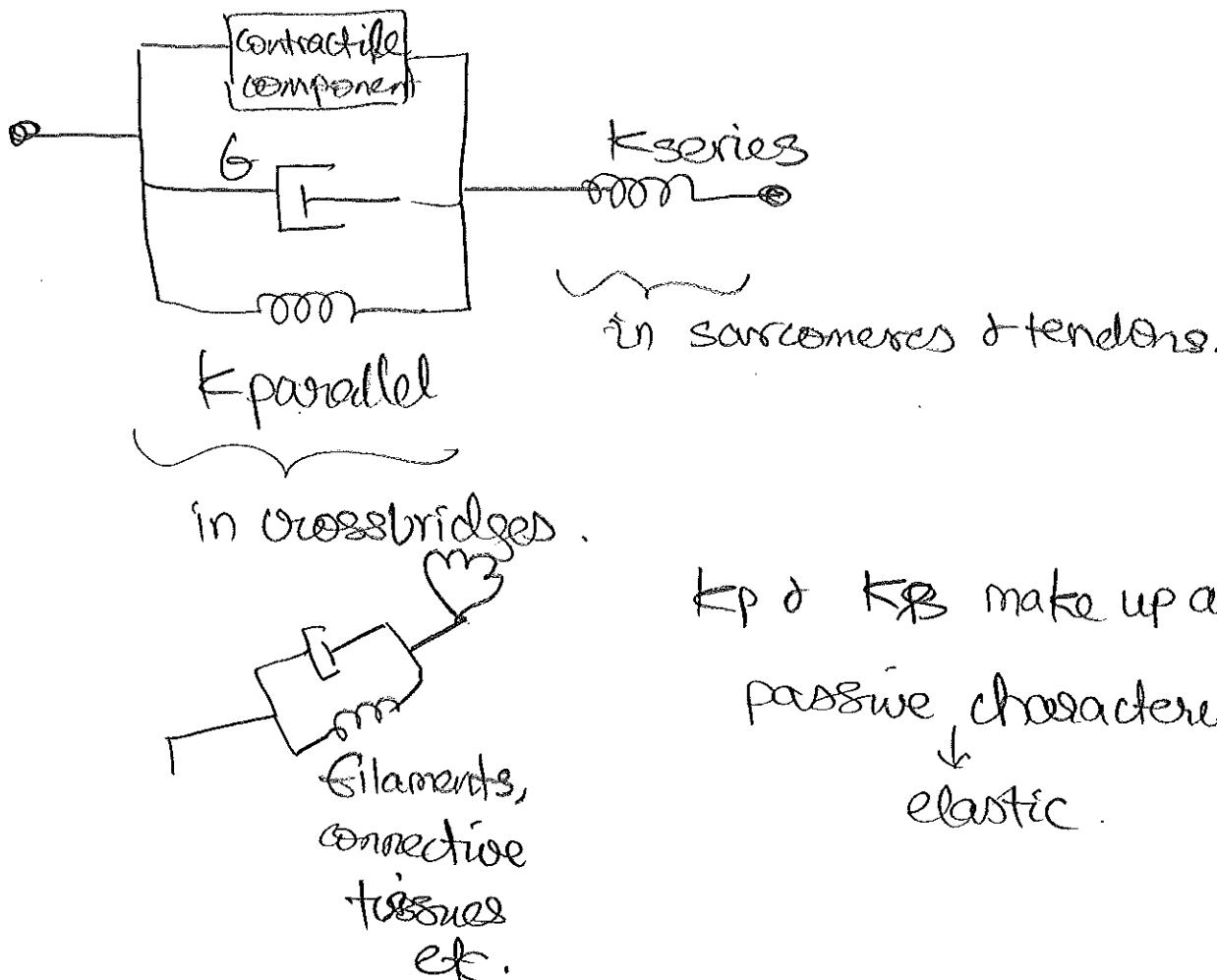
$$\text{When } A \rightarrow 0 \quad T(2C+A) = T_m (1 - e^{-2k_c/b}) \quad (\text{P})$$

continuous stimulation

$$A \rightarrow \infty \quad T(2C+A) = T_m(1 - e^{-k_c/b}) = T(C)$$

Same as if first stimulation did not exist

Full muscle model (also called Hill's model)

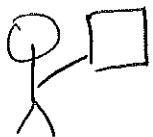
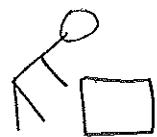


So far we have covered

(P7)

Tension VS time
length.

How about velocity?



lightweight \rightarrow fast

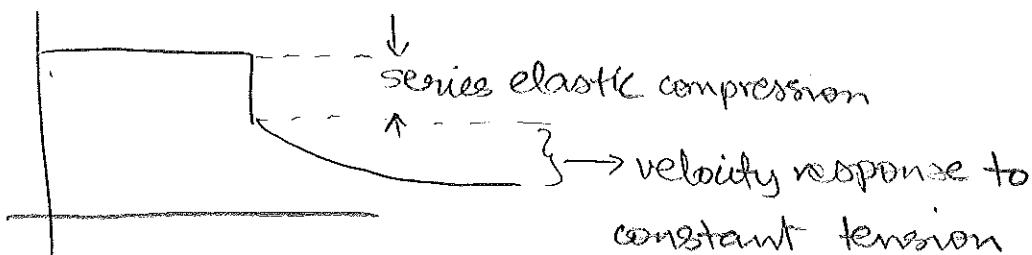
heavy weight \rightarrow slow

Very heavy weight \rightarrow can hold it
but cannot lift

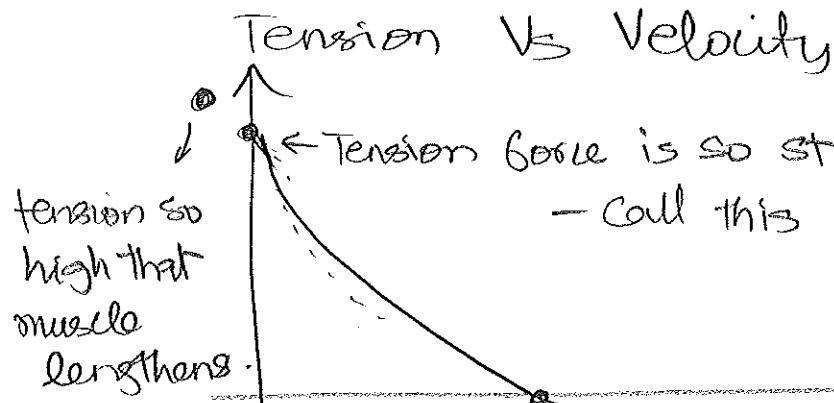
Muscle tension depends on velocity

" can produce more force isometrically than
when actively shortening

Remember in quiet release.



purely in the contractile
+ damper component.



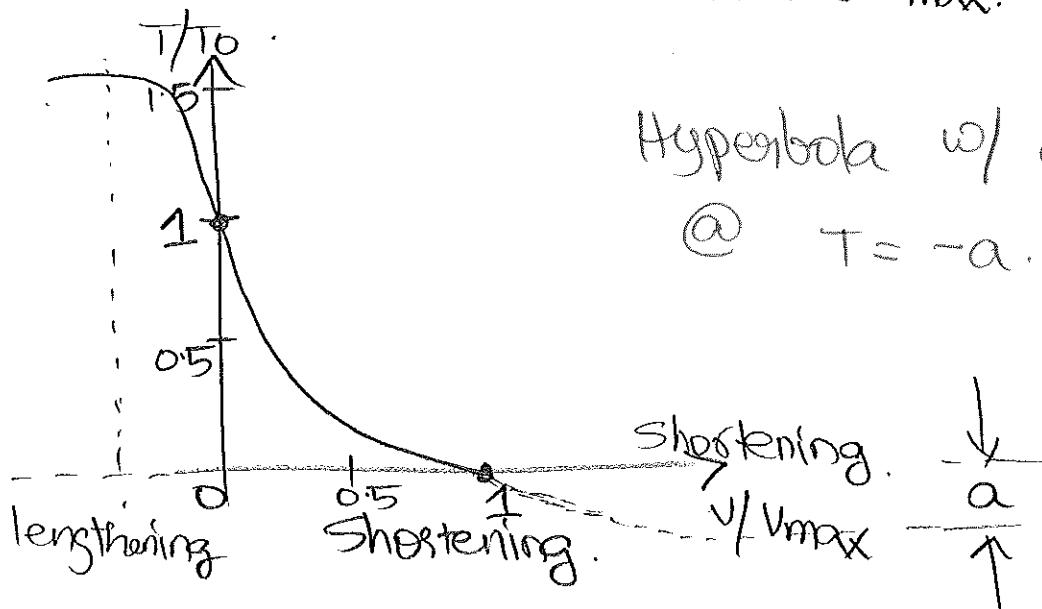
tension so
high that
muscle
lengthens.

← Tension force is so strong that muscle cannot shorten.
— Call this T_0

↑
can move fast
but w/ only
no tension applied

Velocity (rate of muscle length
change).

↳ call this V_{max} .



Hyperbola w/ asymptotes

@ $T = -a$. & $V = -b$.

Normalized Eqn. (Hill's equation)

$$V^l = (1-T^l)/(1+T^l k)$$

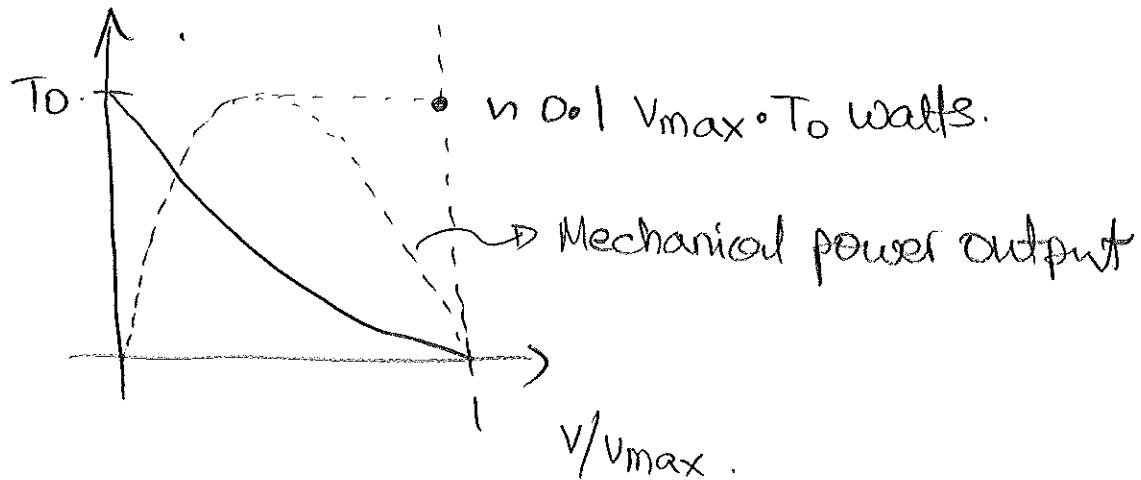
where $V^l = \frac{V}{V_{max}}$, $T^l = T/T_0$, $k = \frac{a}{T_0} = \frac{b}{V_{max}}$

k in the range of $0.15 - 0.25$

works well for vertebrate muscles

$$\text{Power} = FV \Rightarrow T_0 V_0$$

(pq)



Max power @ 20 - 30% of v_{max} .

- ~~standard~~ bicycle gears which allows people to take advantage of this peak.

Muscle lengthening curve is also useful when stopping - explains curve in the negative region

When muscle is @ tetanus activation + applied T is larger than T_0 , muscle actively lengthens