Distributed Wideband Sensing for Faded Dynamic Spectrum Access with Changing Occupancy

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***Abstract*—We propose a distributed compressive sampling technique for cooperative wideband spectrum sensing that requires lesser numbers of measurements while overcoming time-variability of spectrum occupancy and the hidden terminal problem. First, we prove that the wideband spectrum occupancy information can almost surely be recovered with a reduced number of spectrum measurements. Second, we propose non-uniform sensing matrix design that exploits the heterogeneity in the wideband spectrum access to further improve the spectrum sensing recovery accuracy. Using simulations, we confirm our theoretic results and show that cooperation leads to high detection probability, even with each secondary user taking only a small number of measurements. We also show that it is sufficient to consider a subset of close-by secondary users to obtain comparable performances.**

***Index Terms*—Heterogeneous wideband access; distributed compressive sampling; cooperative spectrum sensing.**

I. Introduction

Dynamic spectrum access (DSA) emerges as a key technology for overcoming spectrum shortage problems [1]. Due to its great potential, DSA has already found its way to standardization—e.g., IEEE 802.22 [2] for enabling opportunistic access in the TV bands and 3GPP's Licensed-Assisted Access (LAA) and LTE-U [3] for enabling spectrum access in the unlicensed 5 GHz band. Spectrum sensing is vital to enabling successful DSA, and as a result, has been studied thoroughly in the literature. Most of the sensing technique development effort, however, has been focused on narrow band access, and not until recently, has the focus been shifted towards wideband spectrum access [4–6].

Performing wideband spectrum sensing (WSS) through traditional methods has been shown ineffective, by incurring excessive delays, costly hardware, and/or high energy consumption; for instance, sequential sensing approaches require cheap hardware, but incur high sensing delays, whereas, parallel sensing approaches overcome delay issues, but require more hardware [7]. Frequency-domain analysis methods, on the other hand, require sampling rates that are excessively high for the case of wideband, which can be feasible only through complex hardware circuitry and digital processing algorithms. More insights into the limitations of traditional sensing methods when applied to WSS can be found in [7].

Motivated by the sparsity feature inherent to spectrum occupancy and in an effort to address the high sampling rate limitation, researchers have resorted to exploiting compressive sampling (CS) theory to make WSS possible at reasonable sampling rates (e.g. [4, 5, 8]). In essence, these CS-based sensing approaches require a number of measurements that is much smaller than what traditional non-CS-based approaches require [9]. Despite the ability of these CS-based approaches to overcome the high sampling rate limitation, there remains a number of key challenges that limit their applicability in practice. These challenges are:

- *Limited receiver hardware:* The number of measurements that receiver hardware designs are able to perform is practically way smaller than the number of measurements required by the CS-based sensing approaches. Therefore, multiple sequential sensing scans are often required to enable CS-based spectrum occupancy recovery, which leads to excessive recovery delays, making these CS-based approaches unsuitable for realtime applications.
- *Uncertain and time-varying spectrum occupancy:* The number of measurements required by the CS-based sensing approaches depends on the number of occupied bands (i.e., sparsity level). However, the sparsity level is often unknown in advance and changes over time, making it more challenging for CS-based approaches to achieve accurate and robust recovery without excessive overhead.
- *Measurement inconsistency across the different* SUs*:* Due to impairments of the wireless channel, different secondary users (SUs) may observe different spectrum occupancy, leading to inconsistent measurements across the users. This poses a challenge when using CS-based approaches for cooperative occupancy recovery.

This paper combines user cooperation with compressive sampling to propose a practical WSS technique that overcomes these three aforementioned challenges. In addition, unlike most previous approaches in which the entire wideband is considered as one single block with a fixed, global sparsity level, our work considers a more realistic, *non-homogeneous* WSS. In practice indeed, the wideband spectrum occupancy is rather *heterogeneous*, with different frequency blocks exhibiting different occupancy behaviors and statistics [10–12]. This is mainly because applications of similar types (cellular, TV, etc.) are often assigned spectrum bands within the same (or nearby) frequency block, and different application types show different occupancy patterns, resulting in a non-homogeneous wideband spectrum occupancy. Unlike previous works, our proposed technique exploits the heterogeneity information in wideband spectrum occupancy to provide further improvement of the spectrum recovery efficiency. To this end, the main contributions of this paper are:

- We propose a distributed, cooperative CS-based sensing technique for wideband access in faded environments, and prove that the proposed technique recovers the occupancy information with fewer spectrum measurements.
- We show that the number of required measurements can be reduced even further while maintaining a high recovery accuracy by exploiting user closeness.
- We design efficient sensing matrices that capture and leverage prior knowledge about the spectrum occupancy heterogeneity to improve the occupancy recovery accuracy of the CS-based sensing approaches.

The rest of this paper is organized as follows. Section II describes the system model. Section III presents current CS-based sensing approaches along with their challenges. Section IV presents the proposed techniques. Section V presents the numerical evaluations. Section VI concludes the paper.

II. Wideband Spectrum Sensing Model

We consider a heterogeneous WSS system with $\mathtt{N}$ frequency bands and denote the support of the occupied bands by Ω. We assume that the wideband spectrum accommodates multiple different types of user applications, where applications of the same type are allocated frequency bands within the same block. That is, the $\mathtt{N}$ narrow bands are grouped into $\mathtt{g}$ disjoint contiguous blocks, with each block, $\mathtt{G_i}$, consisting of $\mathtt{N}_i$ contiguous bands, being assigned to one application type. For simplicity, we model the state of each band i using a Bernoulli(p_i) with parameter $p_i \in [0,1]$ where p_i is the probability that band i is occupied by some primary user (PU). We assume every PU can only occupy one band. Let $\bar{\mathtt{K}}_j = \sum_{i\in\mathtt{G_j}} p_i$ be the average number of bands occupied within block j (assuming independence across band occupancies). As observed via real measurement studies [10–12], the band occupancy statistics (e.g., $\bar{\mathtt{K}}_j$) vary from one block to another; that is, the spectrum occupancy in wideband access exhibits a *block-like occupancy behavior* where the spectrum occupancy can vary significantly from one block to another.

We also consider that the WSS system has $\mathtt{J}$ SUs that are able and willing to perform the sensing task. The time-domain signal $\boldsymbol{r}(t)$ received by each SU can be expressed as

$$\boldsymbol{r}(t) = \sum_{i=1}^{\mathtt{N}_{sig}} h_i(t) \otimes s_i(t) + w(t), \tag{1}$$

where $h_i(t)$ is the channel impulse response between the PUs and the SU, $s_i(t)$ is the primary user's signal (with power $\mathtt{P}$), $w(t)$ is an Additive White Gaussian Noise with mean 0 and variance $\mathtt{NN_0}$, $\otimes$ is the convolution operator, and $\mathtt{N}_{sig}$ is the number of active PUs (for simplicity $\mathtt{N}_{sig}$ is assumed to be equal to the number of occupied bands).

The discrete Fourier transform of the received signal $\boldsymbol{r}(t)$ can be expressed as

$$\boldsymbol{r}_f = \boldsymbol{h}_f \boldsymbol{s}_f + \boldsymbol{w}_f = \mathtt{x} + \boldsymbol{w}_f, \tag{2}$$

where $\boldsymbol{h}_f$, $\boldsymbol{s}_f$ and $\boldsymbol{w}_f$ are the Fourier transforms of $\boldsymbol{h}(t)$, $\boldsymbol{s}(t)$ and $\boldsymbol{w}(t)$. Here, we assume that $\mathbb{E}(\boldsymbol{s}(t)) = 0$. The vector $\mathtt{x}$ in Eq. (2) represents the faded version of the PUs' signals being sent on the different bands. Since $\boldsymbol{s}_f$ is independent of $\boldsymbol{h}_f$, $\mathbb{E}(\mathtt{x}) = 0$. The vector $\boldsymbol{r}_f$ is nearly sparse with energy levels in the unoccupied bands equaling $\mathbb{E}(\boldsymbol{w}_f^2) = \mathtt{N_0}$.

III. Compressive Sampling-based Sensing: Current Approaches and their Limitations

Recall that the number of samples needed to recover the occupancy information through classical frequency-domain analysis methods can be excessively large, especially when the spectrum is wideband, making such methods unpractical. To overcome this issue, compressive sampling (CS) theory has been leveraged to take advantage of the sparsity nature of the spectrum occupancy vector $\mathtt{x}$ to reduce the number of required samples [9]. More specifically, the signal resulting from applying CS theory can be written as [5]:

$$\mathtt{y} \quad = \quad \Phi\mathtt{F}^{-1}(\mathtt{x} + \boldsymbol{w}_f) = \Psi\mathtt{x} + \eta, \tag{3}$$

where $\mathtt{y} \in \mathbb{R}^{\mathtt{M}}$ is the measurement vector, $\mathtt{F}^{-1}$ is the inverse discrete Fourier transform (as $\mathtt{x}$ is sparse in the Fourier basis), Φ is the $\mathtt{M} \times \mathtt{N}$ sensing matrix assumed to be full rank, i.e. $\mathtt{rank}(\Phi) = \mathtt{M}$, and $\mathtt{M} = \mathcal{O}(\mathtt{K}\log(\mathtt{N}/\mathtt{K}))$ [9], with $\mathtt{K}$ being the sparsity level of the entire wideband. The coefficients of Φ are drawn from a Bernoulli distribution $\{\frac{\pm 1}{\sqrt{\mathtt{M}}}\}$ and the sensing noise η is equal to $\Phi\mathtt{F}^{-1}\boldsymbol{w}_f$. From a hardware perspective, the number of measurements $\mathtt{M} = \mathcal{O}(\mathtt{K}\log(\mathtt{N}/\mathtt{K}))$ corresponds to the number of hardware branches each SU device needs to have to be able to perform the CS-based sensing, with each branch using a pseudo-random sequence mixer corresponding to a raw of Φ [7, 8].

A. CS-Based Wideband Spectrum Sensing

Broadly speaking, there are two classes of CS-based approaches that can be used to recover the spectrum occupancy vector $\mathtt{x}$ from the measurement vector $\mathtt{y}$ (Eq. (3)). These are (i) heuristic approaches, such as Basis Pursuit (BP) [13] and Orthogonal Matching Pursuit (OMP) [14], which are fast and easy to implement, but may not be very accurate, and (ii) convex relaxation approaches which allow for more robust and accurate recovery, but require more computation. One widely known approach of the latter class is LASSO [9, 15], which recovers the occupancy vector $\mathtt{x}$ by solving

$$\mathscr{P}_{\mathtt{LASSO}} : \min_{\mathtt{z}} \|\mathtt{z}\|_{\ell_1} \quad \text{s.t.} \quad \|\Psi\mathtt{z} - \mathtt{y}\|_{\ell_2} \leq \epsilon \tag{4}$$

where ϵ is a pre-defined error threshold parameter. wLASSO (or weighted LASSO) [16] is another convex relaxation approach which exploits the spectrum occupancy variability observed across the different frequency blocks to allow for a more efficient solution search, thereby requiring lesser numbers of measurements and/or incurring smaller errors when compared to LASSO [16]. Formally, by referring to $\mathscr{P}_{\mathtt{LASSO}}$ (Eq. (4)), re-writing the vector variable $\mathtt{z}$ as $\mathtt{z} = [\mathtt{z}_1^T, \mathtt{z}_2^T, \ldots, \mathtt{z}_{\mathtt{g}}^T]^T$ where $\mathtt{z}_i$ is the $\mathtt{N}_i \times 1$ vector corresponding to block i for $i = 1, 2, \ldots, \mathtt{g}$, and assigning for each block i

a weight $\omega_{\mathtt{i}}$ such that $\omega_{\mathtt{i}} > \omega_{\mathtt{j}}$ when $\bar{\mathtt{K}}_i < \bar{\mathtt{K}}_j$ for all blocks i, j, wLASSO recovers $\mathtt{x}$ by solving

$$\mathscr{P}_{\mathtt{wLASSO}} : \min_{\mathtt{z}} \sum_{i=1}^{\mathtt{g}} \omega_{\mathtt{i}} \|\mathtt{z}_{\mathtt{i}}\|_{\ell_1} \quad \text{s.t.} \quad \|\Psi\mathtt{z} - \mathtt{y}\|_{\ell_2} \leq \epsilon \qquad (5)$$

Here, the idea is to choose the weights in such a way that a block with a higher sparsity level is assigned a smaller weight, and one possible way of meeting this requirement is by setting $\omega_{\mathtt{i}} = (1/\bar{\mathtt{K}}_{\mathtt{i}})/\sum_{\mathtt{j}=1}^{\mathtt{g}}(1/\bar{\mathtt{K}}_{\mathtt{j}})$ for each block i.

B. Challenges with Current CS-Based Sensing Approaches

Recall that the number of measurements needed for the CS-based sensing approaches to successfully recovery the occupancy is $\mathtt{M} = \mathcal{O}(\mathtt{K}\log(\mathtt{N}/\mathtt{K}))$ [7, 8], which depends on the total number of bands, N, and the sparsity level of spectrum occupancy, K. This gives rise to the following two challenges.

- **Challenge 1: Hardware limitation.** The number of hardware branches needed to enable the CS-based recovery can be high and unpractical. For example, even when the number of occupied bands is as small as $K = 6$, the number of needed branches for a total number of bands $N = 50$ can be as high as $\mathtt{M} = 16$ [7]. In practice, however, the number of branches that reasonable receiver designs have is typically in the order of 4 to 8 [17], a number that is much smaller than the number of measurements, M, required by the CS-based approaches. Therefore, hardware presents a major limitation on the applicability of such CS-based approaches.
- **Challenge 2: Uncertain and time-varying sparsity.** The second challenge that these CS-based approaches also face is that the number of occupied bands (i.e., the sparsity level) is time-varying. Most CS-based approaches, however, assume that the sparsity level, K, is fixed, often done by setting it to the overall average occupancy of the spectrum [4, 18]. This time variability of the sparsity of the wideband occupancy makes existing approaches either inaccurate or incur high overhead.

In general, from a practical viewpoint, cooperative spectrum sensing approaches are more effective than non-cooperative approaches, since they are designed to provide spectrum availability information not just to one SU, but to multiple SUs, often located in different geographic locations. Clearly, having each SU perform the CS-based sensing task on its own can be costly and redundant, as it might suffice for one SU to perform sensing and share it with other SUs, thereby saving SUs' energy and computation resources. Despite all the known benefits of cooperation, there is another major challenge that needs to be addressed to enable cooperative CS-based sensing.

- **Challenge 3: Inconsistent observations.** In practice, different SUs may observe different spectrum occupancy due to wireless channel impairments (e.g., fading, multipath, etc.), leading to inconsistent measurements across the different users. This presents a challenge when it comes to enabling and designing cooperative CS-based spectrum sensing approaches. This problem captures the hidden terminal problem as a special case.

IV. The Proposed WSS Technique

In this work, we propose a cooperative, distributed compressed sensing technique for wideband spectrum access that overcomes the three aforementioned challenges. In addition, our proposed technique allows exploiting any prior knowledge about the spectrum occupancy statistics to improve the recovery accuracy further.

A. The Proposed Spectrum Recovery Approach

Although, due to fading, each SU observes a different spectrum occupancy vector x, most SUs observe the same support of the (nearly) sparse occupancy vector. Hence, to be able to detect the support, we propose to compute, for every SU j, the contribution $\xi_{j,n}$ of every column of SU j's sensing matrix, Ψ_j, to $\mathtt{y}_j$ on each band n; i.e., $\xi_{j,n} = \langle \mathtt{y}_j, \psi_{j,n}\rangle^2 = (\mathtt{y}_j^T \psi_{j,n})^2$ for $n = 1..\mathtt{N}$. For this, we define the sample mean ξ_n as

$$\xi_n = \frac{1}{\mathtt{J}} \sum_{j=1}^{\mathtt{J}} \xi_{j,n} = \frac{1}{\mathtt{J}} \sum_{j=1}^{\mathtt{J}} \langle \mathtt{y}_j, \psi_{j,n}\rangle^2 \text{ for } n = 1..\mathtt{N} \qquad (6)$$

Once ξ_n is computed, the indices corresponding to the K highest values among the N statistics are selected iteratively. We refer to this technique as *spectrum occupancy recovery*. Although inspired by the approach proposed in [19], our proposed recovery approach differs in the following aspects: in our work, (i) the signal occupying each band is not Gaussian, but rather follows a mixed Rayleigh and Gaussian distribution in the occupied bands that depends on the distance between each SU and the active PU, and Gaussian with mean 0 and variance $\mathtt{N}_0$ in the unoccupied bands (nearly sparse signal); (ii) the sensing matrices are non-uniform Bernoulli, where elements in column i have mean 0 and variance $\frac{1}{\omega_{\mathtt{i}}^2}$; and (iii) the sensing matrices contain a very small number of measurements M, making their columns highly correlated (orthogonality between columns is hard to meet). Algorithm 1 presents our proposed iterative approach for recovering the occupied support. Recall that we are only interested in detecting the support rather than actual signal values in every band.

Algorithm 1: Spectrum occupancy recovery

Input : $\mathtt{y}_j$, Ψ_j, $\mathtt{r}_{j,0} = \mathtt{y}_j$, $j = 1..\mathtt{J}$, $k = 1$

begin
  while $\|\mathtt{r}_{j,k}\|_{\ell_2} \geq \epsilon \|\mathtt{y}_j\|_{\ell_2}$, $j = 1..\mathtt{N}$ **do**
    $n_k = \arg\max_{n \in \{1..\mathtt{N}\}} \frac{1}{J} \sum_{j=1}^{\mathtt{J}} |\langle \mathtt{r}_{j,k-1}, \psi_{j,n}\rangle|^2$
    $\Omega = \Omega \bigcup \{n_k\}$
    $\mathtt{r}_{j,k} = \mathtt{r}_{j,k-1} - \frac{\langle \mathtt{r}_{j,k-1}, \psi_{j,n_k}\rangle}{\|\psi_{j,n_k}\|_{\ell_2}^2} \psi_{j,n_k}$
    $k = k + 1$
  return Ω

Now that we presented an algorithm, which leverages cooperation to recover the occupied support of a wideband spectrum from only a small number of measurements per SU, we turn our focus, in the next section, to study its correctness. For this, we prove that the proposed algorithm does indeed, with an overwhelming probability, recover the true support Ω.

B. Correctness of the Proposed Spectrum Recovery Approach

The following theorem states that by considering a large number of $\texttt{SU}$s, Ω can almost surely be recovered from only a small number of measurements per $\texttt{SU}$, leading to a high detection probability.

Theorem 1. *Consider* $\texttt{J}$ $\texttt{SU}$*s, and let the measurement matrix* Ψ_j *of* $\texttt{SU}$ j *contain independent Bernoulli elements, with column* i*'s elements being set to* $\{\frac{\pm 1}{\omega_{\rm i}}\}$*. The vector* $\texttt{x}$ *is nearly sparse such that* $\texttt{x}_\ell$ *is i.i.d. Gaussian with zero mean and variance* $\texttt{N}_0$ *if* $\ell \notin \Omega$ *and zero mean and variance* $\mathbb{E}(x_\ell^2) > \texttt{N}_0$*, if* $\ell \in \Omega$*. With* $\texttt{M} > 1$ *measurements per* $\texttt{SU}$*,* **Algorithm 1** *recovers* Ω *with a probability approaching one as* $\texttt{J} \to \infty$*.*

Remark 1. Observe that our proposed sensing matrix is by design chosen to non-uniformly distributed; this is done so that to allow the exploitation of any prior knowledge about the spectrum occupancy statistics to improve recovery accuracy. This will be shown later in Section IV-D.

Proof. The proof is based on Kolmogorov's Strong Law of Large Numbers (SLLN) [20], following the same line of argument as in [19]. The main idea is to show that ξ_n in an occupied band n when $\texttt{J}$ increases is sufficiently high compared to when the band n is not occupied. Due to space limitation, some of the details in the proof are omitted. However, we provide all that is required to guide the reader to the complete proof.

SLLN [20] states that the sample mean $\bar{\texttt{X}}_n = \frac{1}{n}\sum_{i=1}^{n} \texttt{X}_i$ of n independent random variables, $\texttt{X}_1, \texttt{X}_2, \cdots, \texttt{X}_n$, with finite expectations ($\mathbb{E}(\texttt{X}_n) < \infty$ for $n \geq 1$) converges almost surely to $\mathbb{E}(\texttt{X}_n)$; i.e., $\mathbb{P}(\lim_{n\to\infty} \bar{\texttt{X}}_n = \mathbb{E}(\texttt{X}_n)) = 1$, and that SLLN holds if one of the following conditions is satisfied:

1) $\texttt{X}_1, \texttt{X}_2, \cdots, \texttt{X}_n$ are identically distributed.
2) $\mathbb{V}ar[\texttt{X}_n] < \infty$ and $\sum_{n=1}^{\infty} \frac{\mathbb{V}ar[\texttt{X}_n]}{n^2} < \infty$ for all n.

Considering $\xi_{j,n} = \langle \texttt{y}_j, \boldsymbol{\psi}_{j,n}\rangle^2$, first we need to prove that these $\xi_{j,n}$ have finite expectations. Then, since $\xi_{j,n}$ are not identically distributed (due to the presence of fading), we have to prove the second part of Kolmogorov's theorem. Therefore, we start by computing the mean and variance of $\xi_{j,n}$ for every band n to show that both are finite. Without loss of generality, we will assume that the first $\texttt{K}$ bands are the ones that are occupied and the rest are not (contain only noise). The means and variances are given by the following proposition.

Proposition 1. *Consider the* n^{th} *band. The mean of* $\xi_{j,n}$ *is*

$$\mathbb{E}(\xi_{j,n}) = \begin{cases} \displaystyle\sum_{\ell=1}^{\texttt{K}} \frac{\mathbb{E}(x_\ell^2)\texttt{M}}{\omega_\ell^2\omega_{\rm n}^2} + \sum_{\substack{\ell=\texttt{K}+1\\ \ell\neq n}}^{\texttt{N}} \frac{\texttt{N}_0\texttt{M}}{\omega_\ell^2\omega_{\rm n}^2} + \frac{\texttt{N}_0\texttt{M}^2}{\omega_{\rm n}^4}, & \text{if } n \notin \Omega \\ \displaystyle\frac{\mathbb{E}(x_n^2)\texttt{M}^2}{\omega_{\rm n}^4} + \sum_{\substack{\ell=1\\ \ell\neq n}}^{\texttt{K}} \frac{\mathbb{E}(x_\ell^2)\texttt{M}}{\omega_\ell^2\omega_{\rm n}^2} + \sum_{\ell=\texttt{K}+1}^{\texttt{N}} \frac{\texttt{N}_0\texttt{M}}{\omega_\ell^2\omega_{\rm n}^2}, & \text{if } n \in \Omega \end{cases} \quad (7)$$

and the variance of $\xi_{j,n}$*,* $\mathbb{V}ar(\xi_{j,n})$*, is given by Eq.* (14)*.*

To prove Proposition 1, we use the definitions of mean and variance and the following Lemma. However, we did not provide the complete proofs as well as the proof of the lemma due to space limitation.

Lemma 1. *Let* ψ_n *be the* n^{th} *column of the sensing matrix* Ψ *whose elements are Bernoulli with zero mean and variance* $\frac{1}{\omega_{\rm n}^2}$*. Then, we have the following results.*

$$\mathbb{E}(\langle\psi_n, \psi_\ell\rangle^2) = \frac{\texttt{M}}{\omega_{\rm n}^2\omega_\ell^2} \quad (8)$$

$$\mathbb{E}(\langle\psi_n, \psi_\ell\rangle^4) = \frac{\texttt{M}(3\texttt{M}-2)}{\omega_{\rm n}^4\omega_\ell^4} \quad (9)$$

$$\mathbb{E}(\langle\psi_n, \psi_\ell\rangle^2\langle\psi_n, \psi_p\rangle^2) = \frac{\texttt{M}^2}{\omega_{\rm n}^4\omega_{\rm p}^2\omega_\ell^2} \quad (10)$$

$$\mathbb{E}(\|\psi_\ell\|^4\langle\psi_n, \psi_\ell\rangle^2) = \frac{\texttt{M}^3}{\omega_{\rm n}^2\omega_\ell^6} \quad (11)$$

$$\mathbb{E}(\|\psi_\ell\|^4) = \frac{\texttt{M}^2}{\omega_\ell^4} \quad (12)$$

$$\mathbb{E}(\|\psi_\ell\|^8) = \frac{\texttt{M}^4}{\omega_\ell^8} \quad (13)$$

First, we need to show that both the means and the variances of $\xi_{j,n}$ for $n = 1..\texttt{N}$ are finite. It is sufficient to see that $\mathbb{E}(x_\ell^2)$ and $\mathbb{E}(x_\ell^4)$ are finite (upper bounded by the transmit power $\texttt{P}$ and $\texttt{P}^2$) since in practice $\texttt{PU}$s are sending with finite powers. Moreover, $\sum_{j=1}^{\infty} \frac{\mathbb{V}ar(\xi_{j,n})}{j^2}$ is finite (upper bounded by $\max\limits_{j} \mathbb{V}ar(\xi_{j,n}) \sum\limits_{j=1}^{\infty} \frac{1}{j^2}$) which according to Kolmogorov's theorem is sufficient to prove that ξ_n almost surely converges to the mean given by Proposition 1. Finally, we have $\frac{1}{\texttt{J}}\sum_{j=1}^{\texttt{J}} \xi_{j,n}$ converge to $\mathbb{E}(\xi_{j,n})$ for $n = 1..\texttt{N}$. To finish the proof, we only need to show that the two means are sufficiently different. Even with uniform distribution for the sensing matrix, we still have a clear distinction between the two cases. This distinction is more important with non-uniform sensing matrix. For the sake of illustration, we show in Fig. 1 the ratio between the two means: when band n is occupied and when band n is not occupied for different SNRs and different values of $\texttt{M}$. ■

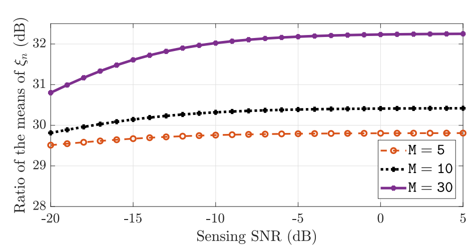

Fig. 1. The ratio between $\mathbb{E}(\xi_n)$ when n is an occupied band and when it is not as a function of the sensing SNR and for a different number of measurements in dB. $\texttt{N} = 256$, $\texttt{K} = 29$, weights in the occupied bands $\omega_{\texttt{in}} = 1/\texttt{K}$, weights in the unoccupied bands $\omega_{\texttt{out}} = 1$, $\texttt{N}_0 = -120dBm$.

$$\mathbb{V}ar(\xi_{j,n}) = \begin{cases} \sum_{\ell=1}^{K} \frac{\mathbb{E}(x_\ell^4)M(3M-2)}{\omega_\ell^4\omega_n^4} + 2\sum_{\ell=1}^{K}\sum_{\substack{m=1\\ m\neq\ell}}^{K} \frac{\mathbb{E}(x_\ell^2)\mathbb{E}(x_m^2)}{\omega_\ell^2\omega_m^2\omega_n^4} + 6\left[\sum_{\ell=1}^{K}\frac{\mathbb{E}(x_\ell^2)M}{\omega_\ell^2\omega_n^2}\right]\left[\sum_{\substack{\ell=K+1\\ \ell\neq n}}^{N}\frac{N_0M}{\omega_\ell^2\omega_n^2} + \frac{N_0M^2}{\omega_n^4}\right] & \\ + \sum_{\substack{\ell=K+1\\ \ell\neq n}}^{N} \frac{{N_0}^2M(3M-2)}{\omega_\ell^4\omega_n^4} + 2\sum_{\substack{\ell=K+1\\ \ell\neq n}}^{N}\sum_{\substack{m=K+1\\ m\neq\ell\\ m\neq n}}^{N} \frac{{N_0}^2M^2}{\omega_\ell^2\omega_m^2\omega_n^4} + 6\frac{N_0M^2}{\omega_n^4}\left[\sum_{\substack{\ell=K+1\\ \ell\neq n}}^{N}\frac{N_0M}{\omega_n^2\omega_\ell^2} + \frac{N_0M^2}{\omega_n}\right] + \frac{{N_0}^2M^4}{\omega_n^8}, & \text{if } n\notin\Omega \\ \sum_{\substack{\ell=1\\ \ell\neq n}}^{K} \frac{\mathbb{E}(x_\ell^4)M(3M-2)}{\omega_n^4\omega_\ell^4} + 2\sum_{\substack{\ell=1\\ \ell\neq n}}^{K}\sum_{\substack{p=1\\ p\neq\ell\\ p\neq n}}^{K} \frac{\mathbb{E}(x_p^2)\mathbb{E}(x_\ell^2)M^2}{\omega_n^4\omega_\ell^4\omega_p^2} + 6\left[\sum_{\substack{\ell=1\\ \ell\neq n}}^{K}\frac{\mathbb{E}(x_\ell^2)M}{\omega_n^2\omega_\ell^2} + \frac{\mathbb{E}(x_\ell^2)M^2}{\omega_n^4}\right]\left[\sum_{\ell=K+1}^{N}\frac{\mathbb{E}(x_\ell^2)M}{\omega_n^2\omega_\ell^2}\right] & \\ +4\sum_{\substack{\ell=1\\ \ell\neq n}}^{K} \frac{\mathbb{E}(x_p^2)\mathbb{E}(x_\ell^2)M^3}{\omega_n^6\omega_\ell^2} + \frac{\mathbb{E}(x_n^4)}{\omega_n^8} + \sum_{\ell=K+1}^{N}\frac{\mathbb{E}(x_\ell^4)M(3M-2)}{\omega_n^4\omega_\ell^4} + 2\sum_{\ell=K+1}^{N}\sum_{\substack{p=K+1\\ p\neq\ell}}^{N}\frac{\mathbb{E}(x_\ell^2)\mathbb{E}(x_p^2)M^2}{\omega_n^4\omega_p^2\omega_\ell^2}, & \text{if } n\in\Omega \end{cases} \quad (14)$$

C. Exploiting User Closeness

While the previous result brings forth the power of cooperation for overcoming the hardware limitation along with the hidden terminal problem, a large number of SUs is needed to do so. In this section, we show that by exploiting the closeness between SUs, the number of required SUs can be significantly reduced. To illustrate this further, consider two SUs with measurement vectors $y_1 = \Psi_1 x_1 + \eta_1$ and $y_2 = \Psi_2 x_2 + \eta_2$. When the received signals at the SUs are quite similar, say $x_2 = x_1 + \delta x$, y_2 can be rewritten as $y_2 = \Psi_2 x_1 + \eta_2 + \Psi_2\delta x$. This is equivalent to having one SU takes twice the number of measurements, i.e., $y_c = [y_1^T \;\; y_2^T]^T$, $\Psi_c = [\Psi_1^T \;\; \Psi_2^T]^T$, and $\eta = [\eta_1^T \;\; \eta_2^T + (\Psi_2\delta x)^T]^T$. With a higher number of measurements, conventional recovery approaches such as LASSO [13] and OMP [14] can be used. Clearly, as the two received signals at the SUs start to differ, it corresponds to the case of having higher noise variance, which yields a worse recovery. This approach will be evaluated in Section V.

D. Non-uniform Sensing Matrix Design

So far we discussed how cooperation could be exploited to overcome the hardware limitation and the hidden terminal problem. We now propose an efficient design of the sensing matrices that leverages prior knowledge about the spectrum occupancy to improve the recovery accuracy. We show that capturing and exploiting the heterogeneity in spectrum occupancy, which is inherent to wideband spectrum access, in the sensing matrix can indeed yield a comparable performance gain to $\mathscr{P}_{\text{wLASSO}}$. Recalling $\mathscr{P}_{\text{wLASSO}}$ given in Eq. (5) and letting $p = Wz$ where $W = \text{diag}(\underbrace{\omega_1, \cdots, \omega_1}_{N_1}, \underbrace{\omega_2, \cdots, \omega_2}_{N_2}, \cdots, \underbrace{\omega_g, \cdots, \omega_g}_{N_g})$, $\mathscr{P}_{\text{wLASSO}}$ could also be reformulated as

$$\mathscr{P}_{\text{wSensing}} : \min_{p} \|p\|_{\ell_1} \quad \text{s.t.} \quad \|\Psi W^{-1}p - y\|_{\ell_2} \leq \epsilon \qquad (15)$$

The new matrix W^{-1} magnifies the columns of the sensing matrix Ψ that correspond to high average sparsity levels (low weights), and diminishes the columns that correspond to low average sparsity levels. By doing so, the sensing energy is better allocated, and more importantly, the error achievable under Algorithm 1 is reduced. Fig. 2 shows the equivalence in terms of performance between the two formulations. To ensure fair comparison under the two scenarios, the elements in the sensing matrix Ψ have variance $\frac{\beta}{M}$, with $\beta = \frac{N}{(M\sum_{j=1}^{N}\frac{1}{\omega_j^2})}$. To avoid confusion, we set $\omega_i = \omega_i/\sqrt{\beta}$. The figure also shows that the new formulation is more robust to noise (better performance at low sensing SNR, with SNR defined as $\frac{\|\Psi x\|^2}{\|\eta\|^2}$).

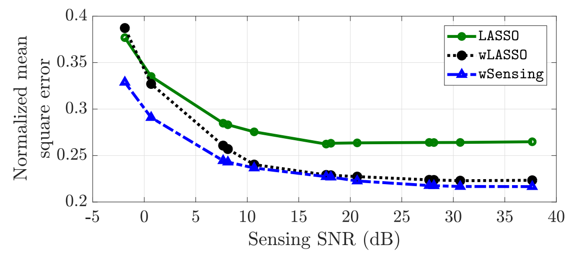

Fig. 2. Recovery performance under nonuniform sensing matrix and weighted recovery using same parameters as in [16]. ($N = 256$, $M = 27$)

V. Performance Evaluation Results

Consider a primary system operating over a wideband consisting of $N = 128$ bands grouped into $g = 4$ blocks with equal sizes. The average probabilities of occupancy in each block are as follows: $\bar{K}_1 = p_1 \times 32$, $\bar{K}_2 = p_2 \times 32$, $\bar{K}_3 = p_3 \times 32$, $\bar{K}_4 = p_4 \times 32$, where $p_1 = p_3 = 0.1$ and $p_2 = p_4 = 0.001$. The PUs are randomly deployed in a cell and for simplicity, we assume that the number of active PUs are equal to the number of occupied bands. We assume all PUs are transmitting with constant power $P = 10$ W, and the received signal in each band is affected by a Rayleigh distributed channel impulse response with mean $1/d^{\alpha/2}$. We also consider Gaussian noise, with each band experiencing Gaussian signal with zero mean and variance $N_0 = -120dBm$.

In Fig. 3, we plot the detection probability as a function of the number of cooperating SUs, J. First, we observe that as the number of cooperating SUs increases, a high detection probability is achieved regardless of the number of measurements each SU is taking, thus confirming our main theorem result. This is mainly because as J increases, $\xi_{j,n}$ converges to its expectation $\mathbb{E}(\xi_{j,n})$, and hence, a better distinction between the bands is achieved. Second, we also observe that for a fixed J, a high detection probability is achieved when each SU is talking a higher number of measurements.

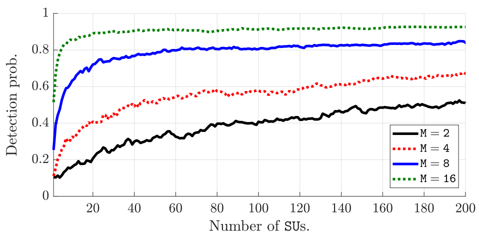

Fig. 3. The detection probability for $\mathtt{M} = 8$ and $N = 128$.

To overcome the need for high numbers of SUs, we investigate the effect of considering only a subset of close-by SUs when performing detection using OMP and LASSO, and compare that to the previous approach. Fig. 4 shows that when considering close-by SUs (6 SUs), the achieved detection probability is close to the one achieved with a high number of SUs, which confirms our observation. Second, our proposed approaches outperform sequential sensing approach proposed in [17], mainly because of their ability to overcome the hidden terminal problem.

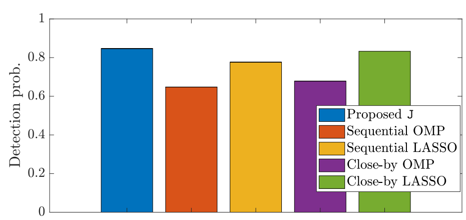

Fig. 4. The detection probability for $\mathtt{M} = 8$ and $\mathtt{N} = 128$.

VI. Conclusions

We leverage user cooperation to overcome receiver hardware limitations as well as time variability of band occupancy during wideband spectrum sensing. We show that cooperation overcomes these issues by enabling distributed compressive sampling-based spectrum sensing, and does so by requiring smaller numbers of measurements by each user only. Also, we consider heterogenous wideband spectrum access environment and design efficient non-uniform sensing matrices suitable for such an environment. Finally, we show that when the impact of fading is not so significant (for instance by considering close-by SUs), comparable performance can still be achieved from a smaller number of SUs.

Acknowledgment

This work was supported in part by the US National Science Foundation under NSF award CNS-1162296.

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