# Power Allocation for Distributed Transmit Diversity with Feedback Loop Delay

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***Abstract*—We study two power allocation (PA) schemes for distributed transmit diversity systems. We first derive the performance of the instantaneous channel gain feedback-based PA (ICG-PA) scheme in the presence of channel variation during feedback delay. We then study channel gain variance feedback-based PA (CGV-PA) to mitigate the performance degradation of ICG-PA caused by feedback delay. Finally, we derive design rules for optimum CGV-PA from a compact and accurate performance expression derived.**

***Index Terms*—Distributed transmit antennas, transmit diversity, power allocation, feedback delay, time varying fading.**

## I. Introduction

DISTRIBUTED transmit diversity (DTD)[1]–[5] has become a key feature of the air interface of cooperative relay systems, distributed multiple-input multiple-output (MIMO) antenna systems, and distributed base station systems. Unlike clustered (non-distributed) transmit diversity (CTD) systems where multiple transmit antennas are co-located on a single transmitter, transmit antennas in DTD systems are typically located far apart; thus each channel will undergo different large-scale fading. This results in nonidentical channel gain variances for each channel. In addition to the diversity from small-scale fading, the different channel variances in DTD systems could be exploited to provide an additional diversity, called 'macro-diversity', which is not available in CTD.

It has been shown that selective power allocation is optimum for DTD [3], that is, all the available power should be allocated to the transmit antenna that has the largest channel gain among the distributed transmit antennas. This conclusion is based on two main assumptions: availability of channel state information (CSI) including the instantaneous channel gain (ICG) and no feedback delay in the power allocation control loop. However, in a practical system, there will be an inevitable feedback delay due to signal generation/processing time and propagation time. Thus, due to channel variation during the feedback period, the transmit antenna with the highest channel gain at the time when measurement was taken might not have the highest gain anymore when the power control signal arrives at the receiver. Another problem of instantaneous-channel-gain-based power allocation (ICG-PA) is that it requires considerable overhead for feedback and processing, which becomes even higher in DTD as the number of simultaneous connections for a receiver increases.

In order to resolve the problems of ICG-PA, channel covariance-based power allocation is proposed for CTD in [6]. Since the correlation among the propagation paths is high in CTD, covariance feedback is a valid approach. In the DTD scenario considered in this work, the paths from distributed transmit antennas are uncorrelated. Therefore, covariance-based power allocation is not an appropriate scheme for DTD. In [1], [2], channel-gain-variance-based optimum power allocation (CGV-PA) is proposed for DTD. The term 'mean channel gain' in these papers refers to the mean-square value of the instantaneous channel gain. When the instantaneous channel gains is zero mean, complex Gaussian distributed, it corresponds to the variance of the instantaneous channel gain. Since the channel gain variance is determined by large-scale fading such as shadowing and path-loss, it varies slowly. Thus, in practical environments, the channel gain variance can be assumed constant during feedback delay. Compared with ICG-PA, the channel gain variance in CGV-PA could be reported to the transmitters with a fairly low frequency, which will significantly reduce the feedback load. CGV-PA is especially suitable for DTD since the instantaneous channel gain fluctuation that is not considered in power allocation is inherently compensated in the receiver through diversity combining of the signals from the distributed transmit antennas.

The effects of feedback delay to the performance of power allocation in code-division multiple-access systems, MIMO diversity systems, and space-time coded transmit diversity systems are analyzed in [8], [9], and [10] and [11], respectively. These works assume a single transmit antenna [8] or clustered transmit antennas such that the channel gain variances from all transmit antennas are identical [9]–[11]; thus, there is no macro-diversity. Optimum CGV-PA in the sense of minimum outage probability for DTD is derived in [1], [2]. The optimum power allocation rule derived in [1], [2] cannot be used to minimize the error rates given a certain modulation because even with the same outage probability, the symbol error rates could be quite different depending on the distribution of the received signal-to-noise ratio (SNR) and the outage threshold settings. Our previous work [12] has studied the performance degradation of small-scale-fading-based power allocation caused by feedback delay. However, the observations made in [12] are mainly obtained

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from simulation, and the impact of feedback delay on macro-diversity gain in DTD systems has not been well understood yet. Also, a systematic comparison of CGV-PA and ICG-PA has not been established by existing work.

The main focus of this paper is to analytically assess the performance degradation of ICG-PA in the presence of instantaneous channel gain variation during the feedback period. We also derive a compact and accurate expression that leads to design rules for optimum CGV-PA. We show that optimum CGV-PA is insensitive to feedback loop delay and establish that CGV-PA is a promising PA solution for DTD.

## II. System Model

### A. Channel and Signaling

Consider a DTD system with $N$ independent channels. The signaling method and channel model are basically the same as described in [1]–[3]. The channels might correspond to a set of relays capable of decoding the message transmitted by the source node or might correspond to a distributed transmit antenna system. We assume that the instantaneous channel gains (amplitude and phase) from the $N$ transmitters (or the nodes in a relay network) $h_i$, $i = 1, \cdots, N$, are independent complex Gaussian random variables with distinct variances $g_i, i = 1, 2, \cdots, N$ [1]–[3]. The variance $g_i$ represents the channel variance from the $i$th transmit antenna and is determined by the large scale fading factors such as path-loss and shadowing. Unlike CTD systems where multiple transmit antennas are co-located on a single transmitter, the transmit antennas in DTD are located apart; thus each channel undergoes a different large scale fading. This results in distinct $g_i$'s for different values of $i$.

The received signal from the $i$th transmit antenna (or node in relay networks) is given by [1], [2]

$$y_i = h_i \sqrt{p_i} s + n_i, \quad i = 1, 2, \cdots, N \tag{1}$$

where the transmitted signal $s$ with energy $E[ss^*] = E_s$ is common to all transmit antennas, $n_i$ is the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2 = N_0/2$, and $p_i$ denotes the normalized transmit power gain of the $i$th transmitter so that $\sum_{i=1}^{N} p_i = 1$.

Let $h_{i,(past)}$ denote the $i$th transmit antenna's latest instantaneous channel gain measurement obtained from the latest channel gain feedback and $h_i$ denote the current instantaneous channel gain at the instant when the power controlled signal according to $h_{i,(past)}$ arrives at the receive antenna after feedback delay. The channel gain variance is assumed to satisfy

$$\sigma^2_{h_{i,(past)}} = \sigma^2_{h_i} = g_i. \tag{2}$$

This assumption is reasonable since the coherence time of the large scale fading factor that determines the channel gain variance is much greater than the feedback delay in practical systems. The correlation coefficient between $h_{i,(past)}$ and $h_i$ is given by [8], [11], [16]

$$\frac{E[h_{i,(past)} h_i^*]}{\sigma^2_{h_{i,(past)}} \left(= \sigma^2_{h_i}\right)} = J_0(2\pi f_D \tau) \quad \equiv \quad \rho \tag{3}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, $f_D$ denotes the Doppler frequency, and $\tau$ denotes the feedback delay.

The correlation model satisfying (2) and (3) can be realized by using the linear combination of two independent variables as [10], [13]

$$h_i = \rho h_{i,(past)} + \sqrt{1-\rho^2} m_i \tag{4}$$

where $m_i$, $i = 1, 2, \cdots, N$, with variance $g_i$ are independent, zero-mean, complex Gaussian random variables.

### B. Diversity Combining in the Receiver

The decision variable $d$ in the receiver is obtained by a maximal ratio combiner (MRC), which combines the received signal from each transmit antenna as [1], [2]

$$d = \sum_{i=1}^{N} h_i^* \sqrt{p_i} y_i. \tag{5}$$

The received SNR is calculated as the sum of the received SNRs from each transmit antenna and is expressed as [1], [2]

$$\gamma_R \quad = \quad \sum_{i=1}^{N} \frac{|h_i|^2 p_i E_s}{N_0} = \gamma_0 \sum_{i=1}^{N} |h_i|^2 p_i \tag{6}$$

where $\gamma_0 = E_s/N_0$, which is the reference mean SNR from one of the transmit antennas when its channel gain variance is 1.

## III. Instantaneous Channel Gain-based Power Allocation (ICG-PA)

### A. Optimum ICG-PA Assuming Constant Instantaneous Channel Gains During Feedback Delay

When the instantaneous channel gain does not change during feedback delay, $\rho = 1$ $(f_D\tau = 0)$. The optimum ICG-PA rule for minimum instantaneous error rate is obtained by solving the following equation

$$\begin{aligned} &\{p_{1,opt}, p_{2,opt}, \cdots, p_{N,opt}\} \\ &\quad = \text{argmin}_{(p_1,p_2,\cdots,p_N)} P_e^{(inst)}(\gamma_R) \text{ subject to } \sum_{i=1}^{N} p_i = 1 \\ &\quad = \text{argmax}_{(p_1,p_2,\cdots,p_N)} \sum_{i=1}^{N} |h_i|^2 p_i \text{ subject to } \sum_{i=1}^{N} p_i = 1 \end{aligned} \tag{7}$$

where $P_e^{(inst)}(x)$ denotes the bit (or symbol) error rate given the instantaneous SNR $x$ in AWGN. If there is no instantaneous channel gain variation during feedback delay, then $h_{i,(past)} = h_i$. It can be easily shown that [3]

$$p_i = \begin{cases} 1, & \text{for } i = \arg\max_j |h_j|^2 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

maximizes $\sum_{i=1}^{N} |h_i|^2 p_i$ in (7). This means that selective power allocation is optimum for ICG-PA.

### B. Error Probability of Selective PA when the Instantaneous Channel Gain Varies During Feedback

In this case, $\rho < 1$ ($f_D\tau > 0$). Selective power allocation is optimum for the ideal case when the instantaneous channel gain does not change during feedback delay, i.e., $f_D\tau = 0$, which is a rather unrealistic assumption for practical mobile communication environments. In this subsection, we derive a closed-form expression of the error probability of selective PA for the case when the instantaneous channel gain varies during feedback, i.e., $f_D\tau > 0$.

Let $i_{\max}$ denote the index of the transmit antenna with the maximum instantaneous channel gain for selective power allocation. Note that $i_{\max}$ is not determined from current instantaneous channel gain; it is determined by using the instantaneous channel gain information $\tau$ seconds earlier, that is, $i_{\max} = \text{argmax}_j |h_{j,(past)}|^2$. Let $p_X(y)$ denote the probability density function (pdf) of $X$. The pdf of the received SNR conditioned on $i_{\max}$ can be obtained by applying Bayes' theorem as

$$p_{\gamma_R}(y) = \sum_{k=1}^{N} p_{\gamma_R}(y|i_{\max}=k)\Pr[i_{\max}=k]. \quad (9)$$

Applying Bayes' theorem again to (9) by taking $|h_{k,(past)}|^2$ as the condition, we rewrite $p_{\gamma_R}(y)$ as

$$\begin{aligned} &p_{\gamma_R}(y|i_{\max}=k)\Pr[i_{\max}=k] \\ &= \int_0^\infty p_{\gamma_R}\left(y|i_{\max}=k, |h_{k,(past)}|^2 = z\right) \times \\ &\quad \Pr\left[i_{\max}=k\Big|\left(|h_{k,(past)}|^2 = z\right)\right] p_{|h_{k,(past)}|^2}(z)dz. \end{aligned} \quad (10)$$

From (6) and (8), the first multiplicand in (10) is written as

$$\begin{aligned} &p_{\gamma_R}\left(y|i_{\max}=k, |h_{k,(past)}|^2 = z\right) \\ &= \text{pdf of } |h_k|^2\gamma_0 \text{ given } \left|h_{k,(past)}\right|^2 = z. \end{aligned} \quad (11)$$

From (4), we can rewrite $|h_k|^2$ as

$$\begin{aligned} |h_k|^2 &= \left|\rho h_{k,(past)} + \sqrt{1-\rho^2}m_k\right|^2 \\ &= \left|\rho\left|h_{k,(past)}\right|e^{j\angle h_{k,(past)}} + \sqrt{1-\rho^2}m_k\right|^2 \\ &= \left|\rho\left|h_{k,(past)}\right| + \sqrt{1-\rho^2}m_k e^{-j\angle h_{k,(past)}}\right|^2 \\ &= \left|\rho\sqrt{z} + \sqrt{1-\rho^2}m_k e^{-j\angle h_{k,(past)}}\right|^2 \end{aligned} \quad (12)$$

where $m_k e^{-j\angle h_{k,(past)}}$ is the phase-rotated version of $m_k$; thus, it is another complex Gaussian r.v. whose distribution is the same as that of $m_k$. Consequently, $|h_k|^2$ is the square of a complex Gaussian r.v. with mean $\rho\sqrt{z}$, that is, $|h_k|^2$ is a Ricean r.v. Thus, (11) is given as [15]

$$\begin{aligned} &p_{\gamma_R}\left(y|i_{\max}=k, |h_{k,(past)}|^2 = z\right) \\ &= \frac{1}{(1-\rho^2)g_k\gamma_0} e^{-\frac{\rho^2 z\gamma_0 + y}{(1-\rho^2)g_k\gamma_0}} I_0\left(\frac{\sqrt{y}\rho\sqrt{z\gamma_0}}{(1-\rho^2)g_k\gamma_0/2}\right) \end{aligned} \quad (13)$$

where $I_0(x)$ is the zeroth-order modified Bessel function of the first kind. The second and third multiplicands in (10) are calculated as

$$\begin{aligned} &\Pr\left[i_{\max}=k|\left(|h_{k,(past)}|^2 = z\right)\right] \\ &= \prod_{j\neq k}\Pr\left[|h_{j,(past)}|^2 < z\right] = \prod_{j\neq k}\left(1 - e^{-z/g_j}\right) \end{aligned} \quad (14)$$

$$p_{|h_{k,(past)}|^2}(z) = \frac{1}{g_k}e^{-z/g_k}. \quad (15)$$

Substituting (13), (14) and (15) into (10) and expanding it, we find that the integrand in (10) is a linear combination of the terms $e^{-az}I_0(b\sqrt{z})$ with $b = \frac{\rho\sqrt{y}}{(1-\rho^2)g_k\gamma_0/2}$ and different $a$'s.

Using the definite integral $\int_0^\infty e^{-az}I_0(b\sqrt{z})\,dz = e^{\frac{b^2}{4a}}/a$, we can obtain the closed-form expression for (10). Eq. (14) contains $2^{(N-1)}$ terms after expansion. In order to avoid tedious expansion, we take the example of $N=3$ to illustrate the closed-form expression next. Generalization to an arbitrary value of $N$ is straightforward. The only difference is that there will be more (or less for $N=2$) terms in the calculation. If we set $N=3$ and $k=1$, Eq. (10) is calculated as

$$\begin{aligned} &p_{\gamma_R}(y|i_{\max}=1)\Pr[i_{\max}=1] = \\ &\frac{e^{-\frac{y}{g_1\gamma_0}}}{g_1\gamma_0} - \frac{g_2 e^{-\frac{(g_1+g_2)y}{g_1\gamma_0(g_1+g_2-g_1\rho^2)}}}{g_1\gamma_0(g_1+g_2-g_1\rho^2)} - \frac{g_3 e^{-\frac{(g_1+g_3)y}{g_1\gamma_0(g_1+g_3-g_1\rho^2)}}}{g_1\gamma_0(g_1+g_3-g_1\rho^2)} \\ &+ \frac{g_2 g_3 e^{-\frac{(g_2 g_1 + g_3 g_1 + g_2 g_3)y}{g_1\gamma_0(g_2 g_1 + g_2 g_3 + g_3 g_1 - g_2 g_1\rho^2 - g_3 g_1\rho^2)}}}{g_1\gamma_0(g_2 g_1 + g_2 g_3 + g_3 g_1 - g_2 g_1\rho^2 - g_3 g_1\rho^2)}. \end{aligned} \quad (16)$$

By appropriately replacing $g_1$, $g_2$ and $g_3$ in (16), we can also obtain the closed-form expressions for $p_{\gamma_R}(y|i_{\max}=2)\Pr[i_{\max}=2]$ and $p_{\gamma_R}(y|i_{\max}=3)\Pr[i_{\max}=3]$. Finally, substituting these expressions for $p_{\gamma_R}(y|i_{\max}=k)\Pr[i_{\max}=k]$ into (9), we have a closed-form solution for the pdf of the received SNR, $p_{\gamma_R}(y)$.

The average error probability is calculated as

$$\begin{aligned} P_e &= \int_0^\infty p_{\gamma_R}(y)P_e^{(inst)}(y)dy \\ &= \sum_{k=1}^{N(=3)}\int_0^\infty p_{\gamma_R}(y|i_{\max}=k)\Pr[i_{\max}=k]P_e^{(inst)}(y)dy \end{aligned} \quad (17)$$

where $P_e^{(inst)}(y)$ denotes the bit error rate (BER) or symbol error rate (SER) for the instantaneous SNR in AWGN and is given as [15]

$$P_e^{(inst)}(y) = \frac{1}{2}\text{erfc}(\sqrt{y}) \text{ for BER of BPSK, QPSK with } y = E_b/N_0 \quad (18)$$

$$P_e^{(inst)}(y) \simeq \begin{cases} \text{erfc}\left(\sqrt{\sin\left(\frac{\pi}{M}\right)y}\right) & \text{for SER of MPSK} \\ \text{erfc}\left(\sqrt{\frac{3}{2(M-1)}y}\right) & \text{for SER of QAM} \end{cases} \text{ with } y = E_s/N_0 \text{ and } M > 4. \quad (19)$$

To derive further for a general expression, we define a unified expression for $P_e^{(inst)}(y)$ as

$$P_e^{(inst)}(y) = \alpha\text{erfc}\left(\sqrt{\xi y}\right) \quad (20)$$

where we appropriately set $\alpha$ and $\xi$ according to (18) and (19) for a specific modulation under consideration. Substituting

(16) and (20) into (17), and expanding it and using the integral $\int_0^\infty e^{-dy}\text{erfc}\left(\sqrt{\xi y}\right)dy = \frac{1}{d}\left(1-\sqrt{\frac{\xi}{d+\xi}}\right)$, we can obtain a closed-form solution of the average bit (or symbol) error probability as a function of $\rho$ as

$$\begin{aligned} P_e &= \alpha\left[X\left((g_1,g_2,g_3),\rho,\xi\gamma_0\right)+X\left((g_2,g_1,g_3),\rho,\xi\gamma_0\right)\right.\\ &\quad \left.+X\left((g_3,g_1,g_2),\rho,\xi\gamma_0\right)\right] \end{aligned} \tag{21}$$

where $X\left((p,q,r),\rho,S\right) = \frac{1}{3} - \frac{\sqrt{pS}}{\sqrt{1+pS}} + \frac{\sqrt{pS}\frac{q}{p+q}}{\sqrt{\frac{p+q}{p+q-\rho^2 p}+pS}} + \frac{\sqrt{pS}\frac{r}{p+r}}{\sqrt{\frac{p+r}{p+r-\rho^2 p}+pS}} - \frac{\sqrt{pS}\frac{rq}{pq+rp+qr}}{\sqrt{\frac{pq+pr+qr}{pq+qr+rp-\rho^2(pq+rp)}+pS}}$. The closed-form expressions (16) and (21) allow us to conveniently and accurately evaluate $p_{\gamma_R}(y)$ and $P_e$ as a function of $\gamma_0$, $g_1$, $g_2$, $g_3$ and $\rho$ without relying on time-consuming simulation for different parameter sets.

## IV. Near-Optimum Channel-Gain-Variance-based Power Allocation (CGV-PA)

### A. Error Probability for CGV-PA

The optimum CGV-PA rule for minimum average error rate is obtained by solving the following equations

$$\begin{aligned} &\{p_{1,opt},p_{2,opt},\cdots,p_{N,opt}\} \\ &\quad = \text{argmin}_{(p_1,p_2,\cdots,p_N)}P_e \text{ subject to } \sum_{i=1}^{N}p_i = 1 \end{aligned} \tag{22}$$

where $P_e$ is average error rate over the instantaneous channel gains given the channel gain variance.

The average error rate expression is obtained by calculating the following integral

$$\begin{aligned} P_e &= E_{\gamma_R}\left[\alpha\text{erfc}(\sqrt{\xi\gamma_R})\right] \\ &= \int_0^\infty \alpha\text{erfc}(\sqrt{\xi x})p_{\gamma_R}(x)dx \end{aligned} \tag{23}$$

where $p_{\gamma_R}(x)$ is the pdf of the received SNR $\gamma_R$ given the channel gain variances and transmit power.

From (6), we note that $\gamma_R$ is the sum of independent nonidentical exponential r.v.'s. Using the formula given in [14], we can write the closed-form expression for the average error rate as

$$P_e = \alpha\left(1-\sum_{l=1}^{N}A_l\sqrt{\frac{\xi\gamma_0 g_l p_l}{\xi\gamma_0 g_l p_l+1}}\right) \tag{24}$$

where $A_l = \prod_{k=1,k\neq l}^{N}\left(1-\frac{g_k p_k}{g_l p_l}\right)^{-1}$. The analytical solution to (22) given the error probability expression in (24) will be very complex. To simplify the analytical solution of (22), we use an approximation of the average error rate, which can be obtained by approximating erfc($\sqrt{x}$) in (23) as $e^{-2x}$. Then, the average error rate is calculated as

$$\begin{aligned} P_e &\simeq \int_0^\infty \alpha e^{-2\xi\gamma_R}p_{\gamma_R}(x)dx \\ &= \alpha\prod_{i=1}^{N}\frac{1}{2\xi\gamma_0 g_i p_i+1}. \end{aligned} \tag{25}$$

There exist better approximations of erfc($\sqrt{x}$) than the approximation as $e^{-2x}$ (e.g., [17]); however, they do not allow us to find an analytical solution to (22) as the one we adopt here. Simulation results in Sec. V reveal that the derived power allocation based on this approximation achieves almost the same BER as the optimum approach obtained from numerical search.

### B. Optimum CGV-PA in the Sense of Minimum Error Probability

The objective function to be minimized is given by (25), where $p_i$'s are the power allocation parameters to be optimized. The Lagrange function conditioned on a fixed total power is given by

$$M(p_1,p_2,p_3,\cdots,\lambda) = P_e + \lambda\left(\sum_i^N p_i - 1\right). \tag{26}$$

By letting the partial derivative of (26) with respect to $p_1,p_2,\cdots,p_N$ and $\lambda$ equal zero, we obtain $N+1$ equations

$$\begin{aligned} \frac{\partial M}{\partial p_i} &= \lambda - \frac{\xi\gamma_0 g_i}{(2\xi\gamma_0 g_i p_i+1)^2\prod_{j\neq i}^{N}(2\xi\gamma_0 g_j p_j+1)} = 0, \\ &\quad \text{for } i=1,2,\cdots,N \end{aligned} \tag{27}$$

$$\frac{\partial M}{\partial\lambda} = \sum_{i=1}^{N}p_i - 1 = 0. \tag{28}$$

The optimum power for the $i$th transmit antenna of CGV-PA with a total of $N$ transmit antennas is determined from (27) and (28) as

$$p_{i,opt}^{(N)} = \frac{1}{N} + \frac{1}{N\xi\gamma_0}\left[\sum_{k=1,k\neq i}^{N}\left(\frac{1}{g_k}-\frac{1}{g_i}\right)\right]. \tag{29}$$

Note that the only condition for this solution is $\sum_{i=1}^{N}p_i = 1$, while the sign of $p_i$ is not enforced. When $g_i$ is much smaller than $g_k$'s, $\forall k\neq i$, the solution of $p_{i,opt}^{(N)}$ could be negative. In this case, we set $p_{i,opt}^{(N)} = 0$, so that only $N-1$ antennas are actually transmitting signals. Therefore, it is important to reduce $N$ to $N-1$ and recalculate (29) for the remaining active channel links. This process must continue until the allocated powers for all active transmit antennas are positive. This power optimization process can be accomplished through the following steps:

- Step 1. Sort channel gains subject to $g_1 > g_2 > g_3 > \cdots > g_N$.
- Step 2. Calculate (29).
- Step 3. Check whether or not $p_{N,opt}^{(N)} > 0$. If yes, then go to Step 5; otherwise go to Step 4.
- Step 4. Set $p_{N,opt} = 0$ and $N = N-1$ and go to Step 2.
- Step 5. Set $p_{i,opt} = p_{i,opt}^{(N)}, i = 1,2\cdots,N$ and exit.

## V. Performance

First, we investigate the effect of feedback delay on the received SNR of selective ICG-PA, which is optimum for the ideal case – no feedback delay ($f_D\tau = 0$). In Fig. 1, we plot the pdf's of the received SNR of selective ICG-PA given in (9) with (16) for $N = 3$ and various values of $f_D\tau$ . In the ideal case, the received SNR equals the maximum

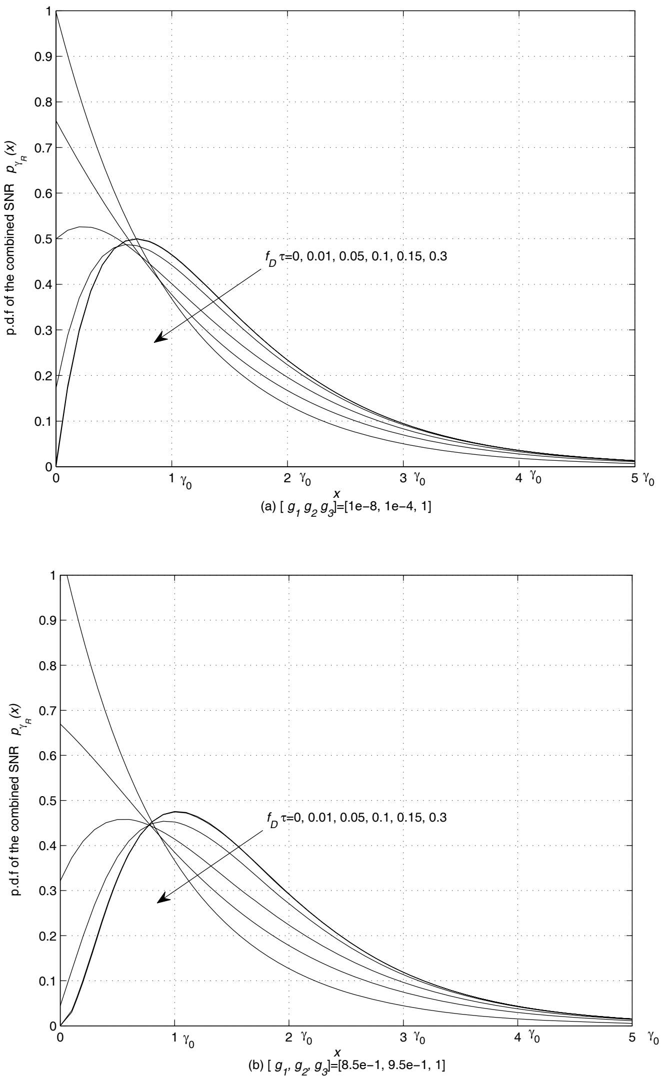


Fig. 1. The probability density functions of the received SNR of ICG-PA for various values of $f_D\tau$.

of the 3 nonidentical exponential random variables and it has a bell-shaped distribution. However, as $f_D\tau$ increases from 0, the pdf's approach the exponential distribution at an accelerated rate. For example, when $f_D\tau = 0.15$, which corresponds to the case that feedback delay is roughly 0.15 of the instantaneous channel gain fading cycle, the received SNR has an exponential-like distribution, which implies that we can hardly exploit the diversity from the different instantaneous channel gains of the multiple transmit antennas. Comparing Fig. 1(a) and Fig. 1(b) where the channel gains are set to [1, $10^{-4}$, $10^{-8}$] and [1, 0.95, 0.85], respectively, we find that the degradation in diversity gains becomes more significant when the channel gain variances do not differ appreciably from one another. When $f_D\tau = 0$, the received SNR reaches its peak around $0.7\gamma_0$ and $\gamma_0$ for the channel gains of [1, $10^{-4}$, $10^{-8}$] and [1, 0.95, 0.85], respectively. On the other hand, when $f_D\tau = 0.3$, the difference between the distributions of the received SNR for channel gains of [1, $10^{-4}$, $10^{-8}$] and [1, 0.95, 0.85] is very small. We thus conclude that the performance of ICG-PA becomes more sensitive to the feedback delay for scenarios where the receiver is located at the intersection of the coverage area of the transmit antennas.

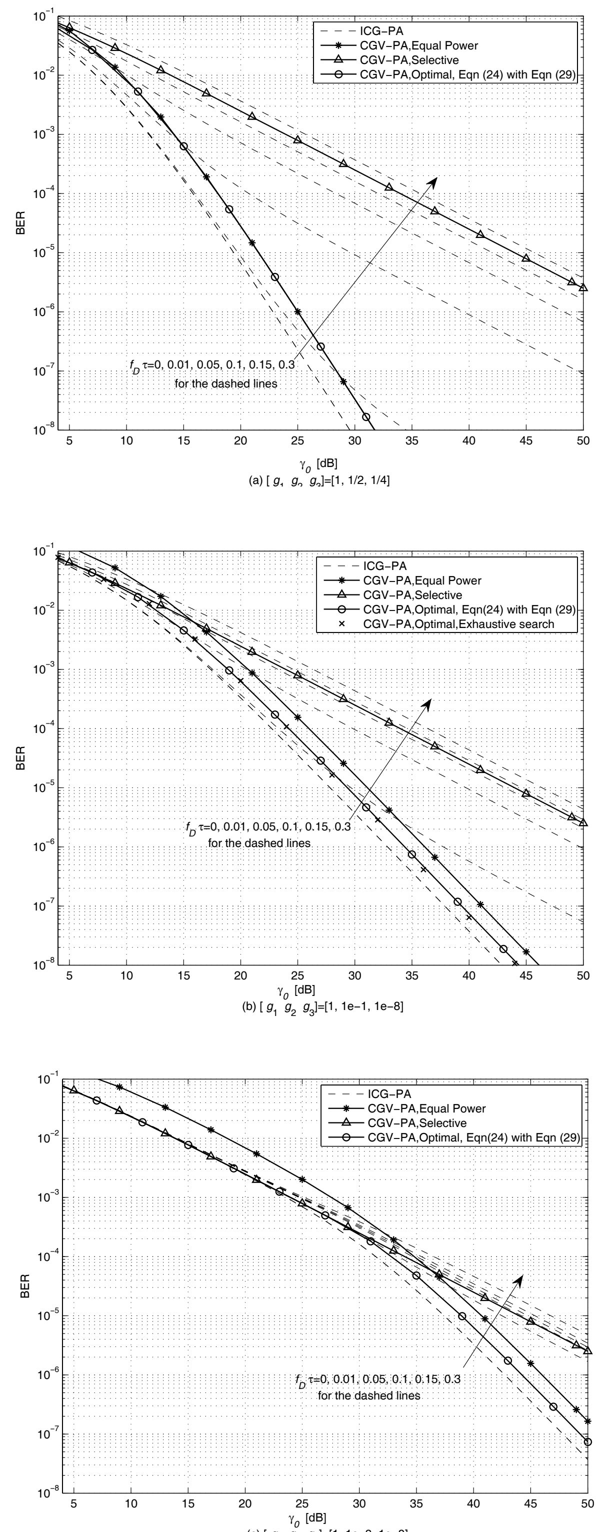


Fig. 2. Comparison of bit-error rate for various PA schemes (BPSK, $N$=3).

Next, we compare the error rates of the ICG-PA and CGV-PA schemes for several typical sets of the channel gain variance combinations. As a reference, two basic CGV-PA schemes are also included in addition to the optimal CGV-PA in (29): the selective CGV-PA, which allocates all transmit power to the transmit antenna with the maximum channel gain variance, and the equal power CGV-PA, which allocates the identical power to all transmit antennas. We consider the following three different scenarios of the receiver locations:

- Scenario 1: In the region where all 3 transmit antennas have comparable channel gain variances (Fig. 2(a)).
- Scenario 2: In the region where one of the transmit antennas has a relatively weak signal (Fig. 2(b)).
- Scenario 3: In the region where only one of the transmit antennas has a strong channel gain variance (Fig. 2(c)).

We plot the bit-error rates of various PA schemes for the three scenarios in Fig. 2(a)-(c). From (20), we note that different modulations result in only different scaling factors, $\alpha$ and $\xi$, to the error rate and SNR $\gamma_0$, respectively. The conclusions drawn for one modulation are thus applicable to other modulations. Therefore, without loss of generality, we limit our numerical examples to BPSK modulation.

First, as the main interest of this study, we observe that ICG-PA degrades drastically as $f_D\tau$ increases. It is important to note that although the difference in SNR distributions between $f_d\tau = 0.01$ and $f_d\tau = 0$ is very small (see Fig. 1), the difference in error probability for these two cases is not negligible. When $f_D\tau = 0.05$, BER degradation becomes unacceptable and the BERs of ICG-PA are greater than those of the optimal CGV-PA scheme. This shows that outdated feedback information significantly degrades the performance of ICG-PA even if the channel changes only slightly during the feedback period. As $f_D\tau$ increases, ICG-PA becomes even worse than the selective CGV-PA. Although this degradation is relatively reduced in Scenario 3, optimal CGV-PA and selective CGV-PA achieve almost the same performance as the ideal ICG-PA that assumes no feedback delay. From these observations, we conclude that even with very frequent and a large amount of channel state information to include instantaneous channel gains, ICG-PA is not a good solution for distributed transmit diversity systems if there is even a small feedback delay.

Regarding CGV-PA, first, it is found that optimal CGV-PA and equal power CGV-PA achieve the same diversity order as ideal ICG-PA without feedback delay ($f_D\tau = 0$). This is because the system still inherently achieves a diversity order $N$ by combining the signals with independent instantaneous channel gains in the receiver. In Scenario 1, equal power CGV-PA achieves nearly the same performance as optimal CGV-PA; in Scenario 3, selective power CGV-PA achieves almost the same performance as optimal CGV-PA in the SNR range of $\gamma_0 < 30$ dB. This agrees with our intuition. However, in Scenario 2 and in the SNR range of $\gamma_0 > 30$ dB in Scenario 3, the optimal CGV-PA rule in (29) achieves a significantly lower BER than equal power and selective CGV-PA schemes. It is observed that optimal CGV-PA achieves more than 1.5 dB SNR gain over equal power CGV-PA although the diversity order is the same. Note that even when the difference the among channel gain variances is large (approximately up to 20 dB), equal power CGV-PA has only a slight SNR loss compared to optimal CGV-PA in the high-SNR region [1], [2]. (See Fig. 5 in [1] and Figs. 2 and 3 in [2].) However, as the difference among channel gain variance exceeds 20 dB, like Scenarios 2 and 3 as shown in Fig. 2(b)(c), the performance gap between optimal CGV-PA and equal power CGV-PA is significant even in the high-SNR region.

It is also observed that the symbol-error-rate gap between selective PA and optimum PA could be substantial for some scenarios (e.g., Scenarios 1 and 2). This shows that simply switching between equal power CGV-PA and selective CGV-PA according to the combination of the channel gain variances is not a proper choice. In other words, an appropriately designed power allocation rule is critical for CGV-PA to achieve the best performance.

In order to assess the effect of the approximation made in (25) for the derivation of the closed-form expression for optimal power, we use exhaustive search to determine the optimal power combination. The corresponding BER results are shown in Fig. 2(b). Note that (24) with the derived power allocation rule in (29) agrees well with the minimum value of (24) obtained by numerical search.

## VI. Conclusions

We have considered transmit power allocation based on the channel variance in a DTD system and derived a simple but accurate optimum power allocation rule to achieve the minimum error rate. The proposed scheme resolves the feedback delay problem in instantaneous channel gain-based power allocation. Even with a small instantaneous channel gain variation, the performance of ICG-PA degrades significantly and the proposed CGV-PA scheme achieves a much lower error rate. In addition, the proposed CGV-PA is simple to implement, since it does not require frequent feedback and the computational complexity for optimal power setting is minimal. This is especially useful for DTD systems where multiple channel connections must be monitored simultaneously.

## References


[1] L. Cao, M. Tao, and P. Y. Kam, "Power control for MIMO diversity systems with nonidentical Rayleigh fading," *IEEE Trans. Veh. Technol.*, vol. 58, no. 2, pp. 998-1003, Feb. 2009.

[2] J. Luo, R. S. Blum, L. Cimini, L. Greenstein, and A. Haimovich, "Power allocation in a transmit diversity system with channel variance information," *IEEE Commun. Lett.*, vol. 9, no. 7, pp. 616-618, July 2005.

[3] L. Dai, S. Zhou, and Y. Yao, "Capacity analysis in CDMA distributed antenna systems," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2613-2320, Nov. 2005.

[4] D. S. Michalopoulos and G. K. Karagiannidis, "Distributed switch and stay combining (DSSC) with a single decode and forward relay," *IEEE Commun. Lett.*, vol. 11, no. 5, pp. 408-410, May 2007.

[5] D. S. Michalopoulos and G. K. Karagiannidis, "Two-relay distributed switch and stay combining," *IEEE Trans. Commun.*, vol. 56, no. 11, pp. 1790-1794, Nov. 2008.

[6] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. Inf. Theory*, vol. 47, no. 6, pp. 2632-2639, Sep. 2001.

[7] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Trans. Signal Process.*, vol. 50, no. 10, pp. 2599-2613, Oct. 2002.

[8] S. D. Fina and P. Lombardo, "Error probability analysis for CDMA systems with closed-loop power control," *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1801-1811, Oct. 2001.

[9] E. N. Onggosanusi, A. Gatherer, A. G. Dabak, and S. Hosur, "Performance analysis of closed-loop transmit diversity in the presence of feedback delay," *IEEE Trans. Commun.*, vol. 49, no. 9, pp. 1618-1630, Sep. 2001.

[10] E. K. S. Au, C. Wang, S. Sfar, R. D. Murch, W. H. Mow, V. K. N. Lau, R. S. Cheng, and K. B. Letaief, "Error probability for MIMO zero-forcing receiver with adaptive power allocation in the presence of imperfect channel state information," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1523-1529, Apr. 2007.

[11] J. Tang, X. Zhang, and Q. Du, "Alamouti scheme with joint antenna selection and power allocation over Rayleigh fading channels in wireless networks," in *Proc. IEEE Globecom'05*, Nov. 2005, pp. 3319-3323.

[12] D. Lim, K. Choi, and H. Liu, "Optimum power allocation for distributed antenna systems with large-scale fading-only feedback," in *Proc. IEEE ITNG'09*, Apr. 2009, pp. 1158-1164.

[13] H. T. Nguyen, J. B. Andersen, and G. F. Pedersen, "Capacity and performance of MIMO systems under the impact of feedback delay," in *Proc. 15th IEEE PIMRC*, vol. 1, 2004, pp. 53-57.

[14] R. M. Radayadeh and M. M. Matalgah, "Results for integrals involving $m$-th power of the Gaussian $Q$-function over Rayleigh fading channels with applications," in *Proc. IEEE ICC'07*, June 2007, pp. 5910-5914.

[15] J. G. Proakis, *Digital Communications,* 4th edition. New York: McGraw-Hill, 2001.

[16] W. C. Jake, Jr., *Microwave Communications.* New York: Wiley, 1974.

[17] M. Chiani, D. Dardari, and M. K. Simon, "New exponential bounds and approximations for the computation of error probability in fading channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 840-845, July 2003.