Unidirectional Flow Throughput Enhancement Through Physical-Layer Network Coding

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***Abstract*— The increasing demands for high data rates necessitate the development of faster schemes of exchanging information along wireless communication links. Physical-Layer Network Coding (PNC) is a promising technique that can improve the achievable data flow rates through higher packet transmission rates, thereby increasing the overall throughput. In this paper, we study the performance of the PNC transmission technique in unidirectional linear flow networks, and compare it with that of the traditional transmission technique. We first derive the bit-error rate (BER) that the PNC transmission scheme achieves, and then using that, we evaluate the end-to-end flow throughput of unidirectional flows. Our results show that PNC has a great potential for enhancing the achievable throughput, especially under medium to high signal-to-noise ratios. We also validate the derived BER results using simulations.**

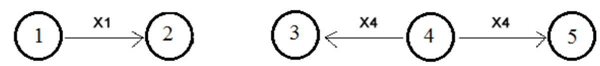

Fig. 1. Traditional transmission on a unidirectional flow network

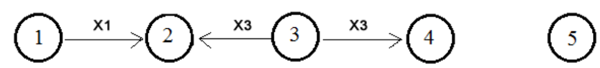

Fig. 2. PNC transmission on a unidirectional flow network

I. Introduction

The need for higher data rates and faster connection speeds of exchanging information in wireless networks have prompted researchers to think of new, efficient techniques that do so by making efficient use of the available wireless resources. Physical-Layer Network Coding (PNC) is one technique that has great potentials for improving the throughput of end-to-end flows through effective use and exploitation of wireless resources [1]. The idea of network coding was first introduced in 2000 by Ahlsweda [2], and then used in many other works (e.g., [3], [4], [5]) and showed great promises for throughput improvements over traditional transmission techniques. Later, PNC emerged also as a promising technique [1], where it is shown to improve performance of three-node bidirectional networks.

At the physical layer, data is transmitted through electromagnetic (EM) waves, and PNC takes advantage of the additive nature of simultaneous arrivals of multiple EM waves. By using a proper modulation, the addition of EM signals can be mapped to $GF(2^n)$ additions of digital bit streams [2], [5]. Symbol-level and carrier-phase synchronization and the use of power control are then assumed in order to be able to receive the two signals with the same phase and amplitude.

In this paper, we study the PNC transmission scheme and compare its performance with the traditional one by considering a unidirectional end-to-end flow in a general linear network with multiple nodes. For simplicity, we assume a fixed distance between any two neighbor nodes. For the sake of illustration, we consider five nodes in a unidirectional flow, where every node is equipped with an omni-directional antenna. The wireless channel is assumed to be half duplex, meaning that the transmission and reception must occur in different time slots. Furthermore, we consider the Decode-and-Forward relaying approach [6] in this work.

Fig. 1 illustrates the traditional transmission scheme in a unidirectional flow network. Here, node 1 and node 4 can both transmit their signals at the same time without interfering with one another, but node 1 and node 3 cannot transmit simultaneously, due to interference.

Fig. 2 illustrates the unidirectional PNC transmission scheme. Unlike the case of the traditional transmission scheme, node 1 and node 3 here can transmit concurrently (i.e., node 1 sends X_1 while node 3 is sending X_3), and provided that node 2 has already received X_3, it can then perform PNC to recover the intended signal/packet coming node 1, even in the presence of the signal coming from node 3. In this case, the performance gain of the PNC scheme over that of the traditional scheme lies in the fact that the number of transmissions to deliver a packet successfully is expected to be lesser under the PNC scheme than under the traditional one. However, due to interference, the BER under the PNC technique is, on the other hand, expected to be worse than that under the traditional one. The objective of this paper is then to investigate whether the degraded BER due to interference pays off by reducing the number of needed transmissions, thereby leading to an increased overall end-to-end flow throughput.

The rest of this paper is organized as follows. Section II derives the BER performance under the PNC scheme. Section III derives and evaluates the achievable throughput under the PNC scheme and compares it with that achievable under the traditional one. Section IV uses simulation to validate the derived BER performances. Finally, we conclude the paper in Section V.

II. Bit-Error Rate

In this section, we derive the bit-error rate (BER) for unidirectional end-to-end flows using the physical-layer network coding (PNC) transmission scheme, and compare it with that of the traditional transmission scheme. We assume an additive white Gaussian noise with power density $N_o/2$, and assume that the received signal energy of one bit (Eb) is unity. We also assume perfect carrier-phase synchronization, and consider the QPSK modulation technique. For the traditional transmission scheme, the BER is the standard $Q(2/N_o)$ [7], where $Q(.)$ is the complementary cumulative distribution function of the zero-mean, unit-variance Gaussian random variable.

Let us refer to the example of Fig. 2 again to illustrate the derivation of the BER of the PNC transmission scheme. Using the PNC scheme, both nodes 1 and 3 are allowed to transmit concurrently; i.e., at a given time slot, node 2 receives, at the same time, two signals: $X_1(t)$ coming from node 1 and $X_3(t)$ coming from node 3, although intended for node 4. As a result, the combined bandpass signal $r_2(t)$ received by node 2 during one symbol period is

$$r_2(t) = X_1(t) + X_3(t)$$

which can also be expressed as

$$r_2(t) = [i_1 \cos(wt) + q_1 \sin(wt)] + [i_3 \cos(wt) + q_3 \sin(wt)]$$

where i_j and q_j are the QPSK modulated information bits of node j, and w is the carrier frequency. Thus, node 2 receives two baseband signals, in-phase (I) and quadrature phase (Q):

$$I = i_1 + i_3 \quad \text{and} \quad Q = q_1 + q_3$$

Here, node 2 encodes the combined bit, $(X_1 + X_3)$, with the already received (stored) bit, X_3, to recover the intended bit, X_1; i.e., $(X_1 \oplus X_3) \oplus X_3 = X_1$. Note that X_3 was already received by node 2 at an earlier transmission time, i.e., when X_3 was transmitted from node 1 to node 2.

The QPSK data stream can basically be considered as two BPSK data streams: an in-phase stream and a quadrature-phase stream. In Fig. 3, we illustrate the PNC mapping, where $X_j \in \{0, 1\}$, and $i_j \in \{-1, 1\}$ represents the in-phase data bit.

As shown in Fig. 3, there are three possibilities of the in-phase space, $\{-2, 0, 2\}$, with corresponding probabilities of 0.25, 0.5, 0.25, respectively. Applying the maximum posterior probability criterion [7] and using the table shown in Fig. 3, $i_2 = -1$ for $i_1 + i_3 = -2$ or $i_1 + i_3 = 2$. Since the error occurs when this criterion is not met, the average probability of error is calculated for all possible cases, and the BER can be written as follows

Modulation Mapping at N3 and N1				Demodulation at N2		
$X^{(I)}_1$	$X^{(I)}_3$	i_1	i_3	$i_1 + i_3$	X_2	i_2
1	1	1	1	2	0	-1
0	1	-1	1	0	1	1
1	0	1	-1	0	1	1
0	0	-1	-1	-2	0	-1

Fig. 3. PNC mapping illustration

$$\begin{aligned} BER_{PNC} = & \frac{1}{4}\int_{\alpha_1}^{\alpha_2} \frac{1}{\sqrt{\pi N_o}} \exp(-\frac{(r+2)^2}{N_o})dr \\ & + \frac{1}{2}\int_{-\infty}^{\alpha_1} \frac{1}{\sqrt{\pi N_o}} \exp(-\frac{r^2}{N_o})dr \\ & + \frac{1}{2}\int_{\alpha_2}^{\infty} \frac{1}{\sqrt{\pi N_o}} \exp(-\frac{r^2}{N_o})dr \\ & + \frac{1}{4}\int_{\alpha_1}^{\alpha_2} \frac{1}{\sqrt{\pi N_o}} \exp(-\frac{(r-2)^2}{N_o})dr \end{aligned} \quad (1)$$

When the received signal is less than α_1, $i_1 + i_3$ is declared to be -2, and when it is more than α_2, $i_1 + i_3$ is declared to be 2. Otherwise, it is assumed to be 0. After some algebraic manipulations, the optimal values of α_1 and α_2 are derived respectively as

$$\alpha_1 = -1 - \frac{N_o}{4} \ln(1 + \sqrt{1 - \exp -(\frac{8}{N_o})})$$

$$\alpha_2 = 1 + \frac{N_o}{4} \ln(1 + \sqrt{1 - \exp -(\frac{8}{N_o})})$$

In Fig. 4, we show the BER of both the PNC and traditional transmission schemes under various values of the signal-to-noise ratio (SNR). The figure shows that the BER of PNC scheme is slightly worse than that of the traditional transmission scheme. However, even though the BER gets worse under PNC, as will be shown and illustrated in the following section, the PNC technique is expected to improve the performance of the system in terms of the overall end-to-end flow throughput by reducing the number of transmissions needed to successfully send packets along the end-to-end flow.

III. Unidirectional Flow Throughput

In this section, we evaluate the end-to-end flow throughput of both the traditional and PNC transmission schemes in unidirectional flows. Consider a unidirectional linear network with n nodes. Nodes are labeled as node 1, node 2,....node n, where node 1 and node n are the source node

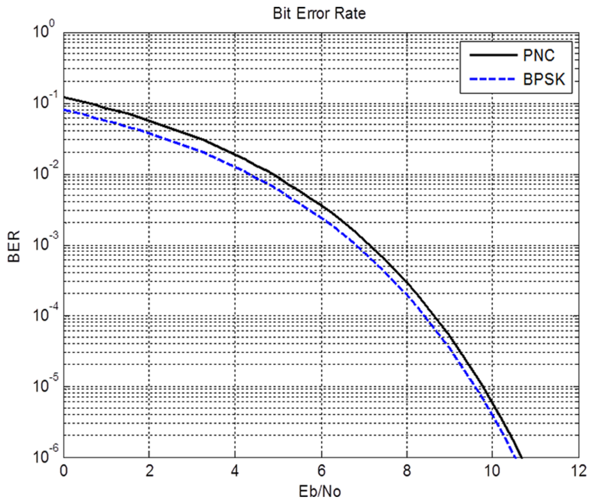

Fig. 4. BER of PNC and traditional transmission schemes

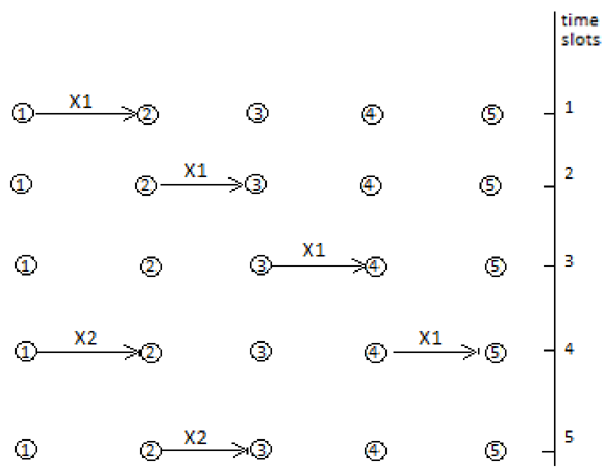

Fig. 5. Unidirectional traditional transmission in a linear network

and the destination node, respectively. We assume that the source node has an infinite number of packets that needs to send to the destination node. We also assume that a packet is received successfully by the destination when all the bits are each received correctly, any erroneous packet is to be retransmitted again and again until it is correctly received. This is done on a per-link basis.

A. Traditional Transmission Scheme

The flow of packets in the traditional transmission scheme when $n = 5$ nodes is illustrated in Fig. 5. Assuming that the packet success probability over a link is p_c and that a packet is to be resent repeatedly until it is delivered successfully, the average number of needed transmissions until a packet is successfully delivered is $1/p_c$. The average transmission time over a link is then $L/(p_c \times C)$, where C is the capacity of the wireless link and L is the length of the packet. Throughout this work, we assume that each packet transmission occurs in one time slot, and hence the length of a time slot is $L/(p_c \times C)$.

Now in order to avoid interference, under the traditional transmission scheme (as shown in Fig. 5), node 1 cannot transmit concurrently with node 3. But when node 4 starts forwarding packet i, node 1 can then transmit packet $i+1$ concurrently with node 4's transmission. This leads to a packet reception rate at the destination node of one packet every three time slots, resulting in a long-term average achievable end-to-end flow throughput of

$$Th_t = \frac{1}{3} p_c^t C$$

where p_c^t is the packet success rate over a link when the traditional scheme is used. For a packet of length L bits, the packet-success rate p_c^t is $(1 - p_e^t)^L$, where p_e^t is the BER under the traditional scheme.

B. PNC Transmission Scheme

The unidirectional PNC transmission scheme is illustrated in Fig. 6. In this case, node 1 and node 3 can send concurrently, and, as explained in previous sections, node 2 will perform PNC to recover the intended packet coming node 1, even in the presence of the signal/interference coming from node 3. Also, even though the BER experienced under the PNC scheme degrades due to the concurrent transmissions (as shown in Fig. 4), the performance gain of the PNC transmission scheme over that of the traditional scheme comes from the fact that it requires fewer number of transmissions than what the traditional scheme does to deliver a packet successfully. As shown in Fig. 6, the concurrent transmissions lead to a packet reception rate at the destination of one packet every two time slots, resulting in a long-term average throughput of

$$Th_{PNC} = \frac{1}{2} p_c^{PNC} C$$

where p_c^{PNC} is the packet success rate over a link when the PNC scheme is used. For a packet of length L bits, the packet-success rate p_c^{PNC} is $(1-p_e^{PNC})^L$, where $p_e^{PNC} = BER_{PNC}$ is the BER under the PNC scheme, and is given by Equation (1).

In Fig. 7, we show the normalized (w.r.t. the capacity of the link) average throughput of the traditional and PNC transmission schemes under various values of the SNR. The throughput basically depends on the packet success rate, which in turn depends on the bit-error rate. Observe that under low SNR values, the throughput obtained under the traditional transmission scheme is slightly higher than that obtainable under the PNC scheme. But under medium to high values of SNR, the PNC throughput is significantly greater than the the traditional one.

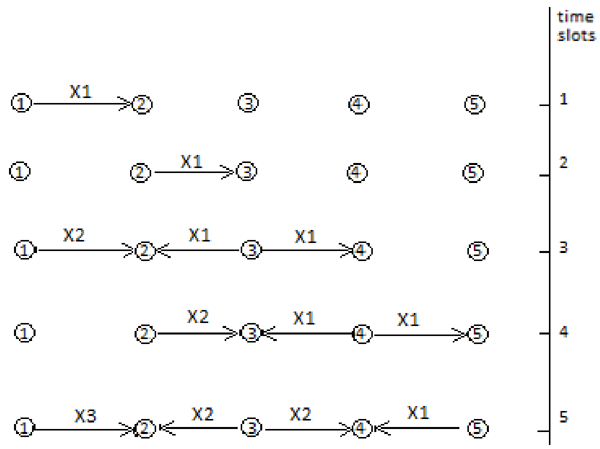

Fig. 6. Unidirectional PNC transmission in a linear network

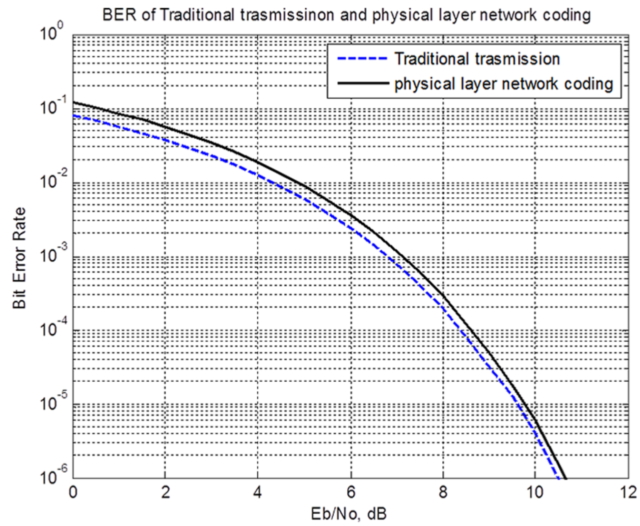

Fig. 8. Simulated BER of PNC and traditional transmission schemes

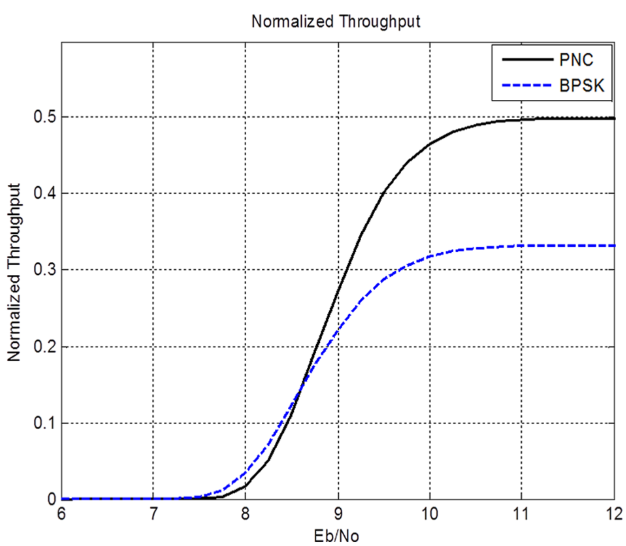

Fig. 7. Normalized throughput of PNC and traditional transmission schemes

IV. Simulation

In this section, we use Matlab to simulate both schemes: PNC and traditional. Specifically, we generated a stream of bits over an additive white Gaussian noise channel, and measured the BER at the destination of a unidirectional flow for various SNR values. Fig. 8 depicts these measured BERs under each of the two studied schemes. The figure shows that the BERs obtained via simulations match well those BERs derived theoretically in Section II. Through these simulation results, we were able then to validate our derived BER results.

V. Conclusion

This paper studies the performance of PNC transmission techniques in unidirectional flow networks, and compares it with that of the traditional transmission technique. We derived BER expressions for unidirectional end-to-end flows under the PNC transmission scheme. We also derived the end-to-end throughput that unidirectional flows can achieve under each of the studied schemes. Results show that the PNC transmission scheme achieves higher overall flow throughput than that achieved under the traditional one when considering medium to high SNR values.

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