A Maximum Likelihood Doppler Frequency Estimator for OFDM Systems

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***Abstract*— This paper derives a maximum likelihood Doppler frequency estimator for orthogonal frequency division multiplexing (OFDM) systems in time-varying multipath channels. The proposed scheme is a frequency-domain approach that utilizes pilot subcarriers, which are commonly implemented in most practical systems. Time-varying fading causes intercarrier interference (ICI) in OFDM systems. Thus, in the proposed estimator, the effect of ICI is taken into consideration with a proper model for accurate results. The estimator can be implemented using a finite impulse response (FIR) filter bank whose coefficients can be pre-calculated and stored in order to lower the computational complexity. We evaluate various methods to improve the estimation accuracy and analyze their complexity-performance tradeoffs. We also derive the Cramér-Rao bound and provide simulation results to quantify the performance of the proposed algorithm.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted by many wireless standards such as IEEE 802.11 and 802.16 and has been implemented in many practical systems. The maximum Doppler frequency, f_d, is the ratio of the speed of the mobile user and the wavelength of the carrier. Knowledge of mobile speeds is critical in improving the performance of multi-cell wireless communication systems. For example, in the pico-cell deployment overlaying with existing macro-cells, the Doppler frequency information of each mobile allows optimization of user assignments to proper base stations, and thus minimizes the number of handover scenarios. The mobile speed information is also very critical for implementing a number of physical- and network-layer functions such as adaptive and fast link adaptation, and accurate channel prediction. Thus, the scheduler gain due to multiuser diversity and spectral efficiency of the system can be increased.

In [1], an autocorrelation-based scheme for maximum-Doppler-frequency estimation was proposed for single-carrier systems, where the estimate is obtained using the envelope of the received signal. In [2], a method based on the differentials of the channel estimates is employed for the estimation process. Another method based on the level-crossing rates was proposed in [3]. In most OFDM systems, a cyclic prefix (CP), which is the replica of the OFDM symbol tail, is used as the guard interval. In [4], the correlation between the tail of the OFDM symbol and the guard interval was exploited to estimate f_d, where the effects of intersymbol interference (ISI) was not considered. In [5], the estimate of f_d is obtained via a maximum-likelihood (ML) based time-domain method for TDMA and CDMA systems. The application of this algorithm to OFDM systems was presented in [6], where time-domain channel estimates were used to obtain the maximum Doppler frequency estimates. In this model, the channel is not estimated based on pilot subcarriers, but by using preambles and inserting frequent mid-ambles. In its frequency domain approach, the ICI is ignored. Thus, an error floor is observed.

In this paper, we propose a ML algorithm of Doppler frequency estimation for OFDM systems. This algorithm is a frequency-domain approach which can be readily applied to any OFDM system since it is based on the already-existing pilot subcarriers, and thus does not increase the system overhead as mid-amble based algorithms do. The estimator can be implemented as a low-complexity finite impulse response (FIR) filter bank whose coefficients can be pre-calculated and conveniently stored in the system memory. It is well known that time-varying fading causes intercarrier interference (ICI) in OFDM systems. However, existing work on Doppler estimation has not considered ICI effects. We provide a proper model of ICI and take into consideration its effects for accurate estimation results in order to avoid an error floor. The algorithm accommodates different choices of design parameters, allowing flexible performance-complexity tradeoffs. We also derive the Cramér-Rao lower bound for the presented algorithm.

This paper is organized as follows. Section II describes the OFDM scheme and the channel under consideration. Section III introduces the proposed ML algorithm and derives the Cramér-Rao lower bound of the mean-square error (MSE). Section IV simulates the performance of the proposed algorithm under different channel conditions and design parameters. Concluding remarks are made in Section V.

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II. SYSTEM MODEL

An OFDM system with K active subcarriers and FFT length N, where $K \leq N$, is considered. Let N_G denote the length of the guard interval, or cyclic prefix, and $d_k,\ k = 1, 2, \ldots, K$, represent the data transmitted over the k-th data subcarrier. The transmitted OFDM signal in the time domain can then be expressed as

$$x(u) = \sqrt{\frac{E_s}{N}} \sum_{k \epsilon \mathcal{K}} d_k e^{j2\pi uk/N}, \quad -N_G \leq u \leq N-1 \quad (1)$$

where E_s is the symbol energy per subcarrier, $\mathcal{K}$ represents the set of active subcarriers, or subcarriers carrying information data and pilots, and $E\{|d_k|^2\} = 1$. Without loss of generality, we assume that the active subcarriers are from 0 to $K-1$. Then Eq. (1) can be rewritten as

$$x(u) = \sqrt{\frac{E_s}{N}} \sum_{k=0}^{K-1} d_k e^{j2\pi uk/N}, \quad -N_G \leq u \leq N-1. \quad (2)$$

We consider a time-varying Rayleigh fading channel with a maximum delay of T_d and an rms delay spread τ_{rms}. The channel is described using a tapped delay line model with an exponentially decaying tap power. We assume that $T_d \leq N_G$, and that the autocorrelation of the channel, the inverse Fourier transform of the Doppler spectrum, can be modeled by a zeroth-order Bessel function of the first kind. The channel coefficient of the l-th tap ($0 \leq l \leq T_d - 1$) at time u is denoted as $h_l(u)$. By stacking vertically all the T_d channel coefficients at time u, we obtain

$$\boldsymbol{h}(u) = [h_0(u)\ h_1(u)\ \cdots\ h_{T_d-1}(u)]^T \quad (3)$$

where $[\cdot]^T$ stands for transpose.

We further assume that the channel taps (i.e., elements of $\boldsymbol{h}(u)$) are independent identically distributed (i.i.d.), zero-mean, circularly symmetric complex Gaussian random variables. The channel autocorrelation function is expressed as

$$E\{\boldsymbol{h}(u+\Delta u)\boldsymbol{h}^H(u)\} = cJ_0\left(\frac{2\pi f_d T \Delta u}{N}\right)\boldsymbol{E} \quad (4)$$

where $(\cdot)^H$ denotes complex conjugate transpose, c is a scaling factor which is used to normalize the channel power, T is the duration of N samples, $J_0(\cdot)$ represents the zeroth-order Bessel function of the first kind, and $\boldsymbol{E}$ is a diagonal $T_d \times T_d$ matrix whose l-th diagonal entry is $e^{-l\tau_{rms}/T_d},\ \ 0 \leq l \leq T_d - 1$. The earlier assumption that the guard interval N_G is not less than the multipath spread ensures ISI-free operations. Our objective is to accurately estimate the normalized maximum Doppler frequency $f_d T$ based on pilot subcarriers of the received signal.

III. ML MAXIMUM DOPPLER FREQUENCY ESTIMATOR

The received signal through a time-varying multipath channel can be written as

$$y(u) = \sum_{l=0}^{T_d-1} h_l(u) x(u-l) + \omega(u) \quad (5)$$

where $\omega(u)$ is the additive white Gaussian noise (AWGN) with zero-mean and variance σ_ω^2. We assume without loss of generality that $E_s = 1$; thus the variance of the AWGN equals the inverse of the signal-to-noise ratio (SNR), i.e., $\sigma_\omega^2 = 1/\text{SNR}$. Once an OFDM symbol is received, the guard interval (the first N_G samples) is discarded, leaving the ISI-free data portion. The received signal during the data portion is expressed as

$$y(u) = \frac{1}{\sqrt{N}} \sum_{k=0}^{K-1} d_k e^{j2\pi uk/N} H_k(u) + \omega(u) \quad (6)$$

where

$$H_k(u) = \sum_{l=0}^{T_d-1} h_l(u) e^{-j2\pi lk/N} \quad (7)$$

represents the Fourier transform of the channel at time u along the delay path. The data signal on the k-th subcarrier of an OFDM symbol at the FFT output is expressed as [7]

$$\begin{aligned} Y_k &= \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} y(u) e^{-j2\pi uk/N} \\ &= d_k H_k + \alpha_k + W_k \end{aligned} \quad (8)$$

where

$$H_k = \frac{1}{N} \sum_{u=0}^{N-1} H_k(u) \quad (9a)$$

$$\alpha_k = \frac{1}{N} \sum_{m=0, m\neq k}^{K-1} d_m \sum_{u=0}^{N-1} H_m(u) e^{j2\pi u(m-k)/N} \quad (9b)$$

$$W_k = \frac{1}{\sqrt{N}} \sum_{u=0}^{N-1} w(u) e^{-j2\pi uk/N}. \quad (9c)$$

The term α_k represents the ICI component. The power of ICI may be negligible when the maximum normalized Doppler frequency $f_d T$ is small (e.g., $f_d T < 0.02$) [7], but ICI should be considered for the general case. Thus, we will include the ICI term in the ML formulation. To allow the use of more than one OFDM symbols for the estimation of the maximum Doppler frequency, we assume that certain amount of latency is acceptable. When multiple OFDM symbols are considered, we can rewrite Eq. (8) by including the index n denoting the n-th OFDM symbol as

$$Y_{k,n} = d_{k,n} H_{k,n} + \alpha_{k,n} + W_{k,n}. \quad (10)$$

Let $\mathcal{P}$ represent the set of pilot subcarriers. Since the values $d_{k,n}$, $k \in \mathcal{P}$, are known, the noisy estimate of the channel can be obtained as

$$\begin{aligned} \tilde{H}_{k,n} &= \frac{Y_{k,n}}{d_{k,n}} \\ &= H_{k,n} + \alpha_{k,n}/d_{k,n} + W_{k,n}/d_{k,n}. \end{aligned} \quad (11)$$

This process can be done for all pilot subcarriers in the M consecutive OFDM symbols[1].

[1] We refer to a set of M consecutive OFDM symbols as an "estimation group."

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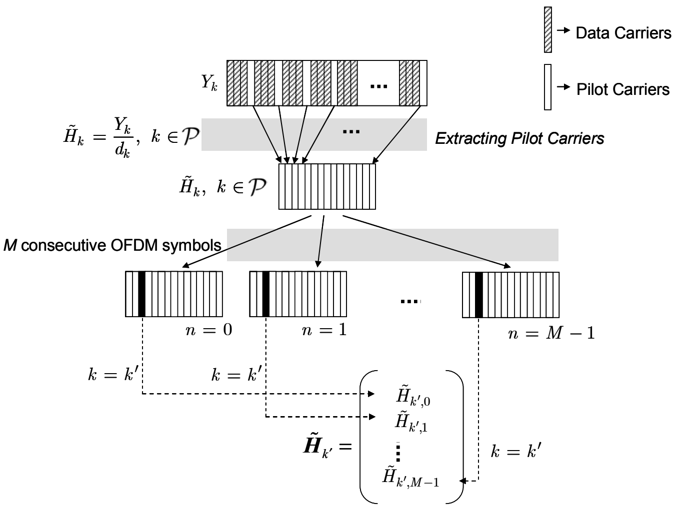

Fig. 1. Detailed block diagram of $\tilde{\boldsymbol{H}}_{k'}$, $k' \in \mathcal{P}$.

As illustrated in Fig. 1, using the specific pilot subcarrier k' of one estimation group, the vector $\boldsymbol{H}_{k'}$ can be obtained as

$$\tilde{\boldsymbol{H}}_{k'} = \left[\tilde{H}_{k',0}\ \tilde{H}_{k',1}, \cdots, \tilde{H}_{k',M-1}\right]^T. \tag{12}$$

The probability density function (pdf) of the ICI component $\alpha_{k,n}$ is a weighted Gaussian mixture pdf. However, by invoking the central limit theorem, we can approximate the ICI term as a complex Gaussian random variable. $\tilde{\boldsymbol{H}}_k$ can thus be modeled as a zero-mean, circularly symmetric, complex Gaussian vector with the following pdf

$$p(\tilde{\boldsymbol{H}}_k) = \left[\pi^M \det(\boldsymbol{R})\right]^{-1} \exp\left(-\tilde{\boldsymbol{H}}_k^H \boldsymbol{R}^{-1} \tilde{\boldsymbol{H}}_k\right) \tag{13}$$

where $\boldsymbol{R}$ is the autocorrelation matrix of vector $\tilde{\boldsymbol{H}}_k$. To calculate each entry of $\boldsymbol{R}$ we need the correlation of $H_{k,n}$ and $H_{k+\Delta k,n+\Delta n}$, which is given in [7] as

$$E\{H_{k+\Delta k,n+\Delta n} H_{k,n}^*\} = \frac{r_f(\Delta k)}{N^2} \sum_{l_1=0}^{N-1} \sum_{l_2=0}^{N-1} J_0\left(\frac{2\pi f_d T(l_1 - l_2 + \Delta n(N+N_G))}{N}\right) \tag{14}$$

where

$$r_f(\Delta k) = c \sum_{l=0}^{Td-1} e^{-l\tau_{rms}/T_d} e^{-j2\pi l \Delta k/N}$$

represents the frequency-domain correlation, and c is a scaling factor to normalize the channel power and is defined as

$$c = \left(\sum_{l=0}^{Td-1} e^{-l\tau_{rms}/T_d}\right)^{-1}. \tag{15}$$

The frequency-domain correlation factor equals to unity if the same pilot subcarriers are chosen (i.e., $\Delta k = 0$) from each OFDM symbol as in the scheme proposed in this paper. However, in the modeling of the correlation of the ICI term $\alpha_{k,n}$, $r_f(\Delta k)$ does not always equal unity and must be taken into account:

$$\begin{aligned} & E\{\alpha_{k,n+\Delta n} \alpha_{k,n}^*\} \\ = \ & \frac{1}{N^2} \sum_{\substack{m_1 \neq k \\ m_1=0}}^{K-1} \sum_{\substack{m_2 \neq k \\ m_2=0}}^{K-1} E\{d_{m_1,n+\Delta n} d_{m_2,n}^*\} \\ & \sum_{l_1=0}^{N-1} \sum_{l_2=0}^{N-1} E\{H_{m_1}(l_1 + \Delta n(N+N_G)) H_{m_2}^*(l_2)\} \\ & e^{j2\pi(l_1+\Delta n(N+N_G))(m_1-k)/N} e^{-j2\pi l_2(m_2-k)/N} \end{aligned} \tag{16}$$

where the term $E\{H_{m_1}(l_1 + \Delta n(N+N_G)) H_{m_2}^*(l_2)\}$ can be obtained through Eq. (14) as

$$\begin{aligned} & E\{H_{m_1}(l_1 + \Delta n(N+N_G)) H_{m_2}^*(l_2)\} = \\ & r_f(m_1 - m_2)\ J_0\left(2\pi f_d T(l_1 + \Delta n(N+N_G) - l_2)/N\right). \end{aligned}$$

Finally, the correlation between the channel term and the ICI component in $\boldsymbol{R}$ is given by

$$\begin{aligned} & E\{H_{k,n+\Delta n} \alpha_{k,n}^*\} = \frac{1}{N^2} \sum_{\substack{m \neq k \\ m=0}}^{K-1} E\{d_{m,n}^*\} \cdot \\ & \sum_{l_1=0}^{N-1} \sum_{l_2=0}^{N-1} E\{H_k(l_1 + \Delta n(N+N_G)) H_m^*(l_2)\} \cdot \\ & e^{-j2\pi l_2(m-k)/N}. \end{aligned} \tag{17}$$

From Eqs. (13)∼(17), the log-likelihood function can be obtained as

$$L(\tilde{\boldsymbol{H}}_k) = \ln(p(\tilde{\boldsymbol{H}}_k)) = \Omega - \ln(\det(\boldsymbol{R})) - \tilde{\boldsymbol{H}}_k^H \boldsymbol{R}^{-1} \tilde{\boldsymbol{H}}_k \tag{18}$$

where Ω is a constant term independent of the Doppler frequency. Maximizing the log-likelihood function is equivalent to minimizing the following cost-function

$$\Lambda_k(f_d T) = \ln(\det(\boldsymbol{R})) + \tilde{\boldsymbol{H}}_k^H \boldsymbol{R}^{-1} \tilde{\boldsymbol{H}}_k. \tag{19}$$

Hence, the maximum-likelihood estimate (MLE) of the normalized Doppler frequency can be obtained as

$$\widehat{f_d T} = arg\ \min_{f_d T} \Lambda_k(f_d T). \tag{20}$$

The MSE of an unbiased estimator is lower bounded by the Cramér-Rao bound [8], which can be found to be

$$\text{CRB} = \frac{1}{\Re\left\{tr\left[\frac{\partial \boldsymbol{R}}{\partial f_d T} \boldsymbol{R}^{-1} \frac{\partial \boldsymbol{R}}{\partial f_d T} \boldsymbol{R}^{-1}\right]\right\}} \tag{21}$$

where $\Re\{\cdot\}$ denotes the real part and $tr[\cdot]$ represents the matrix trace.

Exact calculation of the MLE requires the knowledge of delay profile due to the presence of $r_f(\Delta k)$ term in the ICI denotation. Since accurate delay profile may not be available in practical implementations, there might be a mismatch between the assumed $\boldsymbol{R}$ and the actual $\boldsymbol{R}$. The worst-case of mismatch occurs if the receiver assumes flat fading, i.e.,

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$r_f(m_1 - m_2) = 1$. To better demonstrate the efficiency of the estimator, we assume this worst-case mismatch scenario throughout the paper. However, for the theoretical Cramér-Rao bounds, we assume the delay profile is known and we employ the actual $\boldsymbol{R}$.

In order to improve the accuracy of the estimator, the MLE over multiple pilot subcarriers can be formulated. This requires the knowledge of statistics such as the channel delay spread and delay profile. Instead, we sum the cost function over pilot subcarriers as

$$\Lambda(f_dT) = \sum_{k\in\mathcal{P}} \Lambda_k(f_dT). \tag{22}$$

Further reduction in the MSE can be achieved in the time-domain by summing the cost function over more than one estimation groups at the expense of increased latency. Although such time- and frequency-domain averaging of the cost function is not optimum, the performance of the estimator can be significantly improved.

For a specific estimation algorithm, the number of OFDM symbols in one estimation group, M, will be fixed. Thus, in order to significantly reduce the overall complexity of the system, the terms $\ln(\det(\boldsymbol{R}))$ and $\boldsymbol{R}^{-1}$ can be pre-calculated and stored in the memory for a certain range of normalized Doppler frequency values. The complexity can be further reduced since $\tilde{\boldsymbol{H}}_k^H \boldsymbol{R}^{-1} \tilde{\boldsymbol{H}}_k$ can be evaluated using a filter bank via Cholesky factorization and through low-rank approximation.

The main design parameters of the estimator are P, the number of pilot subcarriers from each OFDM symbol; M, the number of OFDM symbols in one estimation group; and G, the number of estimation groups. By choosing appropriate values for specific communications scenarios, we can achieve flexible performance-complexity tradeoffs. The performance, complexity, and latency aspects of different sets of these parameters will be investigated by simulations in the next section.

IV. Simulation Results

The time-varying channel is obtained by an FIR filter whose spectrum is the same as the one used in [7]. Each simulation is based on the observation of $5000 \times G \times M$ OFDM symbols. Without loss of generality, an OFDM symbol is assumed to have 32 subcarriers. The total length of an OFDM symbol including the guard interval is $112.5\mu s$, and the length of the guard interval is $12.5\mu s$. The maximum number of channel taps is assumed not to exceed the sample length of the guard interval, $N_G = 4$. The search range of the ML estimator is set to be $0 \sim 0.04$ with a step size of 0.001 (in terms of f_dT). We assume every one in four subcarriers is a pilot.

We define the normalized MSE (NMSE) as

$$\text{NMSE} \equiv \frac{\text{MSE}}{(f_dT)^2} = \frac{E\{|\widehat{f_dT} - f_dT|^2\}}{(f_dT)^2}. \tag{23}$$

Fig. 2 compares the NMSE with CRB for different values of M when $P = G = 1$ where the solid curves depict

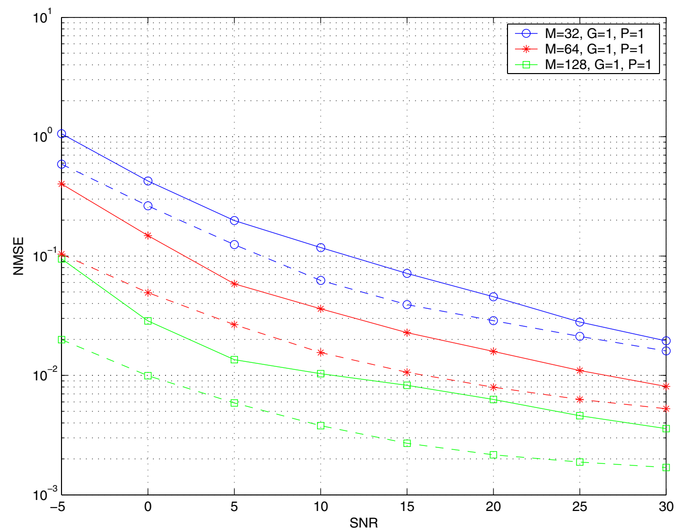

Fig. 2. NMSE vs. SNR for different M when $G = P = 1$ (Solid: Simulation, Dashed: Cramér-Rao lower bound).

the simulated NMSE results and the dashed curves represent the Cramér-Rao bounds. The maximum normalized Doppler frequency f_dT is chosen to be 0.01. The number of channel taps (T_d) equals the sample length of the guard interval and the rms delay spread of the channel is $\tau_{rms} = T_d/4$. It is observed that a larger M reduces both the NMSE and CRB. However, the system complexity and the memory required to store $\ln(\det(\boldsymbol{R}))$ and $\boldsymbol{R}^{-1}$ increases proportional to M. From Eq. (23), with $f_dT = 0.01$, $T = 100\mu sec$ and $f_d = 100$Hz, a 10^{-2} NMSE value of the estimator corresponds to 10Hz standard deviation. Unlike in [6], no error floors are observed in the NMSE and CRB.

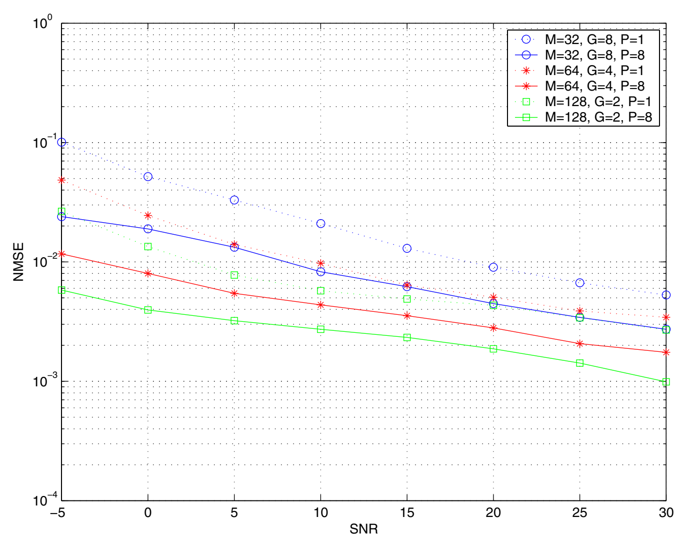

Fig. 3. NMSE vs. SNR for different number of pilot subcarriers for $f_dT = 0.01$.

In Fig. 3, the NMSE values of the estimator for different

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values of P, M, and G are depicted when $f_d T$ is set to 0.01. The product $M \times G$ is kept constant to maintain a fixed estimation latency of 28.8 ms. As expected, this figure clearly demonstrates that the system performance improves when more pilot subcarriers per OFDM symbol are used. Employing more than one pilot subcarriers does not increase the latency since they belong to the same estimation group and the increase in the complexity of the algorithm is negligible.

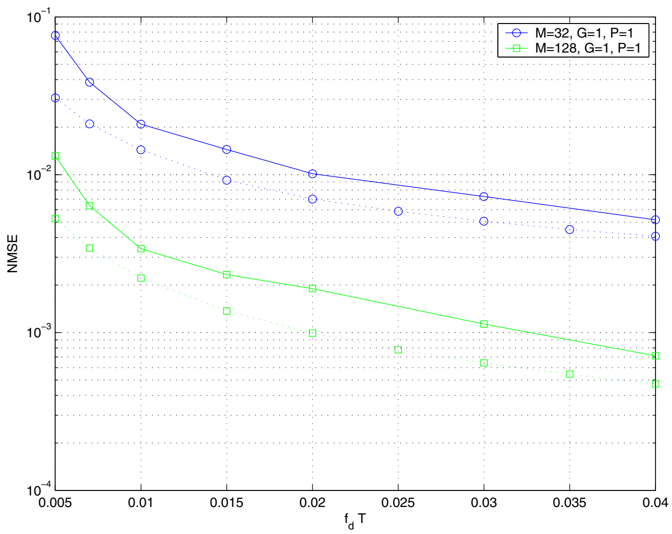

Fig. 4. NMSE and CRB vs. $f_d T$ for SNR = 30dB.

So far we have employed a maximum normalized Doppler frequency value of 0.01. Fig. 4 demonstrates the robustness of the estimator at different levels of ICI (i.e., different values of $f_d T$) when $P = G = 1$, $M = 32$ or 128, and the SNR is 30dB. The theoretical CRB lower bound values are also provided. Since the proposed algorithm takes ICI into account, the performance of the estimator does not deteriorate at higher values of $f_d T$ where significant ICI is present.

V. Conclusion

We have derived a ML algorithm to estimate the maximum Doppler frequency for OFDM systems in time-varying Rayleigh fading channels. The algorithm has no extra overhead since the already-existing pilot subcarriers are employed for the estimation process. The proposed estimator can be implemented via a filter bank whose coefficients can be stored in the system memory for low-complexity implementation. Many design parameters associated with the proposed algorithm are adjustable to meet various performance requirements. We have also given the Cramér-Rao bound for the MSE of the Doppler estimates. Simulation results verified the accuracy of the proposed algorithm even at high maximum Doppler frequency values since the effects of ICI are considered in the algorithm.

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