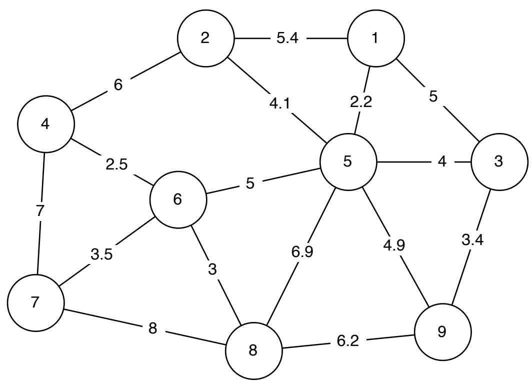
CS325 Assignment #3, Summer 2009, OSU

This assignment is due on Thursday, 7/16/2009. You are required to **type your solution**, and turn it in printed at the beginning of class.

1. Consider the following undirected graph:

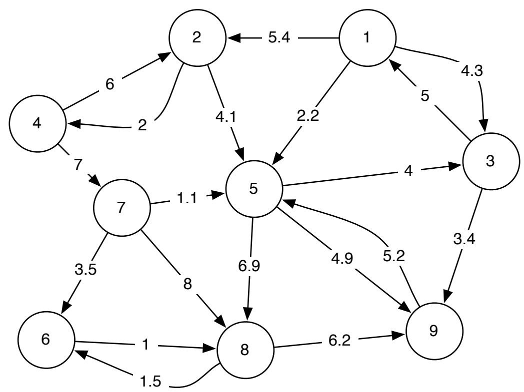


Find a minimum spanning tree using each of the following algorithms:

a. Kruskal's algorithm
b. Prim's algorithm (starting from node 1).
c. Reverse delete

For each algorithm, list the order in which it adds / removes edges to / from the MST.

2. Run Dijkstra's algorithm on the following graph to compute the shortest-path weight for each vertex. Start from vertex 1.



3. For each of the following alphabets and letter frequencies, run Huffman's algorithm to produce a binary tree (show a picture of your tree), and also state the resulting code (i.e. function from letters to binary strings).

   a. $\alpha = \{A, B, C, D\}$ and $f_A = .1,\ f_B = 0.75,\ f_C = .1,\ f_D = .05$

   b. $\alpha = \{A, B, C, D, E, F, G\}$ and $f_A = .1,\ f_B = .1,\ f_C = .1,\ f_D = .1,\ f_E = .35,\ f_F = .05,\ f_G = .2$

4. Huffman's algorithm uses a priority queue. What would its runtime be if we used a naive array implementation for the priority queue (instead of the more efficient binary heap)? Explain your answer.

5. a. You have pennies, nickels, dimes and quarters. Suppose I ask you for X cents in change. Give a greedy algorithm which finds the smallest set of coins with total value equal to X. Explain why your algorithm gives the smallest possible number of coins that add up to X.

   b. Suppose that instead of having coins worth $\{1, 5, 10, 25\}$, you instead have coins that are worth $\{1, 3, 4\}$. Give an example where your greedy algorithm does not produce the smallest set of coins.

6. Houses are placed along a straight line at positions $H = \{h_1, h_2, ..., h_n\}$, where h_i is a real number. A cell phone company would like to place as few base stations as possible, so that all of the houses are within 4 miles of a base station. Your goal is to devise a greedy algorithm that takes as input the positions of the houses, and outputs the positions of the base stations $B = \{b_1, b_2, ..., b_k\}$, such that for all h_i, there exists some b_j such that $|h_i - b_j| \leq 4$, and $|B| = k$ is as small as possible.

   a. Write the psuedocode for your algorithm

b. Analyze the runtime of your algorithm

c. Prove by induction that your algorithm uses the minimum possible number of base stations.