Partial Relay Selection For Hybrid RF/FSO Systems with Hardware Impairments

Elyes Balti¹, Mohsen Guizani¹, Bechir Hamdaoui² and Yassine Maalej¹
¹University of Idaho, USA, ²Oregon State University, USA

Abstract—In this paper, we investigate the performance analysis of dual hop relaying system consisting of asymmetric Radio Frequency (RF)/Free Optical Space (FSO) links. The RF channels follow Rayleigh distribution and the optical links are subject to Gamma-Gamma fading. We also introduce impairments to our model and we suggest Partial Relay Selection (PRS) protocol with Amplify-and-Forward (AF) fixed gain relaying. The benefits of employing optical communication with RF, is to increase the system transfer rate and thus improving the system bandwidth. Many previous research attempts assuming ideal hardware (source, relays, etc.) without impairments. In fact, this assumption is still valid for low-rate systems. However, these hardware impairments are no longer neglected for high-rate systems in order to get consistent results. Novel analytical expressions of outage probability and ergodic capacity of our model are derived taking into account ideal and non-ideal hardware cases. Furthermore, we study the dependence of the outage probability and the system capacity considering the number of relays, the rank of the selected relay and the average optical Signal to Noise Ratio (SNR) over weak and strong atmospheric turbulence. We also demonstrate that for a non-ideal case, the end-to-end Signal to Noise plus Distortion Ratio (SNDR) has a certain ceiling for high SNR range. However, the SNDR grows infinitely for the ideal case and the ceiling caused by impairments no longer exists. Finally, numerical and simulation results are presented.

Keywords—Hardware impairments, SNDR, amplify-and-forward, partial relay selection, outage probability, ergodic capacity.

I. INTRODUCTION

Optical wireless communication technology has emerged as a promising solution to assist the Radio Frequency part of a relaying system that reaches the bottleneck in terms of bandwidth efficiency, power consumption and transfer rate. Such a system is called a hybrid RF/FSO. The vast majority of research work in this area investigated the performance of such systems as the outage probability, bit error rate and ergodic capacity for many scenarios, such as systems with a single relay [1] or multiple relays [2], [3] employing different relaying schemes like Amplify-and-Forward (AF) [4], [5], [6], Decode-and-Forward (DF) [2], [7] and Quantize-and-Forward (QF) [8], [9]. Also, mixed RF/FSO systems with multiple relays employ many relay selection protocols like best relay selection [10], [11] (selecting the relay with the highest end-to-end SNR), partial relay selection [10], [11], [12] (selecting the relay based on the knowledge of RF or FSO channels), Distributed Switch and Stay protocol [13] (selecting the relay that belongs to the shortest RF/FSO link) and all-active relaying [13] which consists of transmitting simultaneously on the overall links. Systems employing the latest selection protocol suffer from the problem of synchronization at the reception since it employs optical signals, that is why such system configuration is no longer used in practice.

Although these works suggested systems without taking into account hardware impairments, this assumption is still valid for low-rate systems. However, hybrid RF/FSO systems are characterized by high transfer rate due to the optical contributions so the assumption of neglecting hardware impairments is no longer valid since for high SNR these impairments significantly affect the system performance.

At the practical level, hardware suffers from many impairment types like I/Q imbalance [14] which mitigates the magnitude and shifts the signal phase. Other impairments like High Power Amplifier (HPA) non linearities [15], [16] cause the signal distortion. Qi et al. [17] concluded that the hardware impairments have a destructive effect on the system performance. In addition, [18] introduced a general model of hardware impairment accounting for many types of impairment unlike [14], [15] that consider only one type of impairment. Although Bjornson et al. modeled the overall impairments into one aggregate type in [18], [19], they applied this model on fully RF systems with a single relay. Our contribution is to consider this aggregate impairment model with a hybrid RF/FSO system of multiple relays employing Amplify-and-Forward (AF) with fixed gain relaying. Our system also employs PRS protocol based on the knowledge of the first hop to select the qualified relay to forward the signal to the destination. To the best of our knowledge, we are the first research group to employ hardware impairments with a mixed RF/FSO system of multiple relays.

The rest of this paper is organized as follows. Section II describes the system model. The outage probability and ergodic capacity analysis are presented in section III. Section IV presents the numerical results and their discussions. The final section presents some concluding remarks and future research directions.

II. SYSTEM MODEL

Our system composed of source (S), destination (D) and N parallel relays linked to S and D as shown in Fig. 1. Those relays employ AF with fixed gain and we refer to the PRS with perfect estimation of CSI (Channel State Information) to select the relay of rank K to forward the communication. In addition, we assume Intensity Modulation and Direct Detection (IM/DD) at the reception.

For each transmission, the source S receives local feedbacks from N relays and arranges the received instantaneous SNR ($\gamma_{i}(t)$ for $i=1,\ldots,N$) of the first hop in an increasing order of magnitude as follows: $\gamma_{1}(t) \leq \gamma_{2}(t) \leq \ldots \leq \gamma_{N}(t)$.

The best scenario is to select the best relay ($K=N$) with the highest first hop SNR. However, the best relay is not always available for forwarding, in this case the source will select the next best available relay.
The received signal at the K-th relay is given by:

\[ y_{k}(t) = h_{k}(\omega + \eta_{s}) + \nu_{1} \]  

(1)

where \( h_{k} \) is the RF fading between S and \( R_{k} \), \( \omega \) is the information signal, \( \eta_{s} \sim \mathcal{CN}(0, \sigma_{s}^{2}) \) is the distortion noise at the source S, \( \kappa_{s} \) is the impairment level parameter in S, \( P_{t} \) is the average transmitted power from S and \( \nu_{1} \sim \mathcal{CN}(0, \sigma_{1}^{2}) \) is the AWGN of the RF channel.

After reception of signal \( y_{k}(t) \), relay \( R_{k} \) assists with amplification gain \( G \) depending on the statistical channel fading of the first hop. According to [1, Eqn. (11)], the gain is given by:

\[ G^{2} \triangleq \frac{P_{2}}{P_{1}E[\vert h_{k} \vert^{2}]} \left( 1 + \kappa_{s}^{2} \right) + \sigma_{1}^{2} \]  

(2)

where \( P_{2} \) is the average transmitted power from the relay and \( E[\cdot] \) is the expectation operator.

The instantaneous SNR of the first hop between S and relay \( R_{k} \) can be written as:

\[ \gamma_{1}(k) = \frac{\vert h_{k} \vert^{2} P_{1}}{\sigma_{1}^{2}} = \frac{\vert h_{k} \vert^{2}}{\mu_{1}} \]  

(3)

where \( \mu_{1} \triangleq \frac{P_{2}}{\sigma_{2}^{2}} \) is the average SNR of the first hop.

After the amplification at the relay level, the optical part of the system modulates the received electrical signal by an optical subcarrier using the SIM (Subcarrier Intensity Modulation) technique. The optical signal at the relay \( R_{k} \) is given by:

\[ y_{opt}(k) = G(1 + \eta_{1})y_{1}(k) \]  

(4)

where \( \eta_{1} \) is the electrical-to-optical conversion coefficient.

At the reception D, our system employs direct detection by eliminating the signal direct component and converts it to the electrical one. The photocurrent after conversion can be expressed as follows:

\[ y_{2x}(k) = \eta_{2} I_{k}(G(1 + \eta_{1})(h_{k}(\omega + \eta_{s}) + \nu_{1}) + \eta_{R_{k}}) + \nu_{2} \]  

(5)

where \( \eta_{2} \) is the optical-to-electrical conversion, \( I_{k} \) is the optical irradiance between \( R_{k} \) and D, \( \eta_{R_{k}} \sim \mathcal{CN}(0, \kappa_{R_{k}}^{2}) \) is the distortion noise at the relay \( R_{k} \), \( \kappa_{R_{k}} \) is the impairment level parameter in \( R_{k} \) and \( \nu_{2} \sim \mathcal{CN}(0, \sigma_{2}^{2}) \) is the AWGN of the optical channel.

The instantaneous SNR of the second hop between \( R_{k} \) and D can be written as:

\[ \gamma_{2}(k) = \frac{|I_{k}|^{2} \gamma_{2} P_{2}}{\sigma_{2}^{2}} = |I_{k}|^{2} \mu_{2} \]  

(6)

where \( \mu_{2} \triangleq \frac{\eta_{2}^{2} P_{2}}{\sigma_{2}^{2}} \) is the average electrical SNR of the second hop. According to Eqn. (8) in [20], the average SNR of the second hop \( \gamma_{2}(k) \) is related to the average electrical SNR \( \mu_{2} \) by the following mathematical expression:

\[ \mu_{2} = \frac{\alpha \beta \gamma_{2}(k)}{(\alpha + 1)(\beta + 1)} \]  

(7)

The end-to-end Signal to Noise plus Distortion Ratio (SNDR) can be expressed as:

\[ \gamma_{e2e} = \frac{|h_{k}|^{2} |I_{k}|^{2}}{d|h_{k}|^{2} |I_{k}|^{2} + |I_{k}|^{2} (1 + \kappa_{R_{k}}^{2}) \sigma_{2}^{2}} \]  

(8)

After mathematical manipulations, the end-to-end SNDR is given by:

\[ \gamma_{e2e} = \frac{\gamma_{1}(k) \gamma_{2}(k)}{d\gamma_{1}(k) \gamma_{2}(k) + (1 + \kappa_{R_{k}}^{2}) \mu_{2}} + A \]  

(9)

where \( \kappa_{R_{k}}^{2} \) is the impairment level, \( A \) is the constant \( A = 1 + \mathbb{E}[\gamma_{1}(k)] \).

Since the first hop is subject to Rayleigh fading, the PDF of the relative instantaneous SNR \( \gamma_{1}(k) \) is given by [13, Eqn. (8)]. After some mathematical manipulations, the PDF of \( \gamma_{1}(k) \) is expressed as follows:

\[ f_{\gamma_{1}(k)}(x) = K \left( \begin{array}{c} N \\ K \end{array} \right) \sum_{n=0}^{K-1} \left( \begin{array}{c} K-1 \\ n \end{array} \right) (-1)^{n} \frac{(N-K+n+1)x}{\mu_{1}} e^{-\frac{(N-K+n+1)x}{\mu_{1}}} \]  

(10)

The Cumulative Distribution Function (CDF) of \( \gamma_{1}(k) \) is given by:

\[ F_{\gamma_{1}(k)}(x) = \int_{0}^{x} f_{\gamma_{1}(k)}(t) \, dt \]  

(11)

After mathematical manipulations, the CDF of \( \gamma_{1}(k) \) can be expressed as follows:

\[ F_{\gamma_{1}(k)}(x) = 1 - K \left( \begin{array}{c} N \\ K \end{array} \right) \sum_{n=0}^{K-1} \left( \begin{array}{c} K-1 \\ n \end{array} \right) (-1)^{n} \frac{\mu_{1}}{N-K+n+1} e^{-\frac{(N-K+n+1)x}{\mu_{1}}} \]  

(12)

The constant \( A \) that appears in the formula of the end-to-end SNDR depends on \( \mathbb{E}[\gamma_{1}(k)] \) which is obtained by:

\[ \mathbb{E}[\gamma_{1}(k)] = \int_{0}^{x} x f_{\gamma_{1}(k)}(x) \, dx = K \left( \begin{array}{c} N \\ K \end{array} \right) \sum_{n=0}^{K-1} \left( \begin{array}{c} K-1 \\ n \end{array} \right) \frac{(-1)^{n} \mu_{1}}{(N-K+n+1)^{2}} \]  

(13)

Using the formula of \( \mathbb{E}[\gamma_{1}(k)] \), the constant \( A \) is obtained by:

\[ A = 1 + K \left( \begin{array}{c} N \\ K \end{array} \right) \sum_{n=0}^{K-1} \left( \begin{array}{c} K-1 \\ n \end{array} \right) \frac{(-1)^{n} \mu_{1}}{(N-K+n+1)^{2}} \]  

(14)

Regarding the second hop, the optical links are affected by atmospheric turbulence which are modeled by Gamma-Gamma
distribution. In this case, the PDF of the instantaneous SNR \( \gamma_{2(k)} \) is given by:

\[
f_{\gamma_{2(k)}}(x) = \frac{(\alpha \beta)^{\frac{n+\delta}{2}} x^{\frac{n+\delta}{2}-1}}{\Gamma(\alpha) \Gamma(\beta) \mu_2^{n+\delta}} K_{\alpha-\beta} \left(2 \sqrt{\frac{\alpha \beta x}{\mu_2}}\right)\]  

(15)

where \( K_{\nu}(.) \) is the \( \nu \)-th order modified Bessel function of the second kind. The parameters \( \alpha \) and \( \beta \) characterize respectively small-scale and large-scale of scattering process in the atmospheric environment, hence, the parameters \( \alpha \) and \( \beta \) can be given by:

\[
\alpha = \left(\exp\left(-\frac{0.49 \sigma_R^2}{(1+1.11 \sigma_R^2)^{\frac{3}{2}}}\right) - 1\right)^{-1}
\]

(16)

\[
\beta = \left(\exp\left(-\frac{0.51 \sigma_R^2}{(1+0.69 \sigma_R^2)^{\frac{3}{2}}}\right) - 1\right)^{-1}
\]

(17)

where \( \sigma_R^2 \) is called Rytov variance and it is a metric of atmospheric turbulence intensity.

### III. Performance Analysis

In this section, we provide analysis of the system performance in terms of outage probability and ergodic capacity. This analysis consists of deriving the analytical expressions of outage probability and system capacity and studying their behavior for a high SNR range. Then, we will show the convergence of the end-to-end SNDR to an accurate ceiling for the case of non-ideal hardware.

#### A. Outage Probability Analysis

The outage probability is defined as the probability that the overall instantaneous SNDR falls below a given outage threshold \( \gamma_{th} \). Its formula can be expressed as follows:

\[
P_{out} = Pr(\gamma_{e2e} < \gamma_{th}) = Pr\left(\frac{\gamma_{1(k)} \gamma_{2(k)}}{d_{\gamma_{1(k)}} \gamma_{2(k)} + (1 + \kappa_{Rk}^2) \gamma_{2(k)} + A < \gamma_{th}}\right)
\]

(18)

where \( Pr(.) \) is the probability notation. After mathematical manipulations, the outage probability can be re-written as:

\[
P_{out} = \int_0^\infty Pr\left(\gamma_{1(k)} < \frac{(1 + \kappa_{Rk}^2) \gamma_{th} \gamma_{2(k)} + A \gamma_{th}}{d_{\gamma_{1(k)}} \gamma_{2(k)} + (1 - d_{\gamma_{1(k)}}) \gamma_{2(k)}}\right) \times Pr(\gamma_{2(k)}) \ d\gamma_{2(k)}
\]

(19)

Since the RF and FSO fadings are independent, the above outage expression can be written as follows:

\[
P_{out} = \int_0^\infty F_{\gamma_{1(k)}}\left(\frac{1 + \kappa_{Rk}^2 \gamma_{th}}{1 - d_{\gamma_{1(k)}}} + \frac{A \gamma_{th}}{1 - d_{\gamma_{1(k)}} \gamma_{2(k)}}\right) \times f_{\gamma_{2(k)}}(\gamma_{2(k)}) \ d\gamma_{2(k)}
\]

(20)

Before deriving the outage probability formula, a necessary condition must be set. This condition is that the outage threshold must be strictly inferior to \( \frac{\gamma_{th}}{2} \), i.e., \( \gamma_{th} < \frac{A}{2} \). Hence, a piecewise form of the outage probability is written as:

\[
P_{out} = \begin{cases} 
  \text{Eqn. (20)}, & \text{if } \gamma_{th} < \frac{1}{2} \\
  1 & \text{otherwise}
\end{cases}
\]

(21)

To derive the outage probability formula, we should first substitute Eqs. (12) and (15) into Eqn. (20) and then using the identities in [21], we finally get the following expression in terms of Meijer’s \( G \) function:

\[
P_{out} = 1 - K\left[N^\frac{K-1}{K} \sum_{n=0}^{N-1} \left(\frac{K-1}{n}\right) \left(\frac{(-1)^{n+1}}{N-K+n+1}\right) \times \frac{\exp\left(\frac{(1+\kappa_{Rk}^2)N-K+n+1}{\mu_1(1-d_{\gamma_{th}})}\right)}{\Gamma(\alpha, \beta) \mu_2^\mu_2} \right]
\]

(22)

#### B. Ergodic capacity analysis

The channel capacity is obtained by [1, Eqn. (32)] as follows:

\[
C \triangleq \frac{1}{2} \mathbb{E} [\log_2(1 + \gamma_{e2e})]
\]

(23)

Since the transmission is done in two time slots, the factor \( \frac{1}{2} \) appears in the channel capacity formula. This formula can be computed by integration referring to the probability density function of \( \gamma_{e2e} \). However, deriving a closed form of the channel capacity in our case is very complex if not impossible. To overcome this problem, we should refer to the approximation given by [1, Eqn. (35)]

\[
\mathbb{E} \left[\log_2 \left(1 + \frac{\psi}{\omega}\right)\right] \approx \log_2 \left(1 + \frac{\mathbb{E}[\psi]}{\mathbb{E}[\omega]}\right)
\]

(24)

#### C. Asymptotic Analysis

In this subsection, we study the behavior of SNDR, outage probability and ergodic capacity for a high SNR regime. For the ideal case, the SNDR can be written as:

\[
\gamma_{e2e} = \frac{\gamma_{1(k)} \gamma_{2(k)}}{\gamma_{2(k)} + A}
\]

(25)

For a high SNR range, SNDR is not upper bounded as shown below:

\[
\lim_{\gamma_{1(k)} \to \infty, \gamma_{2(k)} \to \infty} \gamma_{e2e} = \infty
\]

(26)

We can see that for the ideal case, i.e., without hardware impairments, SNDR has no ceiling which means that the performance is not limited for a high-rate system. Considering the case of hardware impairments, the SNDR is...
upper bounded by a ceiling in the high SNR regime which is shown by the following reasoning:

\[
\lim_{\gamma_1(k), \gamma_2(k) \to 0} \gamma_{c2e} = \frac{\gamma_1(k) \gamma_2(k)}{\gamma_1(k) \gamma_2(k) + (1 + \kappa_2^2 R_k) \gamma_2(k) + A} = \frac{1}{d}
\]

Thereby, we denote the SNDR ceiling \( \gamma_c \) which is given by:

\[
\gamma_c = \frac{1}{d} = \frac{1}{\kappa_2^2 + \kappa_2^2 R_k + \kappa_2^2 R_k^2}
\]

We can see now that the SNDR converges to the ceiling \( \gamma_c \) which means that the performance is limited for a high-rate system.

Regarding the ergodic capacity, the system capacity satisfies the following convergence for high SNR values:

\[
\lim_{\gamma_1(k), \gamma_2(k) \to 0} C = \frac{1}{2} \log_2(1 + \gamma_c)
\]

The above convergence shows the existence of the system capacity ceiling defined as \( C_c = \frac{1}{2} \log_2(1 + \gamma_c) \). Note that for the ideal case, the ergodic capacity grows infinitely without an upper bound for a high SNR regime and so we can see clearly the impairment limitations effect on the system performance.

**IV. NUMERICAL RESULTS**

This section presents numerical and simulation results of the outage probability, the ergodic capacity and the effective SNDR dependence on the system parameters. We use Monte-Carlo simulation to validate the numerical results.

Assuming \( \mu_2 = 20 \) dB, we note that increasing the average SNR of the RF link results in the existence of the outage floor, i.e., increasing the RF SNR has no effect on the outage probability. However, if we assume \( \mu_2 \) grows simultaneously with \( \mu_1 \), the system performance improves better after the 20 dB value and the outage floor disappears.

The variations of the outage probability with respect to Rytov standard deviation are shown in Fig. 3. Both, the best and worst relay selection scenarios are presented for different impairment values. For weak turbulence conditions, the outage probability is better when the best relay is available to forward the communication. For this case, decreasing the impairments level results in further improving the outage performance. If we select the worst relay and decrease the impairment levels, the outage probability does not significantly improve in comparison with the outage performance of the best relay selection scenario. However, either selecting the best relay or decreasing the impairment levels has no effect on the system performance for strong turbulence conditions.

Fig. 4 shows the outage probability dependence on the number of relays for different values of impairment (\( \kappa_1, \kappa_2 \)) and for the best and worst selection scenarios. For low impairment levels, the best relay selection scenario shows a higher performance compared to the worst

![Outage probability versus average SNR](image)

**Fig. 2:** Outage probability versus the average SNR for different values of impairment (\( \kappa_1, \kappa_2 \)) and average electrical SNR for optical link.

![Outage probability versus Rytov Standard Deviation](image)

**Fig. 3:** Outage probability versus \( \sigma_R \) for different values of impairment (\( \kappa_1, \kappa_2 \)) and for the best and worst selection scenarios.
selection scenario. However, increasing the impairment levels results in neglecting the effect of the best selection scenario and thus leading to the same performance for both scenarios.

Fig. 4: Outage probability versus the number of relays $N$ for different values of impairment $(\kappa_1, \kappa_2)$ in different atmospheric turbulence conditions.

Fig. 5: Simulation and approximate ergodic capacity versus level of impairments $(\kappa_1, \kappa_2)$ for the best and worst relay selection scenarios.

Fig. 6 shows the dependence of the ergodic capacity on the average SNR for different impairment values. As expected, the system performs better under low impairment levels. For the ideal case, we observe that the capacity grows infinitely without an upper bound contrary to the non-ideal case where the capacity is limited by a capacity ceiling $C$. Eqn. (29) for a high SNR regime.

The variations of the effective SNDR with respect to the average SNR is presented in Fig. 7. We observe that decreasing the impairment levels, the SNDR grows infinitely for the ideal case and increases with an upper bound for the non-ideal case. For the high SNR regime, Fig. 7 shows the existence of the SNDR ceiling $\gamma_c$. Eqn. (28) that limits the system performance.

V. CONCLUSION

In this work, we introduced an aggregate model of impairments to a mixed RF/FSO system of multiple relays. We also assumed Amplify-and-Forward scheme with a fixed gain relaying. In addition, we proposed the PRS protocol assuming that the best relay is not always available to forward the signal. Moreover, we investigate the effect of the average electrical SNR of the optical link, the rank of the selected
relay, the number of relays and the impairment levels under weak, moderate and strong atmospheric turbulence conditions. The results showed that the system performance depends significantly on the state of the optical link in terms of the optical SNR and atmospheric turbulence conditions. We also proved the existence of the ceilings for the effective SNDR and the capacity which limit the system performance, i.e., the system saturates by the hardware impairments contrary to the ideal case where the system performance grows infinitely without limits. In the future, we plan to extend this work following three major research directions. First, we will consider a more complex optical fading that takes into account the pointing errors and the atmospheric path loss. Second, it is important to consider outdated CSI estimation required for the relay selection rather than considering a perfect CSI estimation. Finally, deriving analytical expressions of the Symbol Error Rate (SER) for different types of modulation is important to get an accurate performance analysis of our system.

REFERENCES


