Feasibility Conditions for Rate-Constrained Routing in Power-Limited Multichannel WSNs

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***Abstract*— This paper develops cross-layer techniques for routing rate-constrained traffic in wireless sensor networks (WSNs) with multichannel access capability. We first derive and prove sufficient conditions that ensure feasibility of data rates in multichannel access WSNs. Then, we use these conditions to devise routing approaches that maximize the network lifetime while ensuring that the obtained routing solutions satisfy the required data rates. Finally, we evaluate and compare the performances of the proposed routing approaches under various network parameters.**

I. Introduction

In Wireless Sensor Networks (WSNs), energy conservation during information exchange is a critical issue for prolonging network lifetime. As a result, researchers have been designing routing techniques for WSNs that are energy aware [1–7]. Although most of these proposed approaches conserve energy, medium access control (MAC) contention constraints associated with the shared wireless medium are often ignored. As a result, the number of flows routed through nodes in the same neighborhood may be such that the shared medium may not be able to provide the data rates required to support these flows. If this happens, data rate requirements of the traffic flows cannot be satisfied by the network. A primary reason for this discrepancy is that the majority of the reported schemes perform network layer optimization without considering the effects of the underlying MAC layer.

To overcome the above limitations, researchers have recently shifted their focus to the development of cross-layer aware techniques [8–11]. For example, in [8], the lifetime maximization problem is considered under cross-layer constraints involving physical, MAC and routing layers. The jointly optimal route, schedule, and power allocation are computed under a general convex programming framework. In [9], a distributed joint routing and MAC algorithm is proposed for lifetime maximization of WSNs, which solves a convex optimization problem by a distributed primal-dual approach, where the network layer problem is solved in the dual domain and the MAC layer problem is solved in the primal domain by relaxing the MAC constraints in the form of a penalty function.

Capacity limitation also presents a major challenge to WSNs, mainly due to the interference, arising from the multihop communication paradigm in the single-channel access environment, which restricts the number of data flows that can occur simultaneously in a given neighborhood, thus limiting the amounts of total achievable network throughput. One key emerging solution to the capacity problem is to enable WSNs with multichannel access capability [12, 13], which has recently been made possible by means of the newly emerging cognitive radio technology that empowers sensor nodes to switch from one channel to another at reasonable costs [14]. Cognitive radio technology has then be viewed as a potential solution to enabling multichannel access, thereby limiting the severity of interference in WSNs.

In this paper, we develop cross-layer frameworks suitable for multichannel access WSNs. More specifically, we propose routing techniques for WSNs with multiple channel access capability that (i) are cross-layer aware by accounting for radio, MAC contention, and network constraints, (ii) guarantee to meet data rate requirements of end-to-end flows, and (iii) are energy efficient by increasing the lifetime of WSNs. We first derive and prove two sufficient conditions (termed rate-based and degree-based conditions) for rate feasibility in multichannel access WSNs that ensure the feasibility of the obtained routing solutions, in that the shared wireless medium is guaranteed to provide these flows with their required rates. Then, we formulate two routing problems as optimization problems (referred as LPM-1 and LPM-2), each with an objective of maximizing the network lifetime subject to the derived rate-feasibility conditions. Using simulations, we compare the performances of the proposed routing schemes in terms of the achievable network lifetime and solution complexity.

The outline of this paper is as follows. Section II describes the network model. Section III derives two sufficient rate-feasibility conditions. Section IV presents the routing schemes and Section V provides the performance evaluation of the proposed schemes with comparative results. Finally, Section VI concludes the paper.

II. Model

We model the WSN as a directed graph $G = (\mathcal{N}, \mathcal{F})$ that consists of a set $\mathcal{N}$ of nodes and a set $\mathcal{F}$ of flows. Each flow in $\mathcal{F}$ corresponds to an ordered pair of distinct transmitter node and receiver node (n, m) such that m is within n's transmission range—i.e., m is a neighbor of n—and n needs to transmit to m. If n is currently transmitting to m, then the flow $f \equiv (n,m)$ is said to be *active*. We assume that for every flow f in $\mathcal{F}$, there are $c(f)$, $c(f) \geq 1$, available channels on which f can communicate. Throughout, we assume that each node has one radio only, and that it can only communicate on one channel at a time.

We also model the set of flows $\mathcal{F}$ as a graph $H = (\mathcal{F}, C_R, C_I)$ where C_R is the set of all unordered, radio-contending pairs of flows and C_I is the set of all ordered, MAC-contending pairs of flows in $\mathcal{F}$. Two flows f and g are said to radio-contend with each other (i.e., $(f,g) \in C_R$) when they cannot be active at the same time due to radio constraints. That is, when the two flows share one node. In this work, we assume that a node can either transmit or receive, but not both, at any given time. A flow f is said to MAC-contend with another flow g (i.e., $(f,g) \in C_I$) when 1) flows f and g do not share a node between them, and 2) flow f cannot be (interference-free) active when g is active because of medium access contention constraints, which are solely dictated by the underlying MAC protocol. Note that if $(f,g) \in C_I$, it does not necessarily mean that $(g,f) \in C_I$. But $(f,g) \in C_R$ implies that $(g,f) \in C_R$. The graph H is referred to as flow contention graph, and depends mainly on the network topology (e.g., node placement, transmit power, connectivity), routing protocol (e.g., paths nodes' traffic routes through), and the MAC protocol (e.g., medium access contention constraints).

For every $f \in \mathcal{F}$, let $\Psi_R(f)$ be the set of flows in $\mathcal{F}$ that radio-contend with f; i.e., $\Psi_R(f) = \{g \in \mathcal{F} : (f,g) \in C_R\}$ denotes the set of all flows that cannot be active with flow f at the same time, due to radio resource limitation. Let $d_R(f) = |\Psi_R(f)|$, representing the number of flows in $\Psi_R(f)$. Likewise, let $\Psi_I(f) = \{g \in \mathcal{F} : (f,g) \in C_I\}$ be the set of flows that MAC-contend with f; i.e., if f is active on one channel and in order for f to be MAC contention-free, no flow in $\Psi_I(f)$ can be active on the same channel at the same time (so as to avoid interference due to MAC contention). Let $d_I(f) = |\Psi_I(f)|$, representing the number of flows in $\Psi_I(f)$.

Given the MAC protocol and a network topology graph $G = (\mathcal{N}, \mathcal{F})$, one can derive the corresponding flow contention graph $H = (\mathcal{F}, C_R, C_I)$. Note that the MAC-contention set C_I depends on the underlying MAC, and hence so do the MAC-contention constraints.

III. Cross-Layer Aware Rate Feasibility Conditions

Let $H = (\mathcal{F}, C_R, C_I)$ be a flow contention graph. Let's assume that each flow f in $\mathcal{F}$ flows data traffic at a rate of x_f bits per second. Let $\mathrm{x} = (x_f)_{f\in\mathcal{F}}$ be the vector, referred to as flow rate vector, representing the data rates of all flows in $\mathcal{F}$. The vector x is said to be feasible flow rate vector in H if there exists a time schedule in which the rates of all flows are satisfied. Formally, x is feasible in H if there exists a time schedule $\mathcal{S} = [0,\tau]$ of length $\tau > 0$ in which every flow $f \in \mathcal{F}$ communicates τx_f bits. For each subset $A \subseteq \mathcal{F}$, we define the weight of A under a given flow rate vector x to be $\delta(A, \mathrm{x}) = \sum_{f\in A} x_f$. Let W denote the capacity of the single channel, and c_{max} denote the total number of available channels that any link can have (i.e., a maximum of $\tau c_{max} W$ bits can be transmitted in the interval $[0,\tau]$ on any link).

In this section, we propose two different sets of sufficient conditions under which a given flow rate vector is feasible in H. The first set takes topology parameters, rates of the flows, and the availability of multiple channels into consideration. Whereas, the second set accounts for the topology parameters and the availability of multiple channels only.

Proposition *1:* (*Rate-based conditions*) x is feasible in H if, for every flow $f \in \mathcal{F}$, $x_f \leq \min\{W - \delta(\Psi_R(f), \mathrm{x}), c(f)W - c(f)\delta(\Psi_R(f), \mathrm{x}) - \delta(\Psi_I(f), \mathrm{x})\}$.

Proof: Let $\mathcal{N}$ denote $|\mathcal{F}|$ and $\mathrm{x} = (x_f)_{f\in\mathcal{F}}$ be a flow vector that satisfies the conditions stated by the proposition. Without loss of generality, let us arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, \mathcal{N}\}$ such that $x_p \leq x_q$ for all $1 \leq p \leq q \leq \mathcal{N}$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. Also, let $\mathcal{S} = [0,\tau]$ be a time schedule of length $\tau > 0$ seconds. We show that for all $n = \{1, 2, \ldots, \mathcal{N}\}$ the flows in the subset $\mathcal{F}^n \subseteq \mathcal{F}$ are schedulable in $\mathcal{S}$. Thus, $\mathcal{F}^n \subseteq \mathcal{F}$ is schedulable. We proceed the proof by induction.
BASIS: $\mathcal{F}^1 = \{1\}$. Since $x_1 \leq W - \delta(\Psi_R(1), \mathrm{x})$ and $x_1 \leq c(1)W - c(1)\delta(\Psi_R(1), \mathrm{x}) - \delta(\Psi_I(1), \mathrm{x})$, then $x_1 \leq W$ and consequently $\mathcal{F}^1$ is schedulable in $\mathcal{S}$.
INDUCTION STEP: Now, assume that all the flows in $\mathcal{F}^{n-1}$ are schedulable in $\mathcal{S}$ for any n, $1 \leq n \leq \mathcal{N}$. We will show that the flows in $\mathcal{F}^n$ are also schedulable in $\mathcal{S}$. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. To prove that both conditions are jointly sufficient we will consider one by one. Let $\Phi_R(n)$ be the set of flows in $\mathcal{F}^{n-1}$ that radio contends with n, i.e., $\Phi_R(n) = \mathcal{F}^{n-1} \bigcap \Psi_R(n)$. Since $\Phi_R(n) \subseteq \Psi_R(n)$ and $x_n \leq W - \delta(\Psi_R(n), \mathrm{x})$, then $x_n \leq W - \delta(\Phi_R(n), \mathrm{x})$. Therefore, even when all the flows in $\Phi_R(n)$ (only previously scheduled flows in $\mathcal{F}^{n-1}$ that radio contend with n) were already scheduled disjointly, there is still a room of at least $(W - \delta(\Phi_R(n), \mathrm{x}))$ for n to be scheduled. So, n is radio schedulable. Next, let's focus on the second condition which indicates that flows can interfere with each other and hinder n to be scheduled in $\mathcal{S}$ due to MAC contention. Consider the worst case scenario, where all the flows in $\mathcal{F}^{n-1}$ radio contend with the flows that are in the MAC contention set $\Psi_I(n)$. The only possible way here is to schedule the flows in $\Psi_I(n)$ concurrently with n in the other $c(n) - 1$ available channels. The available bandwidth for flow n and all the flows in $\Psi_I(n)$ is then $\Delta(n) \equiv c(n)(W - \delta(\Psi_R(n), \mathrm{x}))$. Since $x_n \leq c(n)W - c(n)\delta(\Psi_R(n), \mathrm{x}) - \delta(\Psi_I(n), \mathrm{x})$ or equivalently $x_n + \delta(\Psi_I(n), \mathrm{x}) \leq \Delta(n)$, then all flows in $\Psi_I(n)$ can be scheduled concurrently with flow n. Hence, all flows in the set $\mathcal{F}^n$ are schedulable in $\mathcal{S}$. ■

Proposition *2:* (*Degree-based conditions*) x is feasible in H if, for every flow $f \in \mathcal{F}$, $x_f \leq \min\{\frac{W}{d_R(f)+1}, \frac{c(f)W}{(d_R(f)+1)(d_I(f)+1)}\}$.

Proof: Let $\mathcal{N}$ denote $|\mathcal{F}|$ and $\mathrm{x} = (x_f)_{f\in\mathcal{F}}$ be a flow

vector satisfying the above sufficient conditions, stated by the proposition. Without loss of generality, let us arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, \mathcal{N}\}$ such that $x_p \leq x_q$ for all $1 \leq p \leq q \leq \mathcal{N}$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. Also, let $\mathcal{S} = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show that $\forall n = \{1, 2, \ldots, \mathcal{N}\}$ the flows in the subset $\mathcal{F}^n \subseteq \mathcal{F}$ are schedulable in $\mathcal{S}$. Thus, $\mathcal{F}^n \subseteq \mathcal{F}$ is schedulable. Here, we also prove by induction.
BASIS: We take $\mathcal{F}^1$ = $\{1\}$. Since $d_R(1) \geq 0$ and $d_I(1) \geq 0$, and $x_1 \leq \frac{W}{d_R(1)+1}$ and $x_1 \leq \frac{c(1)W}{(d_R(1)+1)(d_I(1)+1)}$ hold, it follows that $x_1 \leq W$. Thus, $\mathcal{F}^1$ is schedulable in $\mathcal{S}$.
INDUCTION STEP: Let's now assume that all the flows in $\mathcal{F}^{n-1}$ are schedulable in $\mathcal{S}$ for any n, $1 \leq n \leq \mathcal{N}$, and show that the flows in $\mathcal{F}^n$ are also schedulable in $\mathcal{S}$. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let $\Phi_R(n) = \mathcal{F}^{n-1} \bigcap \Psi_R(n)$ (denoting the set of flows in $\mathcal{F}^{n-1}$ that each radio contends with n). Note that for all $k \in \Phi_R(n), x_k \leq x_n$. Since $x_n \leq \frac{W}{d_R(n)+1}$ then $x_k \leq \frac{W}{d_R(n)+1}$. Therefore, each flow k (contending with n and already scheduled in $\mathcal{F}^{n-1}$) needs at most a fraction $\frac{1}{d_R(n)+1}$ of the total schedule. Even when all the $d_R(n)$ flows that radio contend with n happen to be in $\mathcal{F}^{n-1}$ (i.e., $|\Phi_R(n)| = d_R(n)$) and are scheduled disjointly, they can occupy at most $d_R(n) \times \frac{1}{d_R(n)+1}$ of the total schedule, leaving a fraction of $\frac{1}{d_R(n)+1}$ for n, which is the exact requirement to schedule n without overlapping with any of the flows in its radio contention set.

Now when accounting for the MAC contention conditions, we note that the worst case scenario corresponds to when all the flows in $\mathcal{F}^{n-1}$ radio contend with each of the flows in the MAC contention set $\Psi_I(n) \cap \mathcal{F}^{n-1} \equiv \Psi'_I(n)$. The only possible way when this worst case scenario happens is then to be able to schedule the flows in $\Psi'_I(n)$ concurrently with n in the other $c(n)-1$ available channels. That is, an available bandwidth of $\Delta(n) \equiv \frac{c(n)W}{d_R(n)+1}$ is left for scheduling flow n and all the flows in $\Psi'_I(n)$. Since $x_n \leq \frac{c(n)W}{(d_R(n)+1)(d_I(n)+1)}$ or equivalently $x_n(d_I(n)+1) \leq \Delta(n)$ and $\Psi'_I(n) \subseteq \Psi_I(n)$, then $x_n(d'_I(n)+1) \leq \Delta(n)$ where $d'_I(n) = |\Psi'_I(n)|$. Now since $x_k \leq x_n$ for all $k \in \Psi'_I(n)$, $x_n + \sum_{k \in \Psi'_I(n)} x_k \leq x_n(d'_I(n)+1)$. It follows then that $x_n + \sum_{k \in \Psi'_I(n)} x_k \leq \Delta(n)$. Thus, flow n and all the flows in $\Psi'_I(n)$ can all be scheduled concurrently in the available bandwidth of $\Delta(n)$. Hence, all flows in the set $\mathcal{F}^n$ are schedulable in $\mathcal{S}$. ∎

The derived rate-based and degree-based condition sets are useful in network routing problem formulations. Routing problems can often be formulated as optimization problems with the aim to maximize some performance metrics, such as network lifetime, overall achievable network throughput, etc. However, when medium contention constraints are not accounted for, rate solutions provided by these approaches may be infeasible, meaning that the shared wireless medium may not be able to satisfy these rates.

To mitigate the above limitation, we next use the derived sufficient conditions (proposed and proved in this section) to propose energy-efficient routing schemes for multichannel access WSNs that account for the MAC contention constraints, and that by doing so, guarantee to satisfy data rate requirements of end-to-end flows while maximizing the network lifetime.

IV. Rate-Constrained Routing for Lifetime Maximization

Let $G = (\mathcal{N}, \mathcal{F})$ represent a WSN, where $\mathcal{N}$ is the set consisting of a number of sensor nodes (SNs) and a single sink or access node (AN). Let us assume that each SN i generates data traffic destined to the AN at a rate of R_i bits per second. Let $B_i(t)$ denote the energy resources available at SN i for network communications at a given time instant t. Also, let e_{ij} denote the energy required to transmit a bit from node i to node j (this designates the cost of transmitting one bit over a wireless link). Let x_{ij} denote the number of bits per second forwarded by node i to a neighboring node j, and $\mathbf{x} = [x_{ij}]_{1 \leq i,j \leq |\mathcal{N}|}$ be the vector representing the rates of all flows. We define the *network lifetime* to be the amount of time for the first node to die—a node dies if either it runs out of its energy resources, or it becomes disconnected due to the death of other nodes; denote this lifetime by T. Given the required rate vector $R = [R_i]_{1 \leq i \leq |\mathcal{N}|}$, our objective is to find a routing solution that maximizes the network lifetime, T, while meeting the data rate requirements of all the flows. In the remainder of this section, we will describe our proposed routing approaches, which rely on the derived sets of rate feasibility to ensure that the obtained routing solutions indeed meet the flows' data rate requirements. We will first begin by presenting the routing constraints, and then present the routing formulation.

A. Routing Constraints

Given the required rate vector $R = [R_i]_{1 \leq i \leq |\mathcal{N}|}$, the following set of constraints must be satisfied.

- FLOW BALANCE CONSTRAINTS:
  At each SN, the total outgoing traffic rate must equal the sum of all incoming traffic's rate and that of the traffic generated by the SN. That is, for each SN i,

$$\sum_{j \in \mathcal{N}} x_{ji} + R_i = \sum_{j \in \mathcal{N}} x_{ij} \tag{1}$$

  For the AN, the total incoming traffic rate must equal the total traffic generated by all SNs.

$$\sum_{j \in \mathcal{N}} x_{ji} = \sum_{j \in \mathcal{N}} R_j; \quad i = AN \tag{2}$$

  Since no traffic is generated from the AN to SNs, then for each SN j,

$$x_{ij} = 0; \quad i = AN \tag{3}$$

  Finally, all rates must be positive; i.e.,

$$x_{ij} \geq 0; \quad i, j \in \mathcal{N} \tag{4}$$

- ENERGY CONSUMPTION CONSTRAINTS:
Let t_0 be the initial time. If $SN\ i$ has $B_i(t_0)$ amount of energy at time t_0, then the remaining energy at any future time $t_0 + T$ must be greater than or equal to zero. The AN is assumed to have an infinite amount of energy. Formally, for each $SN\ i$,

$$B_i(t_0) \geq T \sum_{j \in \mathcal{N}} e_{ij} x_{ij} \tag{5}$$

- MEDIUM CONTENTION CONSTRAINTS:
For ease of notation, hereafter, a flow f between two nodes i and j will be referred to as (i,j) or simply ij instead of f. As a result, the number of channels $c(f)$ available at flow $f \equiv (i,j)$ will also be denoted by c_{ij}. Now we use the two sets of sufficient conditions we derived in the previous section as a means of ensuring that the routing solutions meet the medium access contention constraints:
  1) **Rate-based Constraints**: The rate vector $\mathbf{x}$ is feasible—i.e., it satisfies the medium access constraints—if for each flow $(i,j) \in \mathcal{F}$ the following medium contention constraints hold.

$$\begin{aligned} x_{ij} &\leq W - \delta(\Psi_R(ij), \mathrm{x}) \\ x_{ij} &\leq c_{ij} W - c_{ij}\delta(\Psi_R(ij), \mathrm{x}) - \delta(\Psi_I(ij), \mathrm{x}) \end{aligned} \tag{6}$$

  2) **Degree-based Constraints**: The rate vector $\mathbf{x}$ is feasible if for each flow $(i,j) \in \mathcal{F}$ the following medium contention constraints hold.

$$\begin{aligned} x_{ij} &\leq \frac{W}{d_R(ij)+1} \\ x_{ij} &\leq \frac{c_{ij} W}{(d_R(ij)+1)(d_I(ij)+1)} \end{aligned} \tag{7}$$

It is worth mentioning that (6) and (7) are all sufficient conditions that ensure rate feasibility while accounting for the medium access contention; i.e., if $\mathbf{x}$ satisfies any of these, then $\mathbf{x}$ satisfies the medium access contention constraints. Recall that none of (6) and (7) implies the other. Also, note that these constraints (i.e., the rate-based and degree-based constraints) are linear and hence their associated routing formulations can simply be written as linear programs (LPs).

B. Routing Formulation

The routing problem consists of determining the rate vector $\mathbf{x}$ that maximizes the network lifetime T subject to (1)–(5) and either (6) or (7). Note that the constraints stated in (5) are not linear, and thus, as they are, the routing problem cannot be formulated as an LP. In order to formulate the routing problem as an LP, we introduce a new variable F, replace the variable T by $\frac{1}{F}$, rewrite constraints (5) as

$$F \geq \frac{1}{B_i(t_0)} \times \sum_{j \in \mathcal{N}} e_{ij} x_{ij} \tag{8}$$

and minimize F as an objective of the optimization formulation. Since minimizing the variable F is equivalent to maximizing the variable T [15], the routing problem can equivalently be formulated as

Minimize F
Subject to:
FLOW BALANCE CONSTRAINTS: $(1) - (4)$
ENERGY CONSUMPTION CONSTRAINTS: (8)
MEDIUM CONTENTION CONSTRAINTS: (6) or (7).

Note that the MEDIUM CONTENTION CONSTRAINTS can be expressed through either (6) or (7). The use of either (6) or (7) yields a linear program that we refer to as LPM-1 and LPM-2. In the next section, we will solve these problems for many instances so as to evaluate the proposed routing formulations with regards to the following performance metrics: network lifetime and solution feasibility.

V. PERFORMANCE EVALUATION

We now evaluate and analyze the performances of the proposed routing schemes (LPM-1 and LPM-2) described in Section IV. The performance metrics that we consider evaluating are: network lifetime, solution feasibility, and execution time. In this evaluation, we use CPLEX and MATLAB as tools to solve the formulated routing optimization problems.

A. Simulation Setting and Method

We generate and simulate random WSNs, each of which has a set of $|\mathcal{N}|$ nodes consisting of a single sink or access node AN, located at the center of a square area, and $|\mathcal{N}|-1$ sensor nodes SNs uniformly distributed in the square area. Each SN is assumed to generate and send data traffic with a fixed rate requirement of R ($R = R_i \quad \forall i \in \mathcal{N}$) to the AN. Without loss of generality, we assume $W = 1$. All simulated WSNs are connected, and each pair of nodes can communicate through c available channels ($c_{ij} = c \quad \forall (i,j)$). Each SN can communicate with the AN either directly or through a set of intermediate relay nodes. We consider the IEEE 802.11 MAC protocol [16] for our simulation; i.e., a node cannot send and receive simultaneously (radio contention) and if node i is in communication with node j, then all nodes within the same communication range of i or j cannot communicate on the same channel (MAC contention). When the total network area is $\mathcal{A}$ and the transmission range is r, the maximum number of concurrent transmissions at any time on a single channel is roughly $\frac{\mathcal{A}}{\pi r^2}$. Therefore, the maximum bit-meter per second per channel the network can support is (roughly) $Wr(\frac{\mathcal{A}}{\pi r^2})$. On the other hand, the aggregate data rates generated from all the $|\mathcal{F}|$ flows, where each flow i using a path of length l_i meters generates a data rate of R bits per second is $R \sum_{i \in \mathcal{F}} l_i$. Taking both of these metrics into consideration, we can then define the normalized network load per channel as

$$\eta = \frac{R.\sum_{i \in \mathcal{F}} l_i}{W.r.(\frac{\mathcal{A}}{\pi r^2})} \tag{9}$$

Throughout, we use this normalized load metric η instead of the aggregated data rate of all the flows as a means of assessing the network traffic load.

B. Network Lifetime Analysis

In this section, we present and analyze the obtained network lifetime performances of the two routing approaches under various network parameters.

1) Impact of Node Degree: We first fix the network area and the transmission range, and vary the number of nodes. With this setting, an increment in the number of nodes leads to an increment in the node degree. Since the transmission range remains the same, the hop length does not change much (it almost remains the same) when increasing the number of nodes. Hence, this allows to study the effect of node degree while masking the impact of hop length and transmission range.

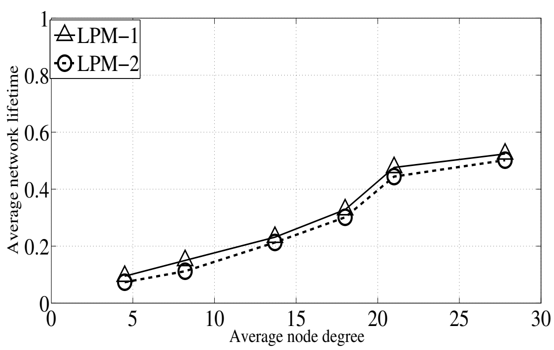

Fig. 1. Average network lifetime (hour) for $r = 30m$, $\mathcal{A} = 100 \times 100m^2$, $\eta = 0.3$, and $c = 3$.

Fig. 1 shows the network lifetime when varying the average node degree (by varying the number of nodes from 20 to 70) while fixing transmission range, area size, number of channels, and network load per channel. First, note that regardless of the routing approach, as the number of nodes increases, the network lifetime increases. This happens because nodes in graphs with higher average node degrees are likely to have more neighbors, which offer them more path options to route their data to the sink. When the average node degree is low, nodes are likely to be forced to route through the same path, thus resulting in shorter lifetimes due to early node failures. Second, LPM-1 always achieves better lifetime performances than LPM-2. Although theoretically LPM-1 and LPM-2 are independent from each other, simulations, however, show that LPM-1 achieves better network lifetime (though the difference is minimal) than LPM-2 when the number of nodes increases (at a moderate network load per channel). Now, note that as indicated via (7), the LPM-2 conditions on a flow (i, j) depend on the node degree (which is reflected via $d_R(ij)$ and $d_I(ij)$), and not on the rates of other flows (i.e., the flows in $\Psi_R(ij)$ and $\Psi_I(ij)$). The LPM-1 conditions on a flow (i, j), on the other hand, depend (as shown via (6)) on the other flows' rates, and not so much on the average node degree[1]. Therefore, increasing the average node degree tightens the constraints on the achievable rates under LPM-2 more than it does under LPM-1, which results in less routing choices, thereby decreasing the network lifetime. This explains why the network lifetime under LPM-2 is shorter than that achievable under LPM-1.

2) Impact of Transmission Range: Here, we fix the network area and the number of nodes, and vary the transmission range. Usually, the higher the transmission range, the greater the interference, but also the higher the node degree. Typically, a higher interference results in less throughput, while a higher node degree yields more throughput. Unlike the previous case, however, the average hop length decreases when the transmission range increases. We consider and simulate the following transmission ranges: 30, 40, 50, 60, and 70m.

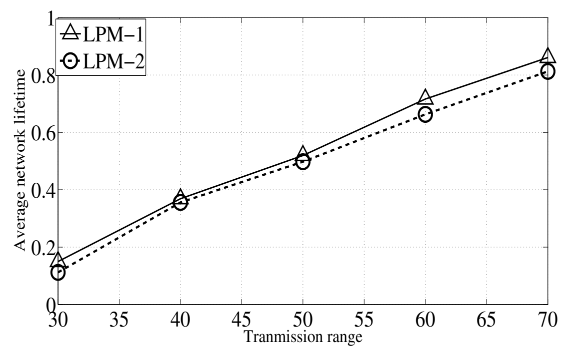

Fig. 2. Average network lifetime (hour) for $|\mathcal{N}| = 30$, $\mathcal{A} = 100 \times 100m^2$, and $c = 3$.

Fig. 2 shows the network lifetime when varying the transmission range while fixing the number of nodes, the number of channels, and the area size. Note that regardless of the routing approach, as the transmission range increases, the network lifetime increases. These results again show that LPM-1 achieves slightly better lifetime performances than LPM-2.

3) Impact of Number of Channels: In this section, we show the impact of the number of channels on network lifetime performances. Results are shown in Fig. 3 for 3 different values of network loads: $\eta = 0.2, 0.4$, and 0.6. First, the figure shows that regardless of the network load η, as the number of channels increases, the average network lifetime achievable under any of the routing approaches also increases at first, but then flattens out. This happens because an increase in the number of channels provides nodes with more routing alternatives/options (i.e., greater solution space), thus resulting in lifetime increases. But after reaching a certain number of channels, a further increase in the number of channels can no longer increase the solution space, which explains why the lifetime remains constant. Also, note that when the number of channels is high, all two approaches result in similar lifetimes, simply

[1]To be exact, LPM-1 conditions also depend on average node degree, but implicitly and loosely

because medium contention constraints are likely to be relaxed when the number of channels is high, and when this is the case, all two approaches become equivalent.

Second, note that for lower normalized network loads, LPM-1 performs better than LPM-2, but as we increase the load, LPM-2 performs better than LPM-1. Although LPM-1 and LPM-2 are independent from each other, (6) and (7) imply that when the network load per channel is increased, the contention condition of LPM-1 (as it depends on the rate of the flows) becomes stricter than LPM-2, yielding shorter lifetimes.

Third and as expected, when the network load increases, the network lifetime decreases, and this is regardless of the routing approach being used.

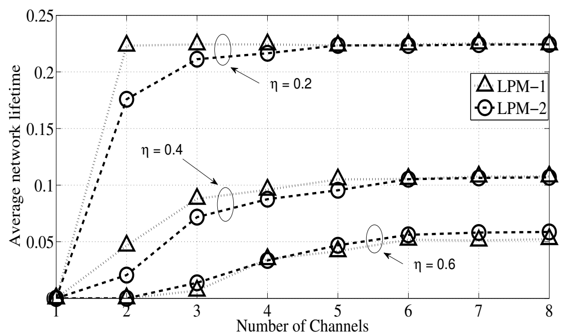

Fig. 3. Average network lifetime (hour) for $r = 30m$, $\eta = 0.3$, and $c = 3$.

C. Complexity Analysis

The objective of this section is to study the tradeoffs between the solution quality (i.e., network lifetime) and the complexity (i.e., execution time) of each approach. Since simulation results show that LPM-1 approach always slightly outperforms LPM-2 in terms of network lifetime (solution quality), it is worth investigating the tradeoffs between the complexity and the quality of the solution of the two approaches. For this, we collect the obtained network lifetime (in seconds) and the execution time (i.e., CPLEX runtime also in seconds) for different numbers of sensor nodes but by fixing the transmission range to $30m$, network area to $100 \times 100m^2$, network load to 0.3, and number of channels to 3.

TABLE I

NUMERICAL RESULTS.

Number of nodes	30		40		50		60	
	LPM-1	LPM-2	LPM-1	LPM-2	LPM-1	LPM-2	LPM-1	LPM-2
Lifetime	539.4	403.3	833	766.8	1121.6	1084.5	1703.9	1598.7
Exec. Time	0.98	0.6	3.543	1.05	6.7	2.17	7.45	2.1

It is notable from Table I that LPM-1 achieves higher lifetime than LPM-2, but at a cost of greater execution time. Although in general the complexity gain increases with the number of nodes, LPM-1 always exhibits better results. If we focus more onto performance degradation rather than reduced complexity (this stipulation is reasonable as complexity is calculated in order of seconds), we can conclude that for a light and medium loaded network LPM-1 is recommended to be used over LPM-2 with a better solution quality at a slightly higher complexity.

VI. CONCLUSION

In this paper, we formulate and prove two sufficient conditions based on the medium access contention constraints for rate-feasibility in a multichannel wireless sensor network. Using the two derived sets of conditions, we propose two routing techniques, LPM-1 and LPM-2, whose objective is to maximize the network lifetime while ensuring that the obtained routing solutions are feasible. Simulation results show that for lightly or medium loaded networks, LPM-1 always achieves better network lifetime than LPM-2, but at a slightly higher complexity cost.

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