PRACTICAL APPLICATIONS OF SENSITIVITY ANALYSIS IN ENVIRONMENTAL MODELING

D. M. Hamby

Department of Environmental and Industrial Health
University of Michigan School of Public Health
Ann Arbor, MI 48109-2029

Email for correspondence: dmhamby@umich.edu

1 INTRODUCTION

Mathematical models, designed to simulate complex physical processes, are often used in scientific and engineering studies. For example, modeling the movement and consequence of radioactive pollutants is extremely important in the nuclear industry for environmental protection and facility control. One of the steps in model development is the determination of the parameters most influential on model results. A sensitivity analysis of these parameters is not only critical to model validation and uncertainty, but also guides future research.

The following is an assessment of several sensitivity analysis methods. It demonstrates calculational rigor and provides a comparison of parameter sensitivity rankings resulting from various sensitivity analysis techniques. The methods under comparison here have been summarized elsewhere [4]. An atmospheric tritium dosimetry model [3] is used as an example, but the techniques described can be applied to many different modeling problems.

2 SENSITIVITY ANALYSIS METHODS

The results of the application of ten sensitivity analysis techniques on an atmospheric tritium dose model [3] are presented. The sensitivity methods include the utilization of the following one-at-a-time sensitivity measures: partial derivatives (PD), one standard deviation increase and decrease of inputs (±SD), a 20% increase and decrease of inputs (±20%), and a sensitivity index (SI). The sensitivity measures investigated that utilize an array of input and output values generated through random sampling include: an importance index (II), a relative deviation of the output distribution (RD), a relative deviation ratio (RDR), partial rank correlation coefficients (PRCC), standardized regression coefficients (SRC), and rank regression coefficients (RRC). A Latin hypercube sampling procedure was used to generate an input array to the 21-parameter dose model with a sample size of 1000 [3].

In the dose model used here, parameter sensitivity is simplest to achieve by first aggregating the mathematical model, i.e., algebraically combining exposure pathway models, evaluating the resulting equation using best-estimate parameter values, and assessing the relative contribution to dose via each pathway component. Total atmospheric tritium dose to a downwind receptor is the sum of the inhalation and ingestion pathway doses and is given by,

$$D = \left\{\frac{4.84\text{x}10^{-9}\ T_e\ f_w\ C^a\ R_{pa}}{M\ H}\right\} \bullet \left\{(2.74\ U_m\ f_m\ f_{pm}\ I_m\ e^{-(\lambda t_m)}) + (2.74\ U_b\ f_b\ f_{pb}\ I_b\ e^{-(\lambda t_b)}) + (1000\ U_v\ f_v) + (1000\ U_l\ f_l) + \left(\frac{(1.5)\ BR\ H}{f_w\ R_{pa}}\right)\right\} \tag{1}$$

where the constants account for unit conversions. Definitions of parameter distributions are given in Table 1. The five components in the right set of brackets represent the five exposure pathways: milk consumption, beef consumption, produce consumption, leafy vegetable consumption, and inhalation; respectively. It is immediately apparent that the model will be sensitive in some degree to three of the parameters in the left set of brackets (T_e, C^a, and M) since their values influence all pathway dose estimates. The three remaining parameters in the left brackets (f_w, R_{pa}, and H) cancel in the inhalation portion of the equation, therefore, they are expected to be sensitive parameters, but to have less influence than T_e, C^a, and M, since all pathway dose estimates are not affected by their values.

Table 1. Parameter definitions and applicable exposure pathway models.

Description	Parameter
Average annual concentration of tritium	C^a
Effective biological half-life of tritium	T_e
Mass of soft tissue in adult male	M
Average annual absolute humidity	H
Percent water in vegetation	f_w
Ratio of plant to atmospheric tritium	R_{pa}
Consumption rate of milk	U_m
Fodder ingestion rate (milk cattle)	I_m
Feed-to-milk transfer factor	f_m
Fraction of fodder from pasture (milk cattle)	f_{pm}
Milk transport time (milking to consumption)	t_m
Consumption rate of beef	U_b
Fodder ingestion rate (beef cattle)	I_b
Feed-to-beef transfer factor	f_b
Fraction of fodder from pasture (beef cattle)	f_{pb}
Beef transport time (slaughter to consumption)	t_b
Consumption rate of produce	U_v
Fraction of produce from home garden	f_v
Consumption rate of leafy vegetables	U_l
Fraction of leafy vegetables from home garden	f_l
Annual average breathing rate of adult male	BR

There are several statistical tests that involve some form of dividing or segmenting input parameters into two or more empirical distributions based on an associated partitioning of the output distribution [2]. In this example, for a given parameter, all input data associated with a dose below a specific partitioning point are said to belong to one random sample while input data associated with a dose above the same partitioning point belong to a second random sample. These two random samples are then used to generate the empirical distributions. Means, medians, variances, and other characteristics of these distributions are compared to determine whether the distributions are statistically identical. Since their results are specific to the partitioning point, the sensitivity tests performed on the segmented data are not compared to the tests discussed above. The author has compared rankings for the Smirnov, Cramer-von Mises, Mann-Whitney, and Squared Ranks tests elsewhere [5].

3 RESULTS

Sensitivity results for each test have been obtained. Since one sensitivity method does not stand out as being universally accepted as the "correct" method, a "composite" sensitivity ranking has been determined. For the sake of comparing methods, the composite sensitivity ranking is based on the sum of ranks over all ten methods. The parameter with the lowest total rank is considered to have the greatest sensitivity. Iman and Conover [6] have presented a measure of "top-down correlation" for similar problems.

The relative performance of each method was determined by comparing the method-specific sensitivity ranking to the composite ranking. A "performance index" was calculated for this comparison. The performance index is a test of trend and is the sum of the squared-differences of the compared ranks, the T statistic in Spearman's ρ [1]. A smaller value for the index indicates a better trending of the method-specific and composite rank orders. The composite sensitivity ranking and the method performance ranking are shown in Table 2. Parameters are listed in decreasing order of sensitivity and the sensitivity techniques are listed in order of increasing performance index. Sensitivity ranks of the top ten parameters for each method are given in the table.

Table 2. Sensitivity ranking based on overall rank, listed in order of the composite ranking.

Parameter	SI	RD	RRC	±SD	PRCC	RDR	PD	±20%	SRC	II
Biological half-life	2	1	2	2	2	1	2	2	2	
Atmospheric concentration	1	2	1	1	1	2.5	2	2	5	
Produce consumption rate	3	3	3	3	3	9	8.5	8.5	1	2
Mass of soft tissue	4	4	4	4	4	2.5	2	2	7	
Plant/Atm HTO ratio	6	5	5	6	5.5	4	5	5.5	6	
Breathing rate	5	6	6	5	5.5	5	7	7	3	
Meat consumption rate	8	8	8	10	7.5				4	3
Leafy veg. consumption rate	7	7	7	9	7.5					4
Frac. Produce from garden	10	10	10	7		8	8.5	8.5	8	
Milk consumption rate	9								9.5	1
Feed-to-milk transfer factor		9			9	10				5
Absolute humidity						7	5	4		
Frac. from pasture (milk)										7
Percent water in vegetation						6	5	5.5		
Feed-to-meat transfer factor			9	8						6
Frac. leafy veg. from garden										
Beef cow ingestion rate										
Milk cow ingestion rate									9.5	
Frac. from pasture (beef)										8
Beef transport time										10
Milk transport time										9
Performance index	29	30	152	190	202	291	371	378	524	1404

The test of trend using Spearman's ρ also was used to calculate a performance index and to compare sensitivity ranks between methods. These comparisons show which tests behave similarly and which tests appear to be inappropriate for sensitivity analysis, at least for the type of model considered in this work. Smaller values indicate better trending of ranks and greater parity between methods. As an example, the performance index for the comparison between the ±20% and PD methods is 1.5, indicating remarkable agreement between the two rank orders.

4 DISCUSSION

As stated earlier, the performance of each method is measured by how closely the method-specific sensitivity rank compares to the composite rank. The performance index (PI) indicates that the SI and RD methods produce ranking results that are most similar to the composite rank (refer to Table 2). It is encouraging to see that all methods (except the importance index) produce the same general ranking of parameter sensitivity. The importance index is meant to be used with simple additive or multiplicative models; it is apparently not appropriate as a sensitivity measure for the model used in this example. The SI method chooses all of the top ten sensitive parameters while the RD method chooses the top six parameters in the composite order. The first five methods choose the top six parameters, but not necessarily in the composite order.

A performance index was calculated for each combination of ten sensitivity techniques discussed to provide a comparison between sensitivity methods. Small values of PI indicate similar sensitivity rankings. The partial derivative method is the most fundamental of the local sensitivity analysis techniques. It is appropriate only for relatively small changes (on the order of several percent) in the input parameter. It is not surprising, therefore, that sensitivity ranks based on the PD and ±20% methods result in very similar orders. The standard deviation increments (±SD) can at times be quite large, therefore, the ±SD ranks are not as similar. The RDR method acts globally, yet produces rankings similar to PD and ±20%. As suggested by Table 2 and confirmed by the performance index, rankings obtained from the sensitivity index (SI) and the relative deviation (RD) are quite similar. And, to a lesser degree, the SI and RD methods produce results similar to the ±SD method. Parameter sensitivity ranks based on the rank regression coefficient (RRC) are similar to the rankings from the SI, ±SD, and PRCC techniques. The importance index (II), meant for simple multiplicative models, produces results unlike any of the other methods; its utility is questionable.

5 CONCLUSIONS

A number of sensitivity analysis techniques have been presented. The majority of the techniques result in similar rankings of the top several sensitive parameters. Since the actual ranking is not as important as the general ranking, most of the techniques would be appropriate for sensitivity analysis for the type of model considered in this report. The criterion most important, therefore, is the ease with which the sensitivity method can be performed. With the proper software, all methods presented here are relatively easy to execute. Given a moderate number of parameters and a hand calculator, however, the sensitivity index is the easiest and most reliable sensitivity measure. The SI can be calculated without detailed knowledge of the parameter distribution and without the use of random sampling schemes or large computer programs.

The relative deviation (RD) is a reliable measure of parameter sensitivity. Calculation of the RD is quite simple if a sampling technique is employed and the output values are stored for the statistical analysis. This analysis requires a one-at-a-time approach, however, and can be labor intensive. Estimating sensitivity based on the relative deviation ratio (RDR) is not recommended since its results are less reliable and it requires more calculational rigor than the RD.

Rank regression coefficients are easily obtained with the use of commercially available software. An electronic spreadsheet and the SAS statistical package were utilized for this analysis. The calculation of sensitivity rankings by varying the parameter over its standard deviation (±SD) is as simple as calculating the sensitivity index with the exception that some knowledge of the parameter distribution must be available. Varying the input parameter by a standard amount (±20%) is an easy test to perform, but its reliability is less desirable than the simpler SI method.

The simplest approach to conceptualize is the one-at-a-time method where sensitivity measures are determined by varying each parameter independently while all others are held constant. These sensitivity techniques, however, become rather time intensive with large numbers of parameters. The most fundamental of sensitivity techniques is the direct method of using partial differentials to calculate the rate of change in the model output with respect to a given input parameter. The one-at-a-time techniques are valid only for small variability in parameter values and the partials must be recalculated for each change in the base-case scenario.

REFERENCES

[1] Conover, W.J. (1980) *Practical nonparametric statistics*. 2nd Ed. New York: John Wiley & Sons.

[2] Crick, M.J., Hill, M.D., Charles, D. (1987) The role of sensitivity analysis in assessing uncertainty. In the *Proceedings of an NEA workshop on uncertainty analysis for performance assessments of radioactive waste disposal systems*. Paris: OECD, 1-258.

[3] Hamby, D.M. (1993) A probabilistic estimation of atmospheric tritium dose. *Health Physics*, 65, 33-40.

[4] Hamby, D.M. (1994) A review of techniques for parameter sensitivity analysis. Accepted for publication in *Environmental Monitoring and Assessment*.

[5] Hamby, D.M. (1995) A comparison of sensitivity analysis techniques, *Health Physics*, 68, 195-204.

[6] Iman, R.L.; Conover, W.J. (1987) A measure of top-down correlation, *Technometrics*, 29, 351-357.