Lecture 17

Last time:

we said curl $F^T \cong 0 \rightarrow$ curl $\tau^T \cong 0$.

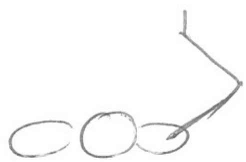
- System is springlike

Then we can talk about V.T.

We can also talk in terms of optimization in these coordinate sys.

You tried 3 different coordinate sys.

x, y $\qquad \theta_1, \theta_2 \qquad \rho_1, \rho_2$



- How they looked like
- What are they like really
- Some examples

None of them are perfect but closest to x., y. optimization

elbow joint → close → open → close to keep hand coordinates smooth.

This is great because

- easiest to relate to the resulting force interaction w/ objects
- Calculation is easiest, no need to worry about kinematic dependency.

But it conveniently neglects

- kinematic variability (arm length)
- motor commands
- external force

One way to incorporate all these effects is to minimize torque change @ muscles.

Minimum torque change model.

(Uno, Kawato, Suzuki, 1989)

$$C = \frac{1}{2} \int_0^{d} \sum_{i=1}^{n} \left(\frac{d\tau_i}{dt}\right)^2 dt ,$$

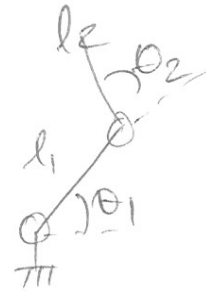
where τ_i is the motor command (in torque) that's fed to the i^{th} muscle.

It's closely related to min jerk theory, becos acceleration is locally proportional to torque @ low speed.

Because getting the exact musculoskeletal system's dynamic eqn is very complex.

They make an approximation, they used a robotic manipulator's torques.

$$\tau_1 = (I_1 + I_2 + m_2 l_1 l_2 c\theta_2 + m_2 l_1^2)\ddot{\theta}_1 + (I_2 + \frac{m_2 l_1 l_2}{2} c_2)\ddot{\theta}_2 - \frac{m_2 l_1 l_2}{2}(2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 s_2 + b_1 \dot{\theta}_1$$



$$\tau_2 = (I_2 + \frac{m_2 l_1 l_2}{2} c_2)\ddot{\theta}_1 + I_2 \ddot{\theta}_2 + \frac{m_2 l_1 l_2}{2}\dot{\theta}_1^2 s_2 + b_2 \dot{\theta}_2$$

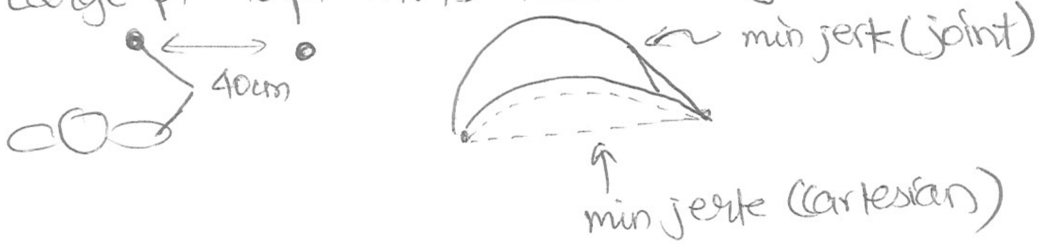
Even w/ this simplicity, τs are non-linear

↳ difficult (impossible) to obtain analytical solution for minimizing pos/vel ~~pos~~ function

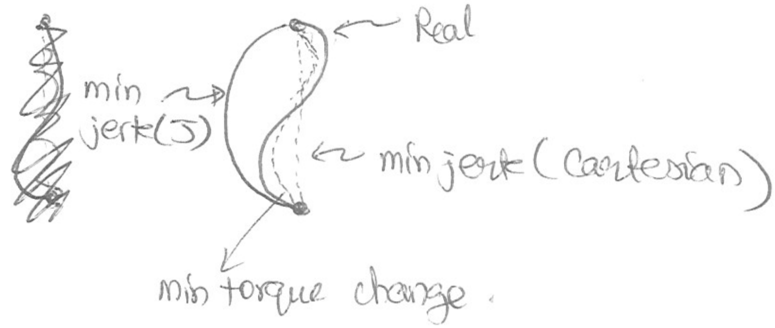
Use iterative learning scheme to solve for them. (why? Read the paper!)

However, after this complex operation, they showed that the predicted mvts. accomodate the variability & shape of mvts slightly better than min. jerk

eg: Large pt. to pt. mvts near body



f^2: Smaller fast mvts.



Summary

Min jerk: focus on behavioral goal

↳ fails to accommodate variability in experimental data

↳ also requires high arm stiffness (which is different from experiment data)

Min torque change: focus on variability + complexity

- no explicit step-by-step transformation of coords

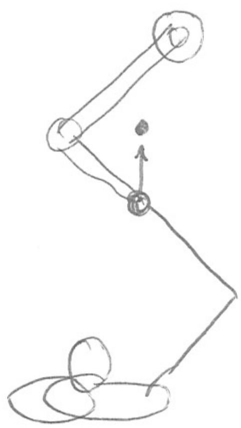
- learn "everything" simultaneously.

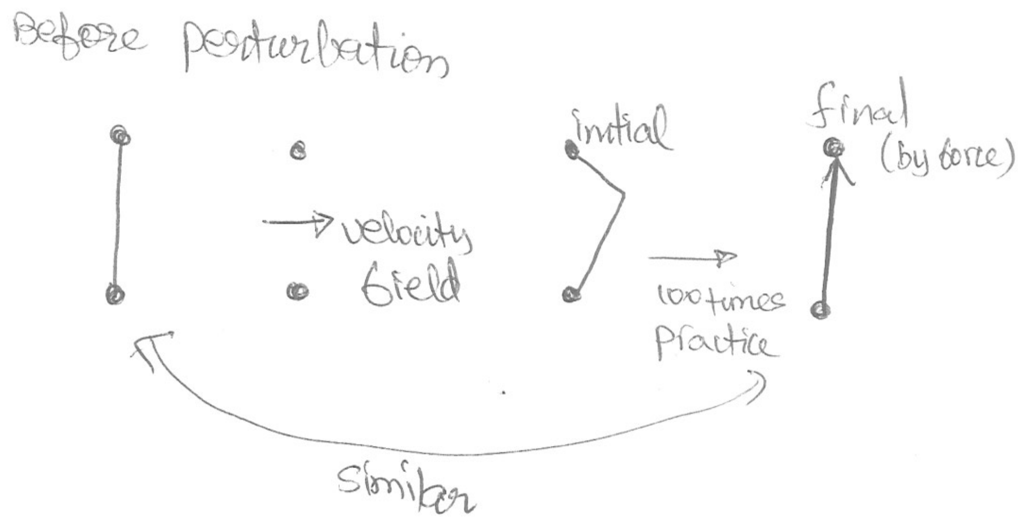
(
* trajectory determination
* coords transformation
* generation of motor command
)

- But fails to explain how goals are achieved in the presence of disturbances or long duration mvts.
(enforces velocity symm. which need not be true)

_ At this point, the min Jerk theory is used most often due to simplicity

Optimization implies that if the sys. is perturbed away from the optimized trajectory, they shd eventually go back to cancel the external perturbation.





Instruction: make continuous mvt. to the target.

Optimized mvt. returns w/in ~ 50 tries & does not change as long as the external perturbation does not change.

~~Back to min. jerk theory?~~

However, a series of work showed that external perturbations or constraints can violate min jerk & min torque change models

1. Matsuoka (1997)

Apply force for only part of the mvt.

Instruction : make continuous mvt. to the target.

Before pert.

Init

50 times practice

Never changed to be straighter

Is it impossible to move straight? No!

When subjects were asked to move straight (goal), they could do it.

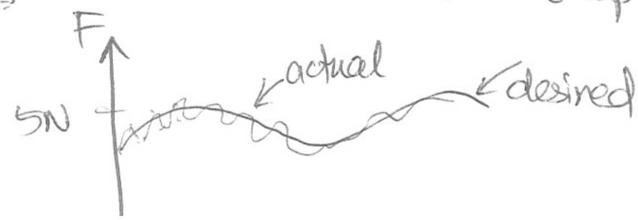
So min jerk @ hand coords may not be what we optimize

How about min torque change?

Latash (1998) say no.

Task : push force plate w/ four fingers

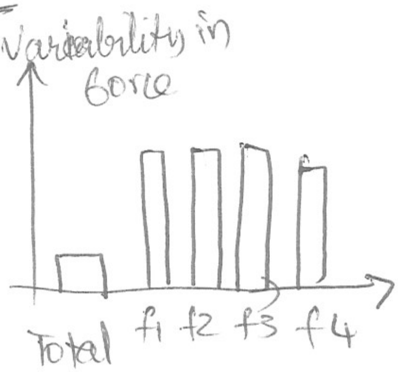
Match the desired force displayed on screen.



Meanwhile they also measured the force produced by each finger & plotted their variability

→ If we optimized torque change @ muscles, then we would see that translate to individual fingers.

BUT



Total task force variability < individual finger force variability

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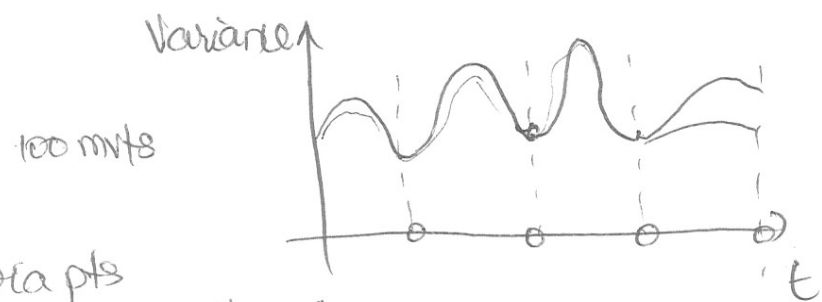
So what are we optimizing?

One more expt: Todorov (1998)

Via pt experiment.

Told to make continuous mvts & try to pass thru' via pts.



Lots of via pts

So turned out Matsuoka (1997)
Latash (1998)
Todorov (1998)

resembles this result if we used "optimal feedback controller in task performance"

Todorov 2001 – does not have official name but can call it "Minimum Variance Theory in Task Space"