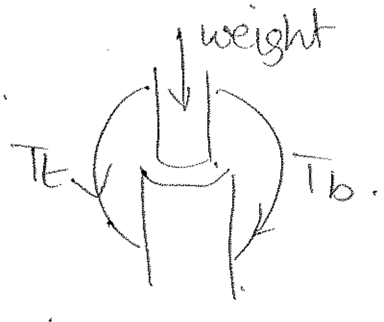
Lecture 12

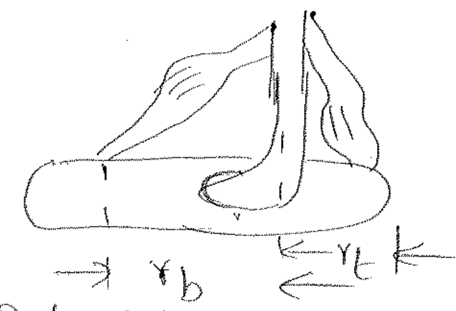
Last time:

$$\tau = I\ddot{\theta} = T_t r_t - T_b r_b + \tau_{ext}$$

grav ext. load

Joint load.





We saw elbow takes ~75 lb w/ NO LOAD!

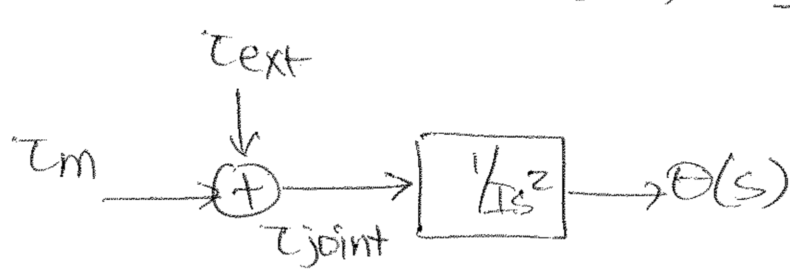
" " hip " 18 MPa → midsize car on a stamp

Now let's see joint stability in the control box diag.

$$\tau_{joint} = \underbrace{T_t r_t - T_b r_b}_{\tau_m} + \tau_{ext} = I\ddot{\theta}$$

In Laplace, $\tau(s) = I s^2 \theta(s)$

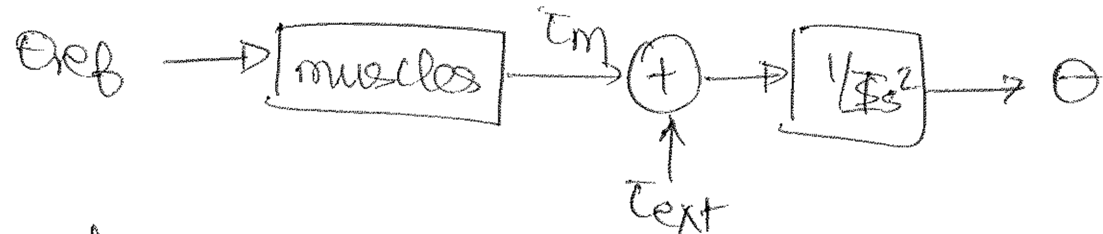
$$\Rightarrow \theta(s) = \frac{\tau(s)}{Is^2}$$



Let's not worry about individual muscle activity for now.

$\alpha's \longrightarrow \boxed{\text{muscles}} \longrightarrow \tau_{musc}$.

α related to $\theta_{desired}$.

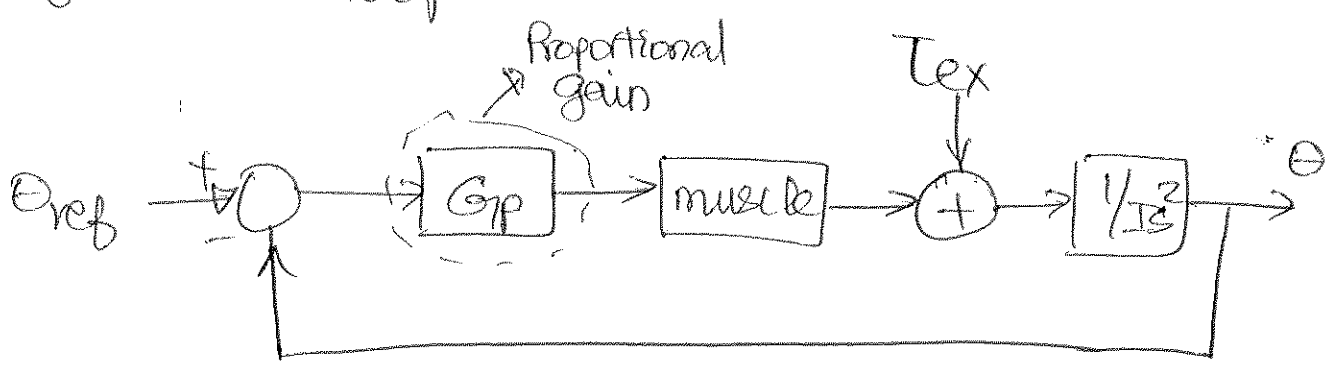


Open loop does not allow joint stability because

$\tau_{ex} \neq 0$.

plant model not perfect.

So Add feedback loop.



Treat muscle as a constant multiplier.

$\hookrightarrow \alpha \propto \tau_m$

$\alpha \xrightarrow{\text{spikes/sec}} \boxed{\text{muscle}} \longrightarrow \tau$ Let's say gain = 1

$C = 1 \frac{\text{sec}}{\text{spikes}}$ N·m

when $\tau_{ex} = 0$

$$\tau_m = G_p(\theta_{ref} - \theta) = Is^2\theta$$

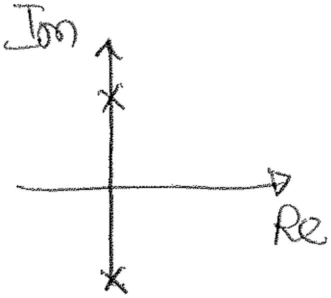
$$\Rightarrow \quad (Is^2 + G_P)\,\theta = G_P\,\theta_{ref}$$

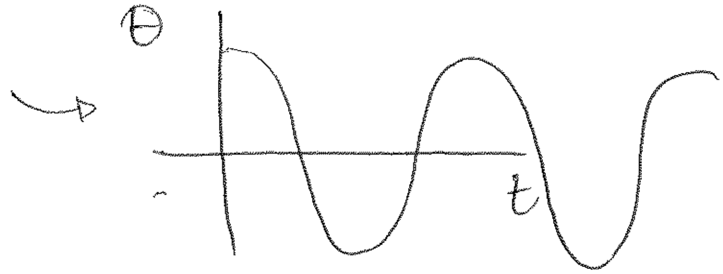
$$\frac{\theta}{\theta_{ref}} = \frac{G_P}{Is^2 + G_P}$$

where are the poles?

Characteristic eqn is $Is^2 + G_P = 0$

$$\Rightarrow \quad s = \pm j\sqrt{\frac{G_P}{I}}$$





oscillates w/ No decay., freq = $\sqrt{\frac{G_P}{I}}$

Good feedback? No.

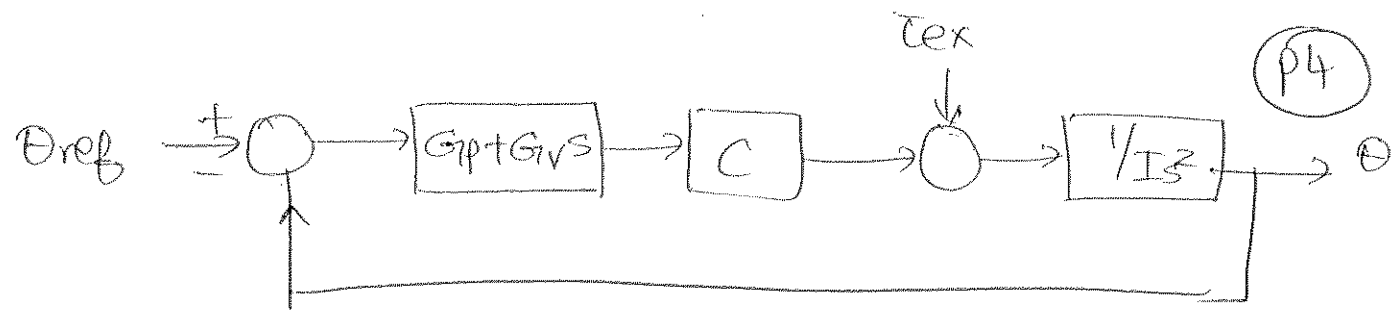
Realistic ? No.

Add Velocity feedback. (more realistic)

$$G_V \frac{d}{dt}(\theta_{ref} - \theta).$$

$$\tau_m = G_P(\theta_{ref} - \theta) + G_V \frac{d}{dt}(\theta_{ref} - \theta)$$

p4



Now T.F.

$$\frac{\theta}{\theta_{ref}} = \frac{G_p + G_v s}{G_p + G_v s + I s^2}$$

Where are the poles? $I s^2 + G_v s + G_p = 0$.

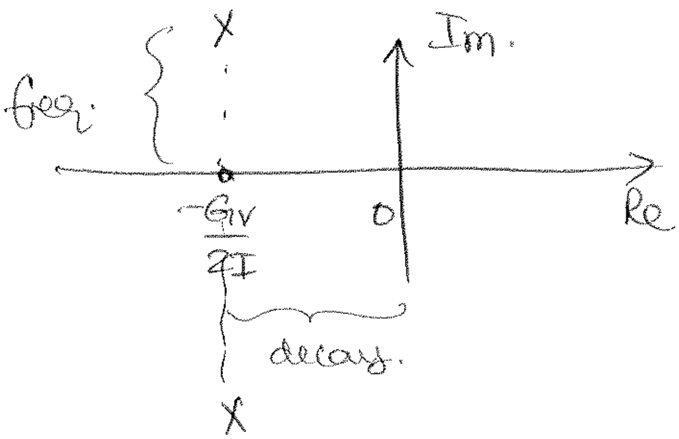
~~a~~ ~~b~~ ~~c~~

$$\text{poles} = \frac{-G_v \pm \sqrt{G_v^2 - 4 I G_p}}{2I}$$

$I > 0$
$G_v > 0$
$G_p > 0$.

Real part always < 0.

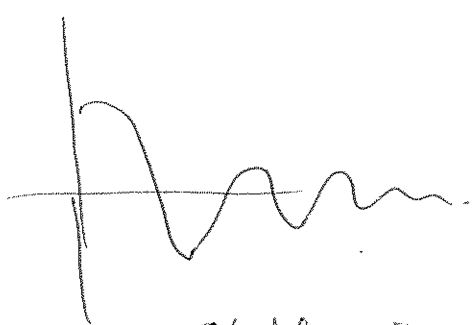
If $G_v^2 < 4 I G_p \Rightarrow$ Imaginary component.



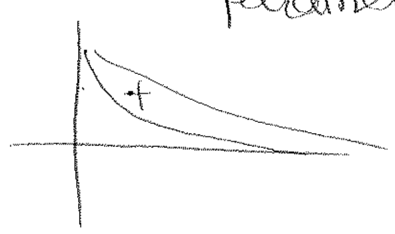
How do they look in the time domain?

~~when $G_v \gg I$~~ Depends on G_p.

~~Dominated by~~ Second order response due to parameters



or



stable

Poles must be on LHP for stability

$$Is^2 - mgh s\beta \cos\theta_0 = 0.$$

$$s = \sqrt{\frac{mgh s\beta \cos\theta_0}{I}} \leftarrow \text{If } > 0 \text{, then unstable.}$$

$mgh s\beta > 0$ always.

$\cos\theta_0 > 0$ when $\underbrace{-90^\circ < \theta_0 < 90^\circ}_{\text{unstable.}}$

- with little perturbation, arm unstable.

Add ~~at~~ closed loop pos & vel. feedback.

$$I\ddot{\theta} + G_V\dot{\theta} + G_P\theta = G_V\dot{\theta}_{ref} + G_P\theta_{ref} + \tau_{ex}$$

plug in again

$$\overbrace{I\ddot{\theta} + G_V\dot{\theta} + (G_P - mgh s\beta\cos\theta_0)\theta}^{\text{charic. eqn}} = G_V\dot{\theta}_{ref} + G_P\theta_{ref} + mgh s\beta \sin\theta_0$$

$$Is^2 + G_V s + \underbrace{G_P - mgh s\beta\cos\theta_0}_{C.} = 0$$

~~always negative~~

$$s = \frac{-G_V}{2I} \pm \frac{\sqrt{G_V^2 - 4IC}}{2I}$$

$G_V > \sqrt{G_V^2 - 4IC}$ for stability

$$G_V^2 > G_V^2 - 4IC$$

$$\Rightarrow \quad 4IC > 0$$

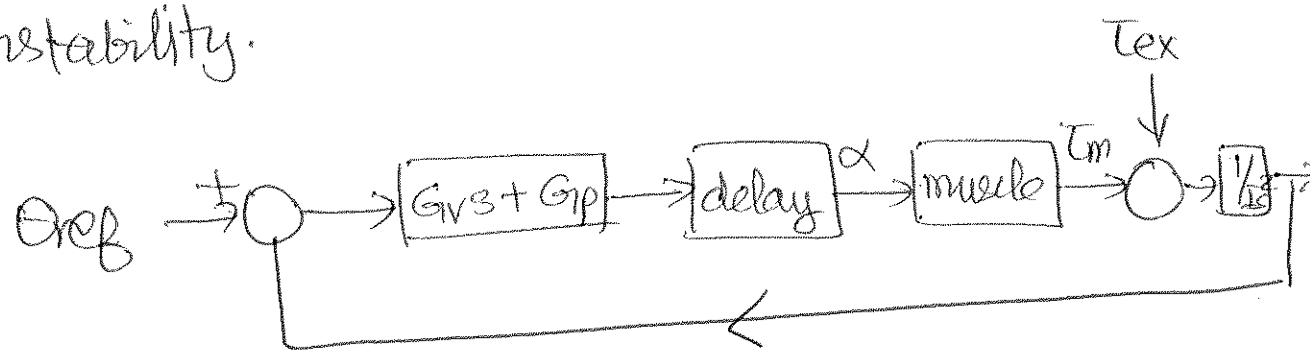
$$\Rightarrow \quad C > 0$$

$$G_p - mgh\,8p\,C\theta_0 > 0$$

↑
tune G_p

So for closed loop control is good for stability

But we have seen that delays & large gain can bring instability.



You'll see this in PS3

for spinal delay. 450ms - 250ms
cortical "

Amazingly slow compared to computers these days (1GHz)

P8

Really slow computers

$$100\ MHz \Rightarrow delay = \frac{1}{10^8} = \underbrace{10^{-8}\ sec}_{\text{1 million times faster than CNS}}$$

So feedback is not very effective when input is high frequency.

How do limbs reject high freq?

Imagine high freq input.

What do you do?