8 Representation
Syntax is Representation

Syntax: Agreed-upon representation for semantic concepts

- doggy
- Hund
- dog
- Wauwau
- chien
Semantics is Representation

Semantic Domain

Abstraction

Topus Uranus

Reality isn’t contained in the physical, material world, but outside of it, in the world of ideas.

~Plato
8 Representation

Representation as a tool of thought
Languages and DSLs are domain representations
Example 1: Spatial Query Language
Example 2: 2D Animations
Example 3: Data Collection & Analysis
A Number Game

- Given: 9 number cards 1, 2, 3, ..., 9
- Two players, A and B, alternate in picking cards
- The player who first can form a sum of 15 wins

Example game:
A takes 8, B takes 2, A takes 4, B takes 3, A takes 5

Question:
Which move should B make next?
Another Game

• Tick-tack-toe: two players, A and B, alternate in placing Xs and Os in a 3-by-3 grid.

• The player who first can form a row, column, or diagonal wins

Example game:

```
X O X
X
O
```

Question:
Which move should B make next?
Both Games Are Isomorphic

• Arrange numbers in the 3-by-3 grid so that they form a magic square.

• Color selected numbers by player

Example game:

Question: Which move should B make next?

Take 6
Multiple Representations

- Mapping the arithmetic representation to a spatial representation makes the task easy for humans.
- Mapping the spatial representation to an arithmetic representation makes the task easy for computers.

*The effectiveness of a representation depends on the cognitive abilities of the “user”*
Textual versus Visual

- Humans are “spatial creatures”
  ⇒ Effective DSLs are often visual

- Computers are “non-spatial machines”
  ⇒ General-purpose languages are textual
Choice of Representation

- The right representation makes problem solving easy
- A good representation captures essential elements and leaves out the rest
- Abstraction is critical: express important aspects in designing a representation

- Cognitive artifacts serve as tools of thought (most important tool: paper + reading and writing)
- “It is things that make us smart” (Don Norman)
DSLs as Tools of Thought

• A DSL is a tool of thought for a particular domain
• A good DSL design facilitates the easy representation of essential domain elements
• Abstraction is critical: A DSL should be designed around the most important domain objects

CREATIVITY CRITERION
An ideal DSL lets you explore new ideas in a domain that were not thought of before
Metalanguages

• BNF and Haskell are metalanguages for describing languages

• BNF and Haskell are metarepresentations

• Advantages of Haskell:
  Can describe semantics (BNF only does syntax)
  Allows more flexible definitions:
  - parameterized types (type constructors)
  - function types in productions
  - higher-order abstract syntax

CREATIVITY ASPECT
... explore new ideas in the domain
Other Metalanguages

- **UML**
  
  *Advantage*: Can be understood by OO programmers
  *Disadvantages*: Limited like BNF, few abstractions, not scalable

- **Prolog**
  
  *Advantage*: Nice parser support, backtracking
  *Disadvantages*: Not higher order (but see Lambda Prolog), untyped

**Strong typing of metalanguage guarantees syntax correctness of represented language**
Other Metalanguages

- **Idris**
  
  *Advantages*: Can model sophisticated dependent type systems and typed semantics, can produce automatic correctness proofs (in some cases)
  
  *Disadvantage*: Niche product, limited support

- **And more:**
  
  C++ (templates)
  FreshML (support for binders)
  MetaOCaml (staged programming)
  Graph Grammars, Multiset Grammars (visual languages)

...
Example 1: Spatial Query Language

Example Query: “Find all cities in a certain area”

⇒ Need types for points and regions
⇒ Need operations and predicates, such as intersection and inside

```haskell
type Point = ...
type Region = ...

(\(\)) :: Region -> Region -> Region
inside :: Point -> Region -> Bool```

Representations
Representation of Regions

A direct, obvious approach:

```plaintext
type Point = (Float,Float)
type Region = [Point]
```

Implementation of inside or intersection is hard and takes a long time!

A functional approach:

```plaintext
type Region = Point -> Bool
(\land\land) :: Region -> Region -> Region
(r \land\land s) p = r p && s p
```

Implementation is trivial and found within in a minute
More Spatial Functions

type Region = Point -> Bool

(\(\setminus\)) :: Region -> Region -> Region
(r \(\setminus\) s) p = r p && s p

(\(\cap\)) :: Region -> Region -> Region
(r \(\cap\) s) p = r p || s p

outside :: Region -> Region
outside r = not . r

inside :: Point -> Region -> Bool
inside p r = r p
Abstraction Opportunities

type Region = Point -> Bool

(/
\) :: Region -> Region -> Region
(r \ s) p = r p && s p

(\/) :: Region -> Region -> Region
(r \ s) p = r p || s p

liftB :: (Bool -> Bool -> Bool) -> Region -> Region -> Region
liftB f g h x = f (g x) (h x)

(/
\) = liftB (&&)
(\/) = liftB (||)
Query Language

A simple database

data City = C {name::String, pos::Point}

cs :: [City]
cs = [C "A" (1,1), C "B" (4,4)]

A simple query: “Find all cities in area r”

cs `select` (pos `isInside` r)  
select = flip filter

isInside :: (a -> Point) -> Region -> (a -> Bool)
isInside f r x = f x `inside` r  
isInside f r x = r (f x)  
isInside f r = r . f
isInside = flip (.)
Query Language Observations

Representation of one basic object (region) directly supports several critical operations.

Functional representation is powerful since functions can be applied.

Higher-order functions can leverage functional representations of objects.
Example 2: Visual Languages

Domain: 2D animations and 2D games on a grid-like background

⇒ Need description for movement of objects

*Example Animation:* “Define a train that follows train tracks”

```
move d t | d==East && t.right==HorTrack = Right
| d==East && t.right==RgtTurn = TurnRight
| ...
```
Visual Rewrite Rules

AgentSheets by Alexander Repenning
Creating Rules by Analogy

“Cars moves on streets like trains move on tracks”

Output: a visual rewrite rule

A form of visual metaprogramming
Movement Built into Background

Automatic bending of movement tiles
Example 3: Data Collection & Analysis

Domain: Science & financial data collections

Example: “Collect and aggregate precipitation data”

```plaintext
corv = [23, 29, 12, 15, 5, 0, 0, 8, 11, 17, 25, 29]
salem = [21, 26, 19, 16, 5, 1, 0, 9, 5, 21, 20, 24]
pdx = [27, 28, 17, 13, 9, 3, 0, 7, 16, 15, 21, 28]
precip = [corv, salem, pdx]

agg x = (maximum x, sum x)
placeAgg = map agg precip
monthAgg = map agg (transpose precip)

placeAgg
[(29, 174), (26, 167), (28, 184)]
```

relationship to data source is lost!
Spatially Embedded Computing

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Syntax of Spreadsheets

Formulas

\[ f \in F \quad ::= \quad v \quad | \quad \omega(f, \ldots, f) \quad | \quad \rho \]

Values

References

\[ \rho \quad ::= \quad (p, p) \]

Operations

Two-dimensional grid

Absolute & relative references

Pointers

\[ p \quad ::= \quad n \quad | \quad +n \quad | \quad -n \]

Natural numbers

Spreadsheets

\[ S \in \Sigma \quad = \quad A \rightarrow F \]

Addresses

\[ a \in A \quad ::= \quad (n, n) \]

(a, f) \in S is called a cell
Spreadsheets vs. Lambda Calculus

\[ f \in F ::= v \mid \omega(f, \ldots, f) \mid \rho \]

\[ e ::= \lambda x.e \mid e \cdot e \mid x \]

\[ S \in \Sigma = A \rightarrow F \]
Computational Limitations of Spreadsheets

\[ f \in F ::= v \mid \omega(f, \ldots, f) \mid \rho \]

\[ e ::= \lambda x.e \mid e \ e \mid x \]

first-order language

no local scope

generally, no recursion