**SOLUTION:** The *Cut Theorem* states that the flow across any cut is equal to the value of the flow (which is defined to be the flow leaving the source). More formally, we wish to show that for *any* flow $f$ and *any* cut $(S, T)$

$$\sum_{x \in S, y \in T} f(x, y) = |f|$$

**Proof:** First, note that:

$$\sum_{x \in S, y \in T} f(x, y) = \sum_{x \in S, y \in V} f(x, y) - \sum_{x \in S, y \in S} f(x, y)$$

where the RH equality is a result of the fact that $T = V - S$. In this form, we can now apply the skew symmetry constraint, yielding:

$$\sum_{x \in S, y \in T} f(x, y) = \sum_{x \in S, y \in V} f(x, y)$$

Observing that $S = \{s\} + S - \{s\}$ facilitates a further transformation:

$$\sum_{x \in S, y \in T} f(x, y) = \sum_{y \in V} f(s, y) + \sum_{x \in S - \{s\}, y \in V} f(x, y)$$

In this form, we can apply the conservation of flow constraint, yielding:

$$\sum_{x \in S, y \in T} f(x, y) = \sum_{y \in V} f(s, y) = |f|$$

which is what we set out to prove.