SEP $= 10^{-3}$, the gain, in decibels, achieved by using $N_r = 4$ receive antennas $(\text{SNR} = -2.7\ \text{dB})$ instead of $N_r = 2$ antennas $(\text{SNR} = 1.4\ \text{dB})$ is about 4.1 dB. This gain is limited to 3.7 dB when $N_r = 3$ receive antennas are employed. Also, it is clear that the analytical results provide a tight lower bound on the SEP for the overall SNR range. These plots demonstrate the effects of N_r on the diversity order. This can be justified by the array gain that is obtained by the use of multiple antennas and the distributed array gain that is achieved by the use of both multiple antennas and the relay node.

From Fig. 5, one can see that an increase in the constellation size M affects the system's error performance. It can be seen that a transition from $M = 16$ to $M = 4$ leads to a performance improvement close to 7.3 dB for SEP $= 10^{-3}$. This is related to the decrease in the Euclidean distance between signals due to the increase in the constellation size in 16-QAM compared with that in 4-QAM. It is clear that the proposed lower bound and the simulation results on the SEP are in excellent agreement. We can also notice that the tightness of the derived lower bound of the SEP improves as the SNR increases; however, this bound loses its tightness at a low SNR.

VII. Conclusion

In this paper, we have used an upper bound for the SNR to analyze the SEP of the MIMO relay channel based on Alamouti STBC transmission. The analysis is achieved using the MGF of the SNR of the cooperative links measured at the destination. A lower bound expression of the SEP is given for MRC M-QAM in Rayleigh flat-fading channels. The validity of our analytical results is confirmed by Monte Carlo simulations. Throughout this paper, we have demonstrated the effectiveness of the lower bound analysis for a high SNR range. Considering the limitation of the analysis for the high-SNR regime, the exact closed-form expression of the SEP with an MRC receiver needs to be studied and compared with STBC MIMO cooperative systems based on the ML receiver.

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Quasi-Synchronous CDMA Using Properly Scrambled Walsh Codes as User-Spreading Sequences

Kwonhue Choi, *Member, IEEE*, and
Huaping Liu, *Senior Member, IEEE*

***Abstract*—An orthogonal spreading code family based on scrambled Walsh–Hadamard (WH) sequences is proposed for quasi-synchronous (QS) code-division multiple-access (CDMA) systems. With ± 1 chip offset, either the top or the bottom half rows of the WH matrix remain orthogonal if they are scrambled using the proposed scrambling patterns. With quadriphase scrambling applied on a quarter row of the WH matrix, the zero-correlation zone (ZCZ) is extended to ± 3 chips. The proposed spreading codes satisfy the theoretical limit of code size for a given code length and ZCZ. Compared with existing spreading codes for QS-CDMA, the proposed spreading codes have several advantages: 1) A special procedure for code family construction is not needed; it only requires a simple modification to the scrambling sequence of the conventional CDMA, which employs WH sequences as user signatures. 2) Flexible code family size; the size can easily be extended for multicell applications without having to consider existing codes used. Moreover, if synchronization is guaranteed to be within ± 1 chip, the code size can be doubled by allocating another quarter member of the WH sequence set. 3) The benefits of long code scrambling from the conventional cellular CDMA systems are inherited, such as robustness against interception and multiaccess interference randomization.**

***Index Terms*—Cellular systems, intercell interference (ICI), orthogonal spreading codes, quasi-synchronous (QS) code-division multiple access (CDMA), scrambling, zero-correlation zone (ZCZ).**

Manuscript received December 2, 2009; revised February 27, 2010 and May 4, 2010; accepted May 10, 2010. Date of publication May 20, 2010; date of current version September 17, 2010. This work was supported by the Ministry of Knowledge Economy, Korea, under the Information Technology Research Center support program supervised by the National IT Industry Promotion Agency under Grant NIPA-2010-C1090-1031-0009. The review of this paper was coordinated by Prof. N. Arumugam.

K. Choi is with the School of Electrical Engineering and Computer Science, Yeungnam University, Gyeongsan 712-749, Korea (e-mail: gonew@yu.ac.kr).

H. Liu is with the School of Electrical Engineering and Computer Science, Oregon State University, Corvallis, OR 97331 USA (e-mail: hliu@eecs.oregonstate.edu).

Digital Object Identifier 10.1109/TVT.2010.2050916

I. Introduction

In third-generation-and-beyond wideband code-division multiple-access (CDMA) systems, synchronization among users on the reverse link is highly desirable to reduce multiple-access interference. Since the chip duration could be increased by employing multicarrier CDMA, potentially with multiple-input–multiple-output antennas or higher order modulation, it is feasible to control the uplink access timing. However, because the signals of users on the reverse link experience different propagation delays and timing adjustments would be imperfect in practice, a slight timing misalignment among users is inevitable. Therefore, it is crucial to design good user-spreading codes that maintain a low correlation in the presence of limited timing offsets.

In [1], generalized orthogonality/generalized quasi-orthogonality codes based on complementary sequence mate for quasi-synchronous (QS) CDMA systems are proposed. The theoretical bounds on the code size and the zero-correlation zone (ZCZ) width are also derived and given in [1]. In [2]–[4], large-area (LA), loosely synchronous (LS), and LA synchronous codes are proposed for QS-CDMA systems. These are ternary codes that require zero insertions in the sequences. Zero insertion in the middle of the chip stream breaks signal continuity and could cause undesired transient responses in the receiver filter output. In addition, the intercode (intersequence set) correlation properties of these codes have not been optimized to achieve low intercell interference (ICI) in cellular QS-CDMA systems.

Systematic construction of spreading sequence design for QS-CDMA that considers ICI is investigated in [5], where new families of generalized LS (GLS) codes that do not have perfectly zero but low intercode cross correlations within a certain window are proposed. As in [1]–[4], the codes in [5] also exclude long code scrambling in the spreading process since scrambling on top of these codes will change their correlation properties. Except for zero timing offset, the correlation properties of the scrambled codes become different from that of the original codes [6], [7]. This is a shortcoming for cellular systems since long code scrambling is essentially required for low probability of interception (LPI) [8]–[10] and for ICI randomization. Due to the lack of long code scrambling, every data symbol is spread with the same pattern of the spreading sequence. Consequently, the spreading sequence can easily be intercepted with a fairly low number of attacking trials [10]. More recently, there has been significant research interest on designing new codes with improved properties (e.g., bound-achieving ZCZ or more flexible code parameters) for QS-CDMA [11]–[14]. These efforts focus on intracode (intrasequence set) correlation properties, and optimality in multicell environments is not studied. In addition, these codes are also deterministic, because randomization (e.g., using long code scrambling) on top of the codes is not allowed.

The main objective of this research is to design spreading codes that are suitable for QS-CDMA with good intra- and intercode cross-correlation properties without sacrificing the long code-scrambling effect, which was not maintained in existing QS-CDMA spreading codes [1]–[5], [11]–[17]. The proposed spreading codes are obtained by scrambling Walsh–Hadamard (WH) sequences. Our previous work in [18] has proposed a basic idea of scrambled WH sequences for QS-CDMA. The work is limited to a single code (a sequence set) design. Since it is not a code family (multiple sequence sets) design, the intercode correlation property has not been analyzed. In addition, the optimality and feasibility of the code for multicell environments have not been studied. Furthermore, ZCZ is limited to ± 1 chip by using a real-valued scrambling pattern.

In this paper, we develop a systematic approach to derive complex-valued scrambling patterns that achieve a ZCZ of ± 3 chips. We analyze the intercode correlation property and evaluate the performance of the codes in multicell fading environments. Our design approach inherits the spreading operation and structure employed in conventional CDMA systems. Consequently, with slight modification, the spreading codes can easily be implemented in a conventional CDMA structure. The resulting spreading codes achieve the theoretical bound of ZCZ in terms of intra- and intercode cross correlations. Compared with conventional QS-CDMA spreading codes, the proposed spreading code family has several advantages due to the scrambling effect, whereas its bit error rate (BER) performance is the same as the existing QS-CDMA code family designed for minimum ICI [5].

This paper is organized as follows: In Section II, we derive scrambling patterns wherein when half of the WH sequence set is scrambled by the proposed scrambling pattern, the scrambled sequences remain orthogonal, even with ± 1 chip of relative shift; when the scrambling pattern is applied to a quarter of the WH sequence set, the scrambled sequences remain orthogonal, even with up to ± 3 chips of relative timing shift. In Section III, we derive the analytical expressions of intracell interference and ICI. In Section IV, we evaluate the BER performance of the proposed QS-CDMA system in a multicell environment. Comparison of BER performance is made between the proposed scheme and the QS-CDMA that employs low ICI GLS code family recently proposed in [5].

II. Derivation of Scrambling Patterns

A. Cross Correlation of Scrambled WH Sequences

We consider real-valued WH sequences as the spreading sequences and design complex-valued scrambling sequences.[1] Let $\mathbf{w}^{(l)} = [w_0^{(l)}, w_1^{(l)}, \ldots, w_{N-1}^{(l)}]$ be the lth row of an N-dimensional WH matrix and $\mathbf{s} = [s_0, s_1, \ldots, s_{N-1}]$ be a scrambling sequence. If we scramble all rows of the WH matrix by $\mathbf{s}$ to form a code $\mathbf{C}$, then the lth sequence of $\mathbf{C}$, i.e., $\mathbf{c}^{(l)}$ (the lth row of the scrambled WH matrix), is expressed as

$$\mathbf{c}^{(l)} = \left[s_0 w_0^{(l)}, s_1 w_1^{(l)}, s_2 w_2^{(l)}, \ldots, s_{N-1} w_{N-1}^{(l)}\right]. \tag{1}$$

When there is a d-chip timing offset between $\mathbf{c}^{(l)}$ and $\mathbf{c}^{(k)}$, the periodic cross correlation $R_{l,k}(d)$ is calculated as

$$\begin{aligned} R_{l,k}(d) &= \sum_{n=0}^{N-1} c_n^{(l)} c_{n+d}^{(k)*} = \sum_{n=0}^{N-1} s_n w_n^{(l)} s_{n+d}^{*} w_{n+d}^{(k)*} \\ &= \sum_{n=0}^{N-1} s_n s_{n+d}^{*} w_n^{(l)} w_{n+d}^{(k)} \end{aligned} \tag{2}$$

where the subscript $+$ denotes modulo-N addition, and $*$ denotes complex conjugate. It can easily be shown that $R_{l,k}(0)$ is equal to 1 for $l = k$ and equal to 0 otherwise since $s_n s_n^* = 1$, and $\sum_{n=0}^{N-1} w_n^{(l)} w_n^{(k)}$ is equal to 1 for $l = k$ and equal to 0 otherwise. Therefore, the scrambled WH sequences are mutually orthogonal for a timing offset of zero chips $(d = 0)$, regardless of the scrambling pattern. When $(d \neq 0)$, the product term $s_n s_{n+d}^*$ is not constant during the correlation period. Consequently, $R_{l,k}(d)$ is different from the case without scrambling, i.e., scrambling changes the correlation of the original sequences, except for in-phase correlation.

[1] In Section II-B, we show that we can make the real-valued scrambling sequences for ZCZ $= 2$.

B. Scrambling Pattern for $R_{l,k}(d) = 0$ With $ZCZ = 2$ $(d = -1, 0, 1)$

Note that ZCZ is defined as the one-sided length of the ZCZ of the correlation function, i.e., $\text{ZCZ} = d_{\max} + 1$ if $R(d) = 0\ \forall d \in \{-d_{\max}, -d_{\max}+1, -d_{\max}+2, \ldots, 0, \ldots, d_{\max}\}$ [13]. For convenience of the derivation, let $L \equiv \{0, 1, \ldots, (N/2) - 1\}$ and $U \equiv \{N/2, N/2+1, \ldots, N-1\}$ represent the indexes of the WH matrix, so that $\{\mathbf{w}^{(k)} | k \in L\}$ and $\{\mathbf{w}^{(k)} | k \in U\}$ represent the bottom half and the top half of the WH matrix, respectively. Note that WH sequences satisfy the following relation:

$$w^{(l)}_{\frac{N}{2}+n} w^{(k)}_{\frac{N}{2}+n+d} = \begin{cases} w_n^{(l)} w_{n+d}^{(k)}, & (l,k) \in (L,L) \text{ or } (U,U) \\ -w_n^{(l)} w_{n+d}^{(k)}, & (l,k) \in (U,L) \text{ or } (L,U) \end{cases} \quad \text{for } 0 \le n \le N/2 - 1. \tag{3}$$

Let us focus on the case of $(l,k) \in (L,L) \cup (U,U)$, i.e., $\mathbf{w}^{(l)}$ and $\mathbf{w}^{(k)}$ belong to the same subgroup. By substituting (3) into (2), we rewrite $R_{l,k}(d)$ as

$$R_{l,k}(d)|_{(l,k)\in(L,L)\cup(U,U)} = \sum_{n=0}^{\frac{N}{2}-1} \left(s_n s^*_{n+d} + s_{\frac{N}{2}+n} s^*_{\frac{N}{2}+n+d} \right) w_n^{(l)} w_{n+d}^{(k)}. \tag{4}$$

In order to maintain $R_{l,k}(d) = 0$, the scrambling pattern must satisfy $s_n s^*_{n+d} + s_{N/2+n} s^*_{N/2+n+d} = 0$ for $0 \le n \le N/2 - 1$, which is rewritten as

$$s_n / s_{\frac{N}{2}+n} = -s^*_{\frac{N}{2}+n+d} / s^*_{n+d}, \qquad 0 \le n \le N/2 - 1. \tag{5}$$

First, we consider the case of a one-chip timing offset, i.e., $d = 1$. We rewrite (5) as $s_0/s_{N/2} = -s^*_{N/2+1} s^*_1 = -s_1/s_{N/2+1}$, where we have assumed that the element of the scrambling sequence has a unit magnitude, i.e., $s_n^* s_n = 1\ \forall n$. Similarly, we have $s_1/s_{N/2+1} = -s_2/s_{N/2+2}$ and $s_2/s_{N/2+2} = -s_3/s_{N/2+3}, \ldots, s_{N/2-1}/s_{N-1} = -s_{N/2}/s_0$. These conditions are written as

$$s_0/s_{\frac{N}{2}} = -s_1/s_{\frac{N}{2}+1} = s_2/s_{\frac{N}{2}+2} = \cdots = -s_{\frac{N}{2}-1}/s_{N-1} = s_{\frac{N}{2}}/s_0. \tag{6}$$

Following the same procedure, we show that (6) holds for all cases when d is an odd integer. The condition (6) is equivalent to the following two general solutions:

$$s_{\frac{N}{2}+n} = (-1)^n s_n \text{ or } -(-1)^n s_n, \quad 0 \le n \le N/2 - 1 \tag{7}$$

which can be rewritten as $[s_{N/2}, s_{N/2+1}, s_{N/2+2}, \ldots, s_{N-1}] = \pm[s_0, -s_1, s_2, \ldots, -s_{N/2-1}]$. From (7), we note that there is no constraint on the first half of the scrambling pattern, and the second half is simply a copy of the first half, with the sign of every other symbol alternated. Therefore, there are $2^{N/2+1}$ binary scrambling patterns and 2^{N+1} quadriphase scrambling patterns that satisfy (7).

By scrambling the WH sequences by one of the patterns that satisfy (7), we can ensure $R_{l,k}(d) = 0$ for $d = 0, \pm 1, \pm 3, \pm 5, \ldots$ if $\mathbf{w}^{(l)}$ and $\mathbf{w}^{(k)}$ are from the same half subset of the WH sequence set. Therefore, we can obtain various codes of $\text{ZCZ} = 2(d = -1, 0, +1)$, length N, and code size $M = N/2$. Note that these codes satisfy the theoretical code size upper bound M_{UB} for a given code length N and ZCZ, i.e., $M = M_{UB} = \lfloor N/\text{ZCZ} \rfloor$ [13, eq. (15)].

C. Scrambling Pattern for $R_{l,k}(d) = 0$ With $ZCZ = 4$ $(d = -3, -2, -1, 0, 1, 2, 3)$

Since the condition in (7) ensures $R_{l,k}(-3, -1, 0, 1, 3) = 0$, ZCZ may be extended to 4 if we can find additional constraints on the scramble patterns that ensure $R_{l,k}(d) = 0$ at $d = \pm 2$. For $\text{ZCZ} = 4$, the maximum code size will be $N/4$ by the theoretical bound. We derive quadriphase scrambling patterns that achieve $R_{l,k}(d) = 0$, with $d = \pm 2$ for a quarter (i.e., $N/4$) of the members of the WH code.

We first consider scrambling patterns for the first quarter subset of the WH sequence set, i.e., $\{\mathbf{w}^{(l)} | l \in Q_1\}$, where Q_i represents the indexes of the ith quarter rows of the WH matrix and is expressed as $Q_i = \{(i-1)N/4 + 1, (i-1)N/4 + 2, (i-1)N/4 + 3, \ldots, (i-1)N/4 + N/4\}$, $i = 1, 2, 3, 4$. From the WH matrix structure, the first quarter subset of the WH sequence set satisfies the following relation: $w^{(l)}_{kN/4+n} = w^{(l)}_n$ for $k = 1, 2, 3$, and $0 \le n \le N/4 - 1$. Thus, $w^{(l)}_{kN/4+n} w^{(k)}_{kN/4+n+2} = w^{(l)}_n w^{(k)}_{n+2}$ for $l = 0, 1, \ldots, N/4 - 1$ and $k = 0, 1, \ldots, N/4 - 1$. Substituting this into (2), we rewrite $R_{l,k}(2)$ as

$$R_{l,k}(2) = 2 \sum_{n=0}^{\frac{N}{4}-1} \left(s_n s^*_{n+2} + s_{\frac{N}{4}+n} s^*_{\frac{N}{4}+n+2} \right) w_n^{(l)} w_{n+2}^{(k)} \tag{8}$$

where we have used the identity $s_{N/2+n} s^*_{N/2+n+2} = s_n s^*_{n+2}$, because s_n should satisfy (7) for $R_{l,k}(\pm 1) = 0$.

In order to ensure that $R_{l,k}(2) = 0$, the scrambling pattern should satisfy

$$s_n s^*_{n+2} + s_{\frac{N}{4}+n} s^*_{\frac{N}{4}+n+2} = 0 \qquad \forall n. \tag{9}$$

By applying $s_n s_n^* = 1\ \forall n$, we express (9) as $s_n/s_{n+2} = -s_{N/4+n}/s_{N/4+n+2}\ \forall n$, which is rewritten as

$$\frac{s_0}{s_{\frac{N}{4}}} = -\frac{s_2}{s_{\frac{N}{4}+2}} = \frac{s_4}{s_{\frac{N}{4}+4}} = -\frac{s_6}{s_{\frac{N}{4}+6}} \cdots = \frac{s_{\frac{N}{4}}}{s_{\frac{N}{2}}} \tag{10a}$$

$$\frac{s_1}{s_{\frac{N}{4}+1}} = -\frac{s_3}{s_{\frac{N}{4}+3}} = \frac{s_5}{s_{\frac{N}{4}+5}} = -\frac{s_7}{s_{\frac{N}{4}+7}} \cdots = \frac{s_{\frac{N}{4}+1}}{s_{\frac{N}{2}+1}}. \tag{10b}$$

In further derivation, we consider the two conditions given in (7). Let us first consider the case of $s_{N/2+n} = -(-1)^n s_n$. Substituting $s_{N/2} = -s_0$ into (10a), we have $s_0/s_{N/4} = s_{N/4}/-s_0$, which results in

$$\frac{s_0}{s_{\frac{N}{4}}} = \pm j. \tag{11}$$

From (10a) and (11), we determine two solutions for the even-indexed chips in the second quarter of the scrambling pattern

$$\left[s_{\frac{N}{4}}, s_{\frac{N}{4}+2}, s_{\frac{N}{4}+4}, s_{\frac{N}{4}+6}, \ldots, s_{\frac{N}{2}-2} \right] = \pm j \left[s_0, -s_2, s_4, -s_6, \ldots, -s_{\frac{N}{4}-2} \right]. \tag{12}$$

Substituting $s_{N/2+1} = s_1$ into (10b) and following a similar procedure, we have the solutions for the odd-indexed chips in the second quarter of the scrambling pattern as

$$\left[s_{\frac{N}{4}+1}, s_{\frac{N}{4}+3}, s_{\frac{N}{4}+5}, s_{\frac{N}{4}+7}, \ldots, s_{\frac{N}{2}-1} \right] = \pm \left[s_1, -s_3, s_5, -s_7, \ldots, -s_{\frac{N}{4}-1} \right]. \tag{13}$$

TABLE I
CONDITIONS ON THE SCRAMBLING PATTERNS TO ACHIEVE ZCZ $= 4$ FOR ALL FOUR QUARTER SUBSETS OF THE WH SEQUENCE SET
($\odot$ DENOTES ELEMENT-BY-ELEMENT MULTIPLICATION)

Code (sequence set) with ZCZ=4	Conditions for scrambling pattern $\mathbf{s}$		
	1st quarter of $\mathbf{s}$ $s_{0:(\frac{N}{4}-1)}$	2nd quarter of $\mathbf{s}$ $s_{\frac{N}{4}:(\frac{N}{2}-1)}$	2nd half of $\mathbf{s}$ $s_{\frac{N}{2}:(N-1)}$
$\{\mathbf{s}\odot\mathbf{w}^{(l)}\vert l\in Q_1\}$ or $\{\mathbf{s}\odot\mathbf{w}^{(l)}\vert l\in Q_2\}$	No constraint	$s_{0:(\frac{N}{4}-1)}\odot\pm[1,j,-1,-j,1,j,-1,-j,\cdots]$ or $s_{0:(\frac{N}{4}-1)}\odot\pm[1,-j,-1,j,1,-j,-1,j,\cdots]$	$s_{0:(\frac{N}{2}-1)}\odot[1,-1,1,-1,\cdots]$
	No constraint	$s_{0:(\frac{N}{4}-1)}\odot\pm[j,-1,-j,1,j,-1,-j,1,\cdots]$ or $s_{0:(\frac{N}{4}-1)}\odot\pm[j,1,-j,-1,j,1,-j,-1,\cdots]$	$s_{0:(\frac{N}{2}-1)}\odot[-1,1,-1,1,\cdots]$
$\{\mathbf{s}\odot\mathbf{w}^{(l)}\vert l\in Q_3\}$ or $\{\mathbf{s}\odot\mathbf{w}^{(l)}\vert l\in Q_4\}$	No constraint	$s_{0:(\frac{N}{4}-1)}\odot\pm[1,j,-1,-j,1,j,-1,-j,\cdots]$ or $s_{0:(\frac{N}{4}-1)}\odot\pm[1,-j,-1,j,1,-j,-1,j,\cdots]$	$s_{0:(\frac{N}{2}-1)}\odot[-1,1,-1,1,\cdots]$
	No constraint	$s_{0:(\frac{N}{4}-1)}\odot\pm[j,-1,-j,1,j,-1,-j,1,\cdots]$ or $s_{0:(\frac{N}{4}-1)}\odot\pm[j,1,-j,-1,j,1,-j,-1,\cdots]$	$s_{0:(\frac{N}{2}-1)}\odot[1,-1,1,-1,\cdots]$

Similarly, for the other case of $s_{N/2+n}=(-1)^n s_n$, we can show that the chips in the second quarter of the scrambling pattern must satisfy the following conditions:

$$\begin{aligned}&\left[s_{\frac{N}{4}}, s_{\frac{N}{4}+2}, s_{\frac{N}{4}+4}, s_{\frac{N}{4}+6},\ldots,s_{\frac{N}{2}-2}\right]\\&\quad=\pm\left[s_0,-s_2,s_4,-s_6,\ldots,-s_{\frac{N}{4}-2}\right]\end{aligned}\tag{14a}$$

$$\begin{aligned}&\left[s_{\frac{N}{4}+1}, s_{\frac{N}{4}+3}, s_{\frac{N}{4}+5}, s_{\frac{N}{4}+7},\ldots,s_{\frac{N}{2}-1}\right]\\&\quad=\pm j\left[s_1,-s_3,s_5,-s_7,\ldots,-s_{\frac{N}{4}-1}\right].\end{aligned}\tag{14b}$$

For the remaining quarter subsets of the WH sequence set, we can follow the same procedure to derive the conditions for the scrambling patterns for ZCZ $= 4$. The results for all four quarter subsets of the WH sequence set are given in Table I.

D. Intercode Correlation Performance

In the proposed QS-CDMA system, each cell/sector uses different long codes to generate the scrambling patterns that satisfy the conditions given in Table I. Fig. 1 shows an example of the spreading process for the kth user in the pth cell, where the first condition in Table I is applied to generate the scrambling pattern for the first quarter subset of the WH sequence set. In a multicell/multisector environment, three rules should be applied in generating the spreading sequence.

1) The same quarter subset of the WH sequence set is employed at every cell/sector as user signature sequences.
2) At each data symbol transmission period, chips in the first quarter of the scrambling sequence come from a long code generator, which will be called the "base scrambling code" in the sequel.
3) For the remaining three quarters of the scrambling sequence, during each symbol interval, the generation pattern that satisfies the conditions given in Table I is employed in every cell, but the base scrambling codes are different for different cells.

In order to analyze the intercode cross correlation, we select two codes (sequence sets) that are obtained by scrambling the first quarter subset of the WH sequence set with scrambling sequences $\mathbf{s}^{(\mathbf{A})}=[s_0^{(A)},s_1^{(A)},\ldots,s_{N-1}^{(A)}]$ and $\mathbf{s}^{(\mathbf{B})}=[s_0^{(B)},s_1^{(B)},\ldots,s_{N-1}^{(B)}]$. The chips in the first quarter of the scrambling sequences $s_{0:N/4-1}^{(A)}$ and $s_{0:N/4-1}^{(B)}$ are the outputs of different long code generators. For the chips of the remaining three quarters of the scrambling patterns, one of the conditions for the first quarter subset of the WH sequence set in Table I is applied.

The cross correlation between the lth WH sequence scrambled by $\mathbf{s}^{(A)}$ and the kth WH sequence scrambled by $\mathbf{s}^{(B)}$ is calculated as

$$R_{l,k}\left(d\middle|\left(\mathbf{s}^{(A)},\mathbf{s}^{(B)}\right)\right)=\sum_{n=0}^{N-1}s_n^{(A)}w_n^{(l)}\left(s_{n+d}^{(B)}w_{n+d}^{(k)}\right)^*.\tag{15}$$

Let us first investigate the in-phase correlation $R_{l,k}(0|(\mathbf{s}^{(A)},\mathbf{s}^{(B)}))$, which is given as $\sum_{n=0}^{N-1}(s_n^{(A)}s_n^{(B)*})\times w_n^{(l)}w_n^{(k)}$. We denote $\odot$ as element-by-element multiplication. From the conditions for the scrambling patterns in Table I, the four quarters of $s_{0:(N-1)}^{(A)}\odot s_{0:(N-1)}^{(B)*}$ are identical. From the WH matrix pattern, the four quarters of $w_{0:(N-1)}^{(l)}\odot w_{0:(N-1)}^{(k)}$ are identical. Thus, $R_{l,k}(0|(\mathbf{s}^{(A)},\mathbf{s}^{(B)}))$ is equal to four times the sum of $(s_n^{(A)}s_n^{(B)*})w_n^{(l)}w_n^{(k)}$, $n=0,1,\ldots,N/4-1$, i.e.,

$$R_{l,k}\left(0\middle|\left(\mathbf{s}^{(A)},\mathbf{s}^{(B)}\right)\right)=4\sum_{n=0}^{\frac{N}{4}-1}\left(s_n^{(A)}s_n^{(B)*}\right)w_n^{(l)}w_n^{(k)}.\tag{16}$$

Similarly to (4), (15) is calculated as $R_{l,k}(d|(\mathbf{s}^{(A)},\mathbf{s}^{(B)}))=\sum_{n=0}^{N/2-1}(s_n^{(A)}s_{n+d}^{(B)*}+s_{N/2+n}^{(A)}s_{N/2+n+d}^{(B)*})w_n^{(l)}w_{n+d}^{(k)}$. The second half of $\mathbf{s}^{(A)}$ and $\mathbf{s}^{(B)}$ is generated by applying the condition (7). If $s_{N/2+n}=(-1)^n s_n$ is applied and d is an odd integer, we have $s_n^{(A)}s_{n+d}^{(B)*}+s_{N/2+n}^{(A)}s_{N/2+n+d}^{(B)*}=s_n^{(A)}s_{n+d}^{(B)*}+(-1)^n s_n^{(A)}(-1)^{n+d}s_{n+d}^{(B)*}=0$. If $s_{N/2+n}=-(-1)^n s_n$ is applied and d is an odd integer, we have $s_n^{(A)}s_{n+d}^{(B)*}+s_{N/2+n}^{(A)}s_{N/2+n+d}^{(B)*}=s_n^{(A)}s_{n+d}^{(B)*}+(-(-1)^n)s_n^{(A)}(-(-1)^{n+d})s_{n+d}^{(B)*}=0$. Therefore, $R_{l,k}(d|(\mathbf{s}^{(A)},\mathbf{s}^{(B)}))=0$ for $d=\pm1,\pm3,\ldots$.

Now, let us consider the case when $d=2$. From (8), we write $R_{l,k}(2|(\mathbf{s}^{(A)},\mathbf{s}^{(B)}))=2\sum_{n=0}^{N/4-1}(s_n^{(A)}s_{n+2}^{(B)*}+s_{N/4+n}^{(A)}s_{N/4+n+2}^{(B)*})w_n^{(l)}w_{n+2}^{(k)}$. For all eight generation methods for the chips in the second quarter of the scrambling patterns given in (12)–(14b), we can easily show that $(s_n^{(A)}s_{n+2}^{(B)*}+s_{N/4+n}^{(A)}s_{N/4+n+2}^{(B)*})=0$.

Therefore, the intercode cross correlation has a ZCZ equal to 4, except for the case of perfect synchronization (i.e., $d=0$). In [5], various families of GLS codes of size 32 and length 131 are provided. The BER performance in a multicell environment is evaluated by choosing the best code family, i.e., the so-called "Type-II" family of eight GLS codes, in terms of intercode correlation and code family size in a network with the six interfering cells. Let us call this code

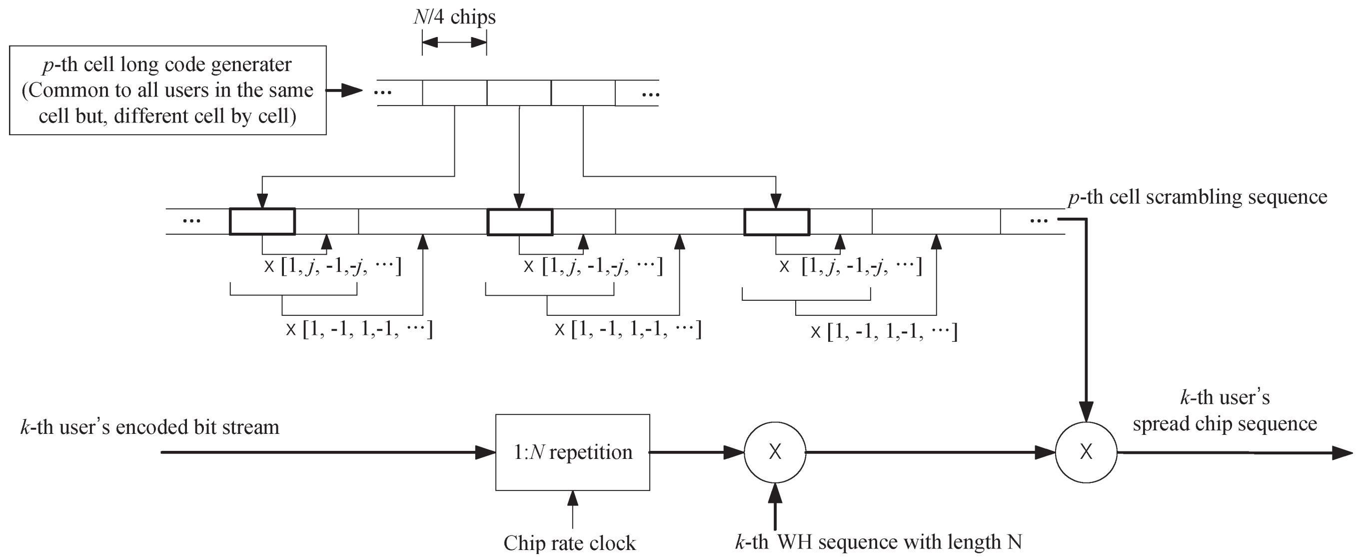

Fig. 1. Spreading process for the kth user in the pth cell of the proposed QS-CDMA system uplink.

family "Type-II (131, 32) GLS" code family in short in the sequel. For comparison with the Type-II (131, 32) GLS code family, we consider the proposed code family of length $N = 128$. We note that our proposed codes have almost identical intercode cross-correlation properties in that we have the following.

1) Property 1: Code size $= 32 (= N/4)$.
2) Property 2: $R_{l,k}(d|(\mathbf{s}^{(A)}, \mathbf{s}^{(B)})) = 0$ for $|d| \leq 3$, except for $d = 0$.[2]
3) Property 3: The mean-square value of $R_{l,k}(d|(\mathbf{s}^{(A)}, \mathbf{s}^{(B)}))$ for $|d| \leq 3$ is 73.14, which is obtained as

$$\begin{aligned}\mathbf{E}&\left[\left|R_{l,k}\left(d|\left(\mathbf{s}^{(A)},\mathbf{s}^{(B)}\right)\right)\right|^2\right]\\ &=\frac{1}{7}\sum_{d=-3}^{3}\mathbf{E}\left[\left|R_{l,k}\left(d|\left(\mathbf{s}^{(A)},\mathbf{s}^{(B)}\right)\right)\right|^2\right]\\ &=\frac{1}{7}\mathbf{E}\left[\left|R_{l,k}\left(0|\left(\mathbf{s}^{(A)},\mathbf{s}^{(B)}\right)\right)\right|^2\right]\\ &=\frac{1}{7}\mathbf{E}\left[\left|4\sum_{n=0}^{\frac{N}{4}-1}\left(s_n^{(A)}s_n^{(B)*}\right)w_n^{(l)}w_n^{(k)}\right|^2\right]\\ &=\frac{16}{7}\sum_{n=0}^{\frac{N}{4}-1}\mathbf{E}\left[\left|\left(s_n^{(A)}s_n^{(B)*}\right)w_n^{(l)}w_n^{(k)}\right|^2\right]\\ &=\frac{16}{7}\frac{N}{4}(= 73.14 \text{ for } N=128)\end{aligned}$$

which is identical to that of the Type-II (131, 32) GLS code in [5] and is four sevenths of the corresponding value of randomly scrambled WH sequences (which is equal to N).

However, the proposed spreading codes have the following advantages for multicell applications over existing QS-CDMA code families.

1) Simple construction (see Fig. 1). Minimal modifications are needed to the conventional scrambling-based spreading.
2) Flexible code family size. Since a different long code (base scrambling code) implies a unique code, the code family size can always be extended without considering already existing codes.
3) Inherent long code scrambling benefit such as LPI and signal randomization.
4) Easy code size expansion when tighter synchronization is possible. If tighter synchronization is guaranteed so that the offset is controlled to be within ± 1 chip, the code size can be doubled by allocating another quarter subset of the WH sequence set.
5) Zero insertion is not required.

III. Exact Expressions of Interference

Although the proposed codes have zero periodic correlation within a shift of up to ± 3 chips, the other users' signals remain nonzero after despreading. This is because the other users' signals are not a cyclic shift in the desired user's symbol duration due to different scrambling sequences and different data symbols over two consecutive data symbol intervals, which interfere in part the desired user's symbol.

For clarity and without loss of generality, we assume that $\mathbf{w}^{(k)}$ and $\mathbf{w}^{(l)}$ are the signature sequences for the kth and the lth user, respectively, and we are interested in despreading the lth user's symbols. First, let us consider the case when d, which is the timing delay of the kth user's symbol timing relative to that of the lth user, is positive. Let m_{k-} and m_k denote the kth user's preceding and current data symbols in the period of the lth user's symbol under consideration, respectively, and $\mathbf{s}^{(-)}$ and $\mathbf{s}$ be the corresponding scrambling sequences for m_{k-} and m_k, respectively. The spreading process is shown in Fig. 1, where spreading is carried out by chipwise complex multiplication between the data symbol (complex-valued for nonantipodal modulations) and complex-valued spreading chip sequences. The complex chip sequences after spreading are then chipwise modulated (quadrature modulation). We can express the correlation between the data-modulated spread chips of the kth user and the spreading sequence of the lth user as

$$X_{l,k}(d) = m_{k-}\sum_{n=0}^{d-1} c_{N+n-d}^{(k-)} c_n^{(l)*} + m_k \sum_{n=d}^{N-1} c_{n-d}^{(k)} c_n^{(l)*} \tag{17}$$

where $c_{0:N-1}^{(k-)}$ and $c_{0:N-1}^{(k)}$ are the corresponding spreading sequences for m_{k-} and m_k, respectively, and $c_{0:N-1}^{(l)}$ is the lth user's spreading sequence, i.e., $c_{0:N-1}^{(k-)} = s_{0:N-1}^{(-)} \odot w_{0:N-1}^{(k)}$, $c_{0:N-1}^{(k)} = s_{0:N-1} \odot w_{0:N-1}^{(k)}$, and $c_{0:N-1}^{(l)} = s_{0:N-1} \odot w_{0:N-1}^{(l)}$. In the proposed QS-CDMA system, the scrambling sequences change symbol by symbol, as shown in Fig. 1, whereas the constraints given in Table I are

[2]Note that, although, unlike [5], we employ *periodic* correlation for code design, simulation results in Section IV reveal that both codes undergo almost the same residual interference from data modulation within the ZCZ window.

maintained; thus, the scrambled WH sequences $c_{0:N-1}^{(k-)}$ and $c_{0:N-1}^{(k)}$ are not identical. The first term on the right-hand side of (17), i.e., $m_{k-}\sum_{n=0}^{d-1} c_{N+n-d}^{(k-)} c_n^{(l)*}$, denotes the interference from symbol m_{k-} that overlaps with the lth user's symbol. In the range of $|d| \leq 3$ that we are interested in, the second sum on the right-hand side of (17) with $N-d$ elements can be reduced into the sum of d elements by applying the zero periodic correlation property as

$$\sum_{n=d}^{N-1} c_{n-d}^{(k)} c_n^{(l)*} = \sum_{n=0}^{N-1} c_{n-d}^{(k)} c_n^{(l)*} \left(= R_{k,l}(d) = 0\right) - \sum_{n=0}^{d-1} c_{n-d}^{(k)} c_n^{(l)*} = -\sum_{n=0}^{d-1} c_{n-d}^{(k)} c_n^{(l)*}. \quad (18)$$

Substituting (18) into (17), we have $X_{l,k}(d) = m_{k-}\sum_{n=0}^{d-1} c_{N+n-d}^{(k-)} c_n^{(l)*} - m_k \sum_{n=0}^{d-1} c_{n-d}^{(k)} c_n^{(l)*}$. Through a similar procedure, we can include the case of negative d values as

$$X_{l,k}(d) = \begin{cases} m_{k-}\sum_{n=0}^{d-1} c_{N+n-d}^{(k-)} c_n^{(l)*} \\ \quad -m_k \sum_{n=0}^{d-1} c_{n-d}^{(k)} c_n^{(l)*}, & 0 < d \leq 3 \\ 0, & d = 0 \\ m_{k+}\sum_{n=0}^{-d-1} c_n^{(k+)} c_{N+n+d}^{(l)*} \\ \quad -m_k \sum_{n=0}^{-d-1} c_n^{(k)} c_{N+n+d}^{(l)*}, & -3 \leq d < 0 \end{cases} \quad (19)$$

where m_{k+} denotes the kth user's data symbol after m_k that overlaps with the lth user's symbol; $c_{0:N-1}^{(k+)}$ is the corresponding spread sequence, i.e., $c_{0:N-1}^{(k+)} = s_{0:N-1}^{(+)} \odot w_{0:N-1}^{(k)}$; and $s_{0:N-1}^{(+)}$ denotes the corresponding scrambling sequence for m_{k+}. Consequently, (19) for $d \neq 0$, although looking complex, is simply equal to the sum of $2|d|$ complex chips. The maximum value is thus $2|d|$, assuming $|m_{k-}| = |m_k| = |m_{k+}| = 1$.

Now, let us consider ICI. Without loss of generality, we assume that the pth code is allocated to the pth cell. Let $m_{k-}^{(p)}$, $m_k^{(p)}$, and $m_{k+}^{(p)}$ denote the preceding, current, and succeeding symbols of the kth user in the pth cell, respectively; $\mathbf{s}_p^{(-)}$, $\mathbf{s}_p$, and $\mathbf{s}_p^{(+)}$ denote their corresponding scrambling sequences; and $\mathbf{s}_q$ denote the scrambling sequence for the lth user symbol in the qth cell. From the property $R_{l,k}(d|(\mathbf{s}^{(A)}, \mathbf{s}^{(B)})) = 0$ for $|d| \leq 3$, except for $d = 0$

$$X_{l,k}^{(q,p)}(d) = \begin{cases} m_{k-}^{(p)}\sum_{n=0}^{d-1} c_{N+n-d}^{(p,k-)} c_n^{(q,l)*} \\ \quad -m_k^{(p)} \sum_{n=0}^{d-1} c_{n-d}^{(p,k)} c_n^{(q,l)*}, & 0 < d \leq 3 \\ m_{k+}^{(p)}\sum_{n=0}^{-d-1} c_n^{(p,k+)} c_{N+n+d}^{(q,l)*} \\ \quad -m_k^{(p)} \sum_{n=0}^{-d-1} c_n^{(p,k)} c_{N+n+d}^{(q,l)*}, & -3 \leq d < 0 \end{cases} \quad (20)$$

where the spreading sequences for $m_{k-}^{(p)}$, $m_k^{(p)}$, $m_{k+}^{(p)}$, and m_l are given as $c_{0:N-1}^{(p,k-)} = s_{p,0:N-1}^{(-)} \odot w_{0:N-1}^{(k)}$, $c_{0:N-1}^{(p,k)} = s_{p,0:N-1} \odot w_{0:N-1}^{(k)}$, $c_{0:N-1}^{(p,k+)} = s_{p,0:N-1}^{(+)} \odot w_{0:N-1}^{(k)}$, and $c_{0:N-1}^{(q,l)} = s_{q,0:N-1} \odot w_{0:N-1}^{(l)}$, respectively. For $d=0$, we have $X_{l,k}^{(q,p)}(0) = 4m_k^{(p)} \sum_{n=0}^{(N/4)-1} c_n^{(p,k)} c_n^{(q,l)*}$ from (16).

IV. Bit Error Rate Performance

In Section II-D, we discussed the merits of the proposed spreading codes over the conventional QS-CDMA spreading codes that exclude long code scrambling in the spreading process. In this section, we provide the BER performance of the proposed spreading code family in a multicell environment as in [5]. For comparison, we select the Type-II GLS code family with $(N, M) = (131, 32)$, which is the most recently proposed QS-CDMA spreading code in [5] with minimum ICI. We first provide the system model for BER evaluation and an analytic expression of the decision variable. Then, we assess the BER performances via mainly a numerical approach. In addition to several advantages over the conventional QS-CDMA spreading codes, the proposed spreading code family achieves the same BER performance as the Type-II GLS code family with minimum ICI.

A. System Model and Decision Variable

Consider a particular cell/sector, e.g., cell 0, with P interfering cell/sector $1, 2, \ldots, P$. Let us denote K as the number of users in each cell and $t_d^{(p,k)}$ as the access timing offset of the kth user in cell p, which is uniformly distributed over $[-1.5T_c, 1.5T_c]$ and independent and identically distributed (i.i.d.) for different k's. The complex baseband equivalent received signal of the lth user in cell 0 during interval $[t_d^{(0,l)}\ NT_c + t_d^{(0,l)}]$ is given as

$$r(t) = \frac{m_l^{(0)} e^{j\phi_{0,l}}}{\sqrt{NT_c}} \sum_{j=1}^{N} c_j^{(0,l)} p_c\left(t - jT_c - t_d^{(0,l)}\right) + \sum_{k=1,k\neq l}^{K} U_k^{(0)}(t) + \sum_{p=1}^{P}\sum_{k=1}^{K} \sqrt{\left(\frac{r_k^{(p)}}{r_k^{(p,0)}}\right)^x} U_k^{(p)}(t) + n(t) \quad (21)$$

where $m_k^{(p)}$ is the transmitted data symbol of the kth user in cell p, with $\mathrm{E}[m_k^{(p)} m_k^{*(p)}] = E_s$ [E_b for binary phase-shift keying (BPSK)]; $p_c(t)$ denotes the chip pulse; $\phi_{p,k}$ is the carrier phase of the kth user in cell p, which is uniformly distributed over $[-\pi, \pi]$ and i.i.d. for different k's; $n(t)$ is the additive white Gaussian noise (AWGN) with power spectral density N_0; and $U_k^{(p)}(t)$ denotes the kth user's signal in cell p expressed as (22), shown at the bottom of the page. Note that $U_k^{(p)}(t)$ is the concatenated spread signal for $(m_{k-}^{(p)}, m_k^{(p)})$ (or $(m_k^{(p)}, m_{k+}^{(p)})$) and extends over the symbol interval $[t_d^{(0,l)}\ NT_c + t_d^{(0,l)}]$. The term $\sqrt{(r_k^{(p)}/r_k^{(p,0)})^x}$ in the last sum of (21) is the amplitude scaling factor of ICI introduced by the uplink power control [24], where $r_k^{(p)}$ is the distance between the kth user in cell p and the base station of cell p, $r_k^{(p,0)}$ is the distance between the kth user in cell p and the base station of cell 0, and x is the path-loss exponent.

$$U_k^{(p)}(t) = \frac{e^{j\phi_{p,k}}}{\sqrt{NT_c}} \begin{cases} m_{k-}^{(p)} \sum_{j=1}^{N} c_j^{(k)} p_c\left(t - jT_c + NT_c - t_d^{(p,k)}\right) + m_k^{(p)} \sum_{j=1}^{N} c_j^{(k)} p_c\left(t - jT_c - t_d^{(p,k)}\right), & \text{if } t_d^{(0,l)} \leq t_d^{(p,k)} \\ m_k^{(p)} \sum_{j=1}^{N} c_j^{(k)} p_c\left(t - jT_c - t_d^{(p,k)}\right) + m_{k+}^{(p)} \sum_{j=1}^{N} c_j^{(k)} p_c\left(t - jT_c - NT_c - t_d^{(p,k)}\right), & \text{if } t_d^{(p,k)} < t_d^{(0,l)} \end{cases} \quad (22)$$

TABLE II
SIMULATED σ^2_{Intra} AND σ^2_{Inter} WITH BPSK MODULATION, FULL LOADING ($K = M = 32$), AND SIX INTERFERING CELLS ($P = 6$). THE PATH-LOSS EXPONENTS ARE SET TO BE $x = 2$ FOR FREE SPACE AND 3.4 FOR INTERNATIONAL TELECOMMUNICATION UNION VEHICULAR A CHANNEL

Code family	σ^2_{Intra}	σ^2_{Inter}	
		free space (x=2)	ITU Vehicular A (x=3.4) [27]
Type II GLS, (N, M)=(131, 32)	0.0012	0.1056	0.0501
Proposed, (N, M)=(128, 32)	0.0012	0.1053	0.0499

The despread output for a symbol of the lth user in cell 0, i.e., $m_l^{(0)}$, is obtained by taking the correlation between $r(t)$ and the corresponding spreading chip waveform as

$$D = \frac{1}{\sqrt{NT_c}} \int_{t_d^{(0,l)}}^{NT_c+t_d^{(0,l)}} r(t) \times e^{-j\phi_{0,l}} \left\{ \sum_{j=1}^{N} c_j^{(0,l)} p_c\left(t - jT_c - t_d^{(0,l)}\right) \right\}^* dt$$

$$= m_l^{(0)} + \sum_{k=1,k\neq l}^{K} I_{l,k}^{(0)} + \sum_{p=1}^{P}\sum_{k=1}^{K} I_{l,k}^{(p)} \sqrt{\left(\frac{r_k^p}{r_k^{p,0}}\right)^x} + n \quad (23)$$

where

$$n = \frac{1}{\sqrt{NT_c}} \int_{t_d^{(0,l)}}^{NT_c+t_d^{(0,l)}} n(t) \times e^{-j\phi_{0,l}} \left\{ \sum_{j=1}^{N} c_j^{(0,l)} p_c\left(t - jT_c - t_d^{(0,l)}\right) \right\}^* dt \quad (24)$$

is Gaussian distributed with zero mean and variance $N_0/2$, and

$$I_{l,k}^{(p)} = \frac{1}{\sqrt{NT_c}} \int_{t_d^{(0,l)}}^{NT_c+t_d^{(0,l)}} U_k^{(p)}(t) \times e^{-j\phi_{0,l}} \left\{ \sum_{j=1}^{N} c_j^{(0,l)} p_c\left(t - jT_c - t_d^{(0,l)}\right) \right\}^* dt \quad (25)$$

denotes the correlation between the kth user's signal in cell p and the lth user's spread sequence waveform in cell 0. Assuming that $p_c(t)$ is a rectangular pulse over the interval $[0 \quad T_c]$, the lth user's chips are partially interfered by two consecutive chips of the kth user with weights $T_c - (t_d^{(p,k)} - t_d^{(0,l)}) \bmod T_c$ and $(t_d^{(p,k)} - t_d^{(0,l)}) \bmod T_c$, respectively. Consequently, we can simply write the correlation as the weighted sum of $X_{l,k}^{(0,p)}(\lfloor (t_d^{(p,k)} - t_d^{(0,l)})/T_c \rfloor)$ and $X_{l,k}^{(0,p)}(\lfloor (t_d^{(p,k)} - t_d^{(0,l)})/T_c + 1 \rfloor)$ as [25]

$$I_{l,k}^{(p)} = \frac{e^{j(\phi_{p,k} - \phi_{0,l})}}{NT_c} \times \left\{ \left(T_c - \left(t_d^{(p,k)} - t_d^{(0,l)}\right) \bmod T_c\right) \times X_{l,k}^{(0,p)}\left(\left\lfloor \frac{\left(t_d^{(p,k)} - t_d^{(0,l)}\right)}{T_c} \right\rfloor\right) + \left(\left(t_d^{(p,k)} - t_d^{(0,l)}\right) \bmod T_c\right) \times X_{l,k}^{(0,p)}\left(\left\lfloor \frac{\left(t_d^{(p,k)} - t_d^{(0,l)}\right)}{T_c} + 1 \right\rfloor\right) \right\} \quad (26)$$

where $X_{l,k}^{(0,p)} = X_{l,k}$ in (19) if $p = 0$.

B. BER Simulation Results

Since the ICI and intracell interference in (23) are a sum of independent random variables with uniformly distributed phases, they are modeled as complex Gaussian random variables, as commonly accepted in BER analysis for cellular CDMA systems. For BPSK modulation, $m_l^{(0)}$ is equal to $\sqrt{E_b}$ or $-\sqrt{E_b}$. The BER expression conditioned on $m_l^{(0)} = \sqrt{E_b}$ is

$$P_b = Pr\left[\Re(D) > 0 | m_l^{(0)} = \sqrt{E_b}\right] \quad (27)$$

where $\Re(\cdot)$ denotes the real part. From (23) and (27), P_b is expressed as

$$P_b = Q\left(\sqrt{1/\left(\sigma^2_{\text{Intra}} + \sigma^2_{\text{Inter}} + (2E_b/N_0)^{-1}\right)}\right) \quad (28)$$

where σ^2_{Intra} and σ^2_{Inter}, which are the intracell interference and ICI variances when the bit energy is normalized, are given as $\sigma^2_{\text{Intra}} = (1/E_b)\text{E}[\Re(\sum_{k=1,k\neq l}^{K} I_{l,k}^{(0)})^2]$ and $\sigma^2_{\text{Inter}} = (1/E_b)$ $\text{E}[\Re(\sum_{p=1}^{P}\sum_{k=1}^{K} I_{l,k}^{(p)} \sqrt{(r_k^{(p)}/r_k^{(p,0)})^x})^2] = (1/E_b)\text{E}[(r_k^{(p)}/r_k^{(p,0)})^x] \times \text{E}[\Re(\sum_{p=1}^{P}\sum_{k=1}^{K} I_{l,k}^{(p)})^2]$, respectively. This means that we first compute σ^2_{Intra} and σ^2_{Inter} by averaging the power of the intracell interference and ICI over the interfering users' message symbols, carrier phases, locations, and scrambling sequences. Then, we compute BER by substituting these variances into the Q-function given in (28). This technique is well justified, especially for fully loaded systems, which is the case considered in this paper since the interference terms are the sum of a number of independent terms, resulting in a valid Gaussian approximation to its distribution. Table II shows the simulated σ^2_{Intra} and σ^2_{Inter} with a full loading ($K = M = 32$), six interfering cells, and uniform user distribution in each cell. The simulated intracell interference variance is 29 dB lower than the bit energy. This implies that the residual interference due to data modulation is negligible. Note that, even for a large path-loss exponent ($x = 3.4$), ICI dominates the total interference.

For quadrature phase-shift keying (QPSK), $m_l^{(0)} \in \{\pm\sqrt{E_b} \pm j\sqrt{E_b}\}$ with equal probability. Conditioned on $m_l^{(0)} = \sqrt{E_b} + j\sqrt{E_b}$, $P_b = Pr[\Re(D) > 0 | m_l^{(0)} = \sqrt{E_b} + j\sqrt{E_b}]$, which is equal to $Pr[\Im(D) > 0 | m_l^{(0)} = \sqrt{E_b} + j\sqrt{E_b}]$ (where $\Im(\cdot)$ denotes the imaginary part), because the interference in (23) is a circular symmetric complex Gaussian random variable. Consequently, the BER expression for QPSK is identical to (28) with different values of σ^2_{Intra} and σ^2_{Inter}. Since the interfering users' symbols are also QPSK symbols, whose energy is twice of that of BPSK symbols, σ^2_{Intra} and σ^2_{Inter} are twice of the values given in Table II.

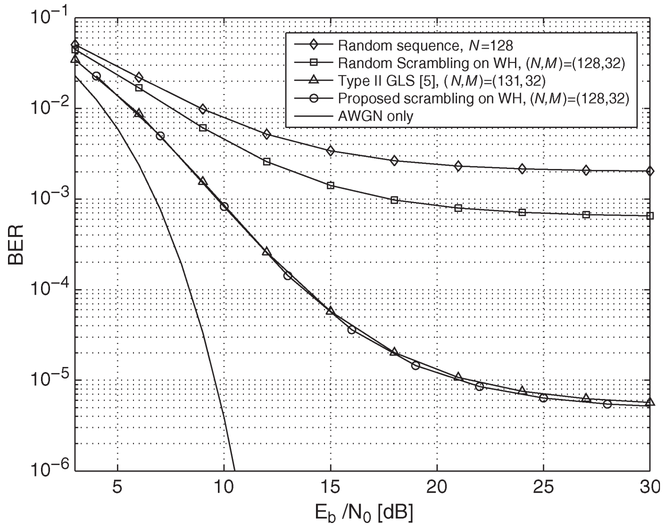

Fig. 2. BER with various spreading codes with BPSK modulation, full loading ($K = M = 32$), and six interfering cells ($P = 6$). The path-loss exponent chosen is $x = 3.4$.

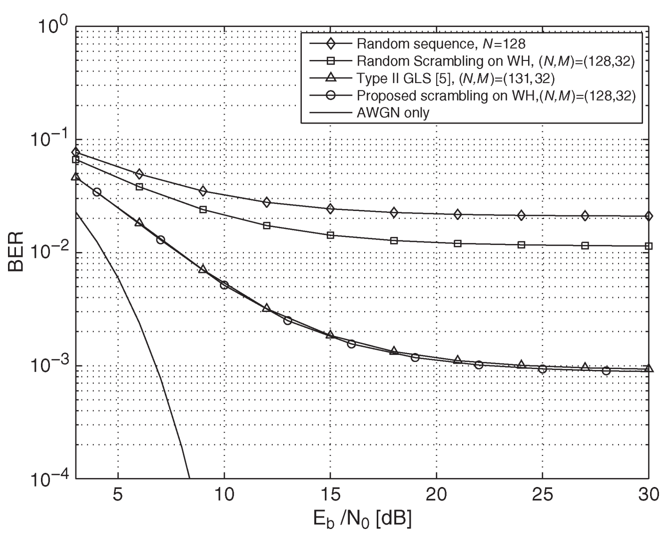

Fig. 3. BER with various spreading codes with QPSK modulation, full loading ($K = M = 32$), and six interfering cells ($P = 6$). The path-loss exponent chosen is $x = 3.4$.

In Figs. 2 and 3, we compare the BER of the proposed spreading scheme with those of other spreading schemes, including Type-II GLS codes in a multicell environment for BPSK and QPSK modulations, respectively. As a reference, we include the cases for the AWGN only, completely random spreading codes, and randomly scrambled WH codes. As expected, the randomly scrambled WH codes perform better than the completely random spreading codes; however, the former performs poorly for QS-CDMA systems, because zero correlation is achieved only for zero timing offset among the user-spreading sequences. On the other hand, the proposed spreading code family and Type-II (131,32) GLS code family greatly reduce the BER close to the AWGN-only case, compared with the other codes. In Fig. 3, we find that the performance trends among the various codes in case of QPSK hold as done in Fig. 2, although there is a 3-dB increase in the interference energy for the same bit energy, compared with BPSK modulation.

V. Conclusion

In this paper, we have proposed spreading codes that are suitable for QS-CDMA systems. The proposed codes retain the long code-scrambling effects and have good intra- and intercode cross-correlation properties. Compared with the conventional QS-CDMA spreading codes, the proposed spreading codes have several advantages, such as simple code construction, flexible code family sizes, and inheritance of long code-scrambling benefits. The proposed spreading codes could also be employed in the recent broadband CDMA systems, such as multicarrier direct-sequence CDMA or code-spread orthogonal frequency-division multiple access. We leave this extension for further study.

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User-Selection Algorithms for Multiuser Precoding

Ngọc-Dũng Đào, *Member, IEEE*, and
Yong Sun, *Senior Member, IEEE*

***Abstract*—A general framework for user selection in the broadcast channel with multiuser linear and nonlinear precoding techniques is investigated. Assuming full knowledge of channel-state information at the transmitter and using a minimum-mean-square-error (MMSE) criterion, we propose several user-selection algorithms based on the conventional incremental and decremental search approaches. Furthermore, a novel iterative user selection approach is introduced, offering a flexible performance–complexity tradeoff. New user grouping algorithms are also developed for orthogonal frequency-division multiple-access systems. Simulation results show that the proposed methods outperform well-known algorithms, which select users based on the users' orthogonality or sum rate bound.**

***Index Terms*—Broadcast channel, multiuser (MU) precoding, orthogonal frequency-division multiple access (OFDMA), Tomlinson–Harashima (TH) precoding, user selection.**

I. Introduction

A downlink broadcast channel of wireless systems [1] has been extensively studied recently. In this channel, a base station (BS) with multiple transmit antennas simultaneously sends several data streams to uncoordinated users. Assuming that full channel-state information is available at the transmitter, dirty paper coding (DPC) techniques can be employed to precancel the interuser interference (see [1] and references therein). Since DPC has high implementation complexity, low-complexity linear multiuser (MU) precoding methods have been proposed, e.g., zero-forcing (ZF) and minimum-mean-square error (MMSE) precoder [2], [3]. Although having low complexity, the performance of linear precoders may be severely degraded if the users' channels are highly correlated. Nonlinear precoding techniques, such as the Tomlinson–Harashima (TH) spatial precoding (see [4] and the references therein) and vector perturbation (VP) precoding techniques [5], are proposed to improve the performance of linear precoding. Nonlinear precoding techniques have manageable complexity and can also be optimized by the MMSE criterion [6].

In wireless communication systems, the number of active users is often larger than the number of users that can be served at the same time. Therefore, user scheduling is an important task. A number of user selection methods with different cost metrics have been proposed for linear precoding. In [7], the user-selection criterion is to maximize the orthogonality among selected users, whereas the upper bound on the users' sum rate [8] is suggested in [9]. The sum-rate bound is also employed in [10] for VP precoding at low signal-to-noise ratio (SNR). A user-selection method for minimizing mean-square error (MSE) is presented in [11], in which the uplink–downlink duality is exploited [12]. This method requires alternative optimization, whose computational complexity may highly be dependent on the channel matrix. Recently, new user-selection methods, which also employ MMSE criterion, have been presented in [13] for the MMSE linear precoding [2].

In this paper, we extend the results in [13] and present a general framework for user selection for both linear and nonlinear precoding using MMSE criterion. First, we show that the MSE of the MMSE-TH precoding in [6] is bounded by the MSE of the linear MMSE precoding [2]. Hence, the MMSE-based user selection methods for the linear MMSE precoding can be well applied for the nonlinear MMSE precoding. Then, we prove that the user selection for the linear MMSE precoder [2] in point-to-multipoint MIMO communications can be translated into the receive antenna selection for linear MMSE receivers in point-to-point MIMO communications. Thus, the incremental and decremental receive antenna selection algorithms, e.g., in [14], or the uplink user selection algorithms in [15] (see also [16]) can be modified for user selection in the downlink and user selection. The decremental search gives better performance than the incremental search. However, when the number of users is large, the complexity of the former is much higher than the latter. We thus propose a novel iterative user-selection approach that is characterized by its flexibility in balancing the performance and complexity. The proposed methods are applicable for ZF precoding and extended to assign users in resource blocks (RBs) of orthogonal frequency-division multiple-access (OFDMA) systems. Simulation results show that our proposed schemes outperform well-known algorithms, which select users based on the user orthogonality [7] and sum rate upper bound [9]. The proposed algorithms can approach the performance of the optimal exhaustive search but with much less complexity.

II. System Model

Assume that the channel exhibits frequency-flat fading. The BS has M transmit antennas. There are a total of K_t single-antenna users, of which K $(K \leq M)$ users are to be selected. Let $\mathbf{h}_k$ be the row channel vector of user k $(k = 1, \ldots, K)$ and $\mathbf{H} = [\mathbf{h}_1^\top\ \mathbf{h}_2^\top, \ldots, \mathbf{h}_K^\top]^\top$ be the channel matrix of all users, where superscript $\top$ denotes the matrix

Manuscript received November 4, 2009; revised March 19, 2010 and May 28, 2010; accepted May 31, 2010. Date of publication June 7, 2010; date of current version September 17, 2010. This paper was presented in part at the IEEE Global Communications Conference 2009, Honolulu, HI, November 30–December 4, 2009. The review of this paper was coordinated by Dr. C. Yuen.

The authors are with Toshiba Research Europe Ltd., BS1 4ND Bristol, U.K. (e-mail: ngoc.dao@toshiba-trel.com; yong.sun@toshiba-trel.com).

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Digital Object Identifier 10.1109/TVT.2010.2052293