

Performance of Frequency Hopped Noncoherent GFSK in Correlated Rayleigh Fading Channels

Huaping Liu*, Vinod Venkatesan*, Curt Nilsen[†], Ron Kyker[†] and Mario E. Magaña*

*Department of Electrical and Computer Engineering
Oregon State University, Corvallis, OR 97331 USA
Email: hliu@ece.orst.edu, Telephone: +1 541 737 2973

[†]Instrumentation Systems Engineering Department
Sandia National Laboratories
MS 9101, PO Box 969, Livermore, CA 94550 USA
Email: canilse@sandia.gov

Abstract—An analytical treatment of the performance of frequency hopped Gaussian frequency shift keying (GFSK) with differential detection in time-varying Rayleigh fading channels corrupted by additive white Gaussian noise (AWGN) is presented. Closed-form expressions for the bit error rate (BER) as a function of E_b/N_0 , modulation index, channel Doppler shift, and the time-bandwidth product of the pre-modulation filter are derived. A pilot symbol-assisted scheme to lower the error floor caused by frequency hopping is analyzed.

I. INTRODUCTION

Gaussian baseband filtered frequency shift keying (GFSK) is an attractive modulation scheme for wireless communications because of its constant envelope, compact spectrum, and flexible bandwidth-performance tradeoffs. In this scheme, decreasing the time-bandwidth product BT results in a more compact spectrum at the expense of a higher BER for the same E_b/N_0 due to the increased level of inter-symbol interference (ISI). In a similar way, varying the modulation index effectively changes the signal separation at the sampling instants, thus provides tradeoffs between bandwidth and the system performance. As a special case when modulation index $h = 0.5$ is adopted, it results in the Gaussian minimum shift keying (GMSK). GMSK signals can be detected by a coherent detector, a differential detector [3], or a limiter discriminator [4]. Similar to GMSK, Gaussian FSK can also be detected by any of these three schemes. The differential detection scheme is robust to phase and frequency offsets and, at the same time, enables a low complexity design.

Frequency hopping (FH) provides excellent immunity to narrowband interference and frequency-selective fading. GFSK combined with FH schemes have been used in wireless personal communications systems [1] [2]. The error performance of GMSK systems in fading channels corrupted by AWGN has been analyzed extensively [3]-[13]. However, a tractable closed-form BER expression as a function of the E_b/N_0 , modulation index h , time-bandwidth product of the pre-modulation Gaussian filter BT , and the normalized

Doppler shift¹ $f_d T$ for a general frequency hopped GFSK system in time-varying Rayleigh fading channels has not been found. In this paper, we derive the closed-form expressions of BER as functions of $(E_b/N_0, h, BT, f_d T)$ for frequency hopped GFSK systems with a one-bit differential detection in time-varying Rayleigh fading channels.

This paper is organized as follows. Section II describes the models of the transmitter, the channel, and the receiver. Closed-form expressions for the BER are derived in Section III. Numerical results obtained from the analytical BER expressions and simulation results are given in Section IV, followed by concluding remarks in Section 5.

II. SYSTEM MODEL

A. Transmitter model

GFSK is a special case of continuous-phase modulated (CPM) signals employing frequency shift keying with spectral shaping of the input bits by a pre-modulation Gaussian filter before frequency modulation. The 3-dB bandwidth B of the pre-modulation filter is one of the parameters that controls the bandwidth of the GFSK spectrum; the lower the BT product (T is the symbol period), the narrower the GFSK spectrum. But decreasing BT results in higher ISI, and thus increases the BER. Another parameter that affects the spectrum width is the modulation index h ; the lower the modulation index, the narrower the spectrum. But decreasing h results in a shorter distance between signals at any sampling instant, and thus increases the BER.

In a frequency hopped GFSK system, the binary input bits are first converted into nonreturn to zero form. The rectangular antipodal bits of duration T are filtered by a Gaussian filter and then sent to the frequency modulator. The hop frequencies are generated locally at the frequency modulator and then radio frequency (RF) modulated with a fixed carrier frequency.

¹As in most papers, a normalized Doppler shift is adopted as it makes more sense in quantifying the channel fading rapidity.

The impulse response of the Gaussian low-pass filter $h(t)$ is expressed as

$$h(t) = \frac{1}{\sqrt{2\pi\sigma T}} \exp\left(-\frac{t^2}{2\sigma^2 T^2}\right) \quad (1)$$

where $\sigma = \frac{\sqrt{\ln(2)}}{2\pi BT}$, B is the 3-dB bandwidth of the filter, and T is the bit duration. The shaping pulse $g(t)$ is obtained by filtering the rectangular pulse with the Gaussian filter $h(t)$ as

$$g(t) = h(t) * \text{rect}\left(\frac{t}{T}\right) \quad (2)$$

where the rectangular function is defined by

$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1/T & \text{for } |t| < \frac{T}{2} \\ 0 & \text{otherwise.} \end{cases}$$

The shaping pulse can be written as

$$g(t) = \frac{1}{2T} \left[Q\left(\alpha \frac{BT}{T} \left(t - \frac{T}{2}\right)\right) - Q\left(\alpha \frac{BT}{T} \left(t + \frac{T}{2}\right)\right) \right] \quad (3)$$

where $\alpha = 2\pi/\sqrt{\ln(2)}$ and $Q(\cdot)$ is the Q -function defined as

$$Q(t) = \int_t^\infty \frac{1}{\sqrt{2\pi}} e^{-\tau^2/2} d\tau.$$

The phase of the CPM modulated signal is

$$\theta(t) = 2\pi h \int_{-\infty}^t \sum_{i=-\infty}^{\infty} b_i g(u - iT) du \quad (4)$$

where h is the modulation index and $b_i \in \{\pm 1\}$ is the i^{th} information bit. From (4), it is clear that the phase of the GFSK signal at any time instant is found by integrating the output of the filter until that time instant. The phase is not only affected by the current bit but also by all the previous and future bits due to the tail of the shaping pulse that is longer than one bit interval.

Assume that the channel is frequency-nonselctive during each hop. Then the frequency-hopped GFSK signal in the m^{th} hop can be represented as

$$s(t) = \sqrt{2E_b/T} \cos[2\pi(f_c + f_h(m))t + \theta(t)] \quad (5)$$

where f_c is the fixed carrier frequency, $f_h(m)$ is the hopping frequency for the m^{th} hop, and E_b is the signal energy per bit.

B. Channel model

A Rayleigh fading channel model with excellent frequency hopping capabilities is described in [14]. In this model, the fading processes at two different frequencies f_1 and f_2 are given by

$$c_1(t) = c_{i1}(t) + jc_{q1}(t) \quad \text{at } f_1 \quad (6a)$$

$$c_2(t) = c_{i2}(t) + jc_{q2}(t) \quad \text{at } f_2 \quad (6b)$$

where $c_{il}(t)$ and $c_{ql}(t)$, $l = 1, 2$, are zero-mean real Gaussian processes. $c_1(t)$ and $c_2(t)$ are correlated but gradually become independent when the absolute value of frequency separation $F = |f_1 - f_2|$ increases. For fading processes with normalized

power, the correlation properties between $c_{il}(t)$ and $c_{ql}(t)$, $l = 1, 2$, have been shown to be [15]

$$r_{c_{i1}c_{i1}}(\tau) = r_{c_{q1}c_{q1}}(\tau) = 0.5J_0(2\pi f_d \tau) \quad (7a)$$

$$r_{c_{i1}c_{q1}}(\tau) = 0 \quad (7b)$$

$$r_{c_{i1}c_{i2}}(\tau, F) = r_{c_{q1}c_{q2}}(\tau, F) = \frac{0.5J_0(2\pi f_d \tau)}{1 + (2\pi\gamma F)^2} \quad (7c)$$

$$r_{c_{i1}c_{q2}}(\tau, F) = -r_{c_{q1}c_{i2}}(\tau, F) = -(2\pi\gamma F)r_{c_{i1}c_{i2}}(\tau, F) \quad (7d)$$

where γ is the delay spread, f_d is the Doppler shift, and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind.

C. Receiver model

The GFSK signal has a continuous phase. The transmitted signal propagates through the time-varying Rayleigh fading channel and arrives at the receiver. The block diagram of the 1-bit differential demodulator can be found in [3]. Assume that the additive white Gaussian noise process $n(t)$ with a one-sided power spectrum density N_0 is added at the receiver input. The equivalent receive low-pass filter $h_r(t) \leftrightarrow H_r(f)$ is assumed to be an n^{th} -order Butterworth filter that yields an effective two-sided noise bandwidth given by [11]

$$B_{rn} = \beta B_r \quad (8)$$

where B_r is the two-sided 3-dB bandwidth of the filter $H_r(f)$ and

$$\beta = \frac{\pi/2n}{\sin(\pi/2n)} \quad (9)$$

is the transformation coefficient relating the 3-dB bandwidth B_r to the equivalent noise bandwidth B_{rn} . Further assume that B_r is appropriately chosen so that distortion to the signal component by the filter is negligible. Then the complex baseband received signal in the m^{th} hop can be modeled as

$$r(t) = \sqrt{2E_b/T} c_m(t) e^{j\theta(t)} + n_r(t) \quad (10)$$

where $c_m(t)$ is a complex Gaussian process of zero-mean representing the channel fading process in the m^{th} hop interval and $n_r(t)$ is the complex additive Gaussian noise process whose real and imaginary parts have zero mean and equal variance $\sigma_n^2 = N_0 B_{rn}$.

The received signal is sampled at $t = t_0 + iT$ and multiplied by its delayed version (delayed by T) after a 90° phase shift. For the differential detector to work properly given the pulse shape expression in (3), it is important that $t_0 = T/2$ must be chosen.

The output of the differential demodulator represents the phase change over the bit interval of interest. Depending on the phase change over the one bit interval a decision as to whether a "1" or a "0" was sent is made. If the phase change is positive, then a decision "1" is made. Otherwise, a decision "0" is made.

III. PERFORMANCE ANALYSIS

A. Decision variable

Let us introduce the following notations: $r_i = r(iT + T/2)$, $r_{i-1} = r((i-1)T + T/2)$. The decision variable of the one-bit differential detector for the i^{th} bit can be written as

$$\lambda_i = \Im\{r_i r_{i-1}^*\} \quad (11)$$

where $\Im\{\cdot\}$ denotes the imaginary part.

This decision variable in (11) can be re-written in a quadratic matrix form as

$$\lambda_i = \mathbf{v}^H \mathbf{R} \mathbf{v} \quad (12)$$

where

$$\mathbf{v} = \begin{bmatrix} r_i \\ r_{i-1} \end{bmatrix}$$

and

$$\mathbf{R} = \begin{bmatrix} 0 & j0.5 \\ -j & 0 \end{bmatrix}$$

where $j = \sqrt{-1}$. The decision rule is

$$\lambda_i \begin{cases} > 0, & \text{decide "1"} \\ < 0, & \text{decide "0"} \end{cases} \quad (13)$$

We assume, without loss of generality, that the i^{th} transmitted bit is a bit "1" for the derivation of the bit error rate. The covariance matrix of \mathbf{v} can be obtained as

$$\begin{aligned} \mathbf{V} &= E\{\mathbf{v}\mathbf{v}^H\} = E\left\{\begin{bmatrix} r_i \\ r_{i-1} \end{bmatrix} \begin{bmatrix} r_i \\ r_{i-1} \end{bmatrix}^H\right\} \\ &= \begin{bmatrix} 2E_b/T + 2N_0\beta B_r & (2E_b/T)J_0(2\pi f_d T)\rho \\ (2E_b/T)J_0(2\pi f_d T)\rho^* & 2E_b/T + 2N_0\beta B_r \end{bmatrix} \\ &= \frac{2N_0}{T} \begin{bmatrix} \frac{E_b}{N_0} + \beta B_r T & \frac{E_b}{N_0} J_0(2\pi f_d T)\rho \\ \frac{E_b}{N_0} J_0(2\pi f_d T)\rho^* & \frac{E_b}{N_0} + \beta B_r T \end{bmatrix} \end{aligned} \quad (14)$$

where $E\{\cdot\}$, $[\cdot]^H$, and $[\cdot]^*$ denote statistical expectation, Hermitian transpose, and complex conjugate, respectively, β is a constant defined in (9), B_r is the two-sided 3-dB bandwidth of the pre-detection receive filter $h_r(t)$, and ρ is given as

$$\rho = E\{e^{j[\theta_i - \theta_{i-1}]}\} = E\{e^{j\Delta\phi_i}\} \quad (15)$$

where

$$\Delta\phi_i = \theta_i - \theta_{i-1} = 2\pi h \sum_n b_n \int_{t_0+(i-1)T}^{t_0+iT} g(u-nT) du \quad (16)$$

is a function of h , the pre-modulation Gaussian filter time-bandwidth product BT , and the bit sequence. Again in (16) $t_0 = \frac{T}{2}$ must be chosen for the differential detector.

The expectation in (15) conditioned on $b_i=1$ depends on previous bits $(b_{-\infty}, \dots, b_{i-1})$ and future bits $(b_{i+1}, \dots, b_{\infty})$. In the absence of ISI with $h = 0.5$ and $BT = \infty$ for MSK, the phase change within the i^{th} symbol interval (conditioned on $b_i = "1"$) is exactly $\pi/2$ resulting in the expectation of $\rho = j$.

For all practical cases, the BT product is in this range (0.1 – 0.9). The average phase change over one bit interval as a function of BT considering all possible combinations of

bits around the bit of interest can be obtained by simulation. Such an average phase change conditioned on that the bit of interest is a "1" has been shown in Fig. 1. This curve can be approximated by the following n^{th} order polynomial as a function of BT

$$\Delta\phi = 2\pi h [a_0 + a_1(BT) + \dots + a_n(BT)^n]. \quad (17)$$

Fig. 1 shows the various polynomial fits to the conditional phase change $\Delta\phi$ normalized by $2\pi h$ along with the raw data. As can be seen any approximation with a 4th order or higher gives very accurate results for all practical cases of interest (BT from 0.1 to 0.9). For the 4th order approximation, the polynomial coefficients are $[a_0 \ a_1 \ a_2 \ a_3 \ a_4] = [-0.0240 \ 1.9854 \ -3.5938 \ 3.0993 \ -1.0219]$.

B. Error probability

For all bits except the first bit in each hop, the matrix $\mathbf{V}\mathbf{R}$ can be calculated as

$$\mathbf{V}\mathbf{R} = \frac{-jN_0}{T} \begin{bmatrix} \frac{E_b}{N_0} J_0(2\pi f_d T)\rho & -(\frac{E_b}{N_0} + \beta B_r T) \\ \frac{E_b}{N_0} + \beta B_r T & -\frac{E_b}{N_0} J_0(2\pi f_d T)\rho^* \end{bmatrix}. \quad (18)$$

As given in (15), ρ is a function of h and BT . Therefore, $\mathbf{V}\mathbf{R}$ is a function of $(E_b/N_0, h, BT, f_d T)$.

Since \mathbf{V} is a correlation matrix, all of its eigenvalues are non-negative and real. $\mathbf{V}\mathbf{R}$ has real eigenvalues, however, exactly half of its eigenvalues are negative, except for the extreme cases when $E_b/N_0 \rightarrow \infty$ and $f_d T \rightarrow 0$. The probability of error of the quadratic decision variable in (12) is [16]

$$P_e|_{\text{"1" sent}} = P\{\lambda_i < 0\} = \sum_{\gamma_l < 0} \prod_{\substack{m=1 \\ m \neq l}}^2 \frac{\gamma_l}{\gamma_l - \gamma_m}, \quad (19)$$

in which γ_1 and γ_2 are the eigenvalues of the matrix $\mathbf{V}\mathbf{R}$. Note that the probability of error given above is conditioned on a "1" being transmitted at the i^{th} bit interval. Assuming that a "1" or a "0" being transmitted is equally probable, the average BER P_e can be shown to have the same expression as $P_e|_{\text{"1" sent}}$.

In the case of frequency hopping, the correlation between one hop and the other can be modeled by (7c) and (7d). For the differential detector, the first bit in each hop will be based on a reference signal from the previous hop. When the average frequency separation between hops becomes large, the error probability of the first bit of each hop will be very high. With the fading process correlation properties given in (7), the correlation between channel coefficients for the last bit in a hop and the first bit in the next hop is determined to be $\frac{J_0(2\pi f_d T)}{1+(2\pi\gamma F)^2}$. Hence, the performance analysis for the first bit in every hop is done differently from the other bits. The covariance matrix for the first bit in every hop is given by

$$\mathbf{V}\mathbf{R} = \frac{-jN_0}{T} \begin{bmatrix} \frac{E_b}{N_0} J_0(2\pi f_d T) & -(\frac{E_b}{N_0} + \beta B_r T) \\ \frac{E_b}{N_0} + \beta B_r T & -\frac{E_b}{N_0} J_0(2\pi f_d T) \\ \frac{E_b}{N_0} J_0(2\pi f_d T) & -(\frac{E_b}{N_0} + \beta B_r T) \\ -\frac{E_b}{N_0} + \beta B_r T & \frac{E_b}{N_0} J_0(2\pi f_d T) \end{bmatrix} \rho^* \quad (20)$$

where ρ is still given by (15) and can be approximated by (17). If there are N bits in a hop, then the average BER is given by

$$P_e = \frac{N-1}{N}P_{e(N-1)} + \frac{1}{N}P_{e1} \quad (21)$$

where P_{e1} is the BER of the first bit in each hop obtained using the covariance matrix given in equation (20) and $P_{e(N-1)}$ is the BER for the rest of the bits in each hop obtained using the covariance matrix given in equation (18).

The Rayleigh fading channel introduces a random phase to the GFSK signal. The phase change between adjacent bits within a hop is small for a low Doppler shift. The phase continuity between the last bit in any hop and the first bit of the next hop is, however, invariably lost due to the sudden and random change in the phase of the channel. As will be confirmed by the theoretical evaluation and simulations later, the probability of error for the first bit in every hop increases as the frequency separation between two adjacent hops increases. This will cause an error floor even for a low Doppler shift.

The remedy proposed here is to insert a pilot symbol at the beginning of every hop. This will lower the spectral efficiency. However, depending on the hopping rate relative to the bit rate, the penalty in spectral efficiency can be very small. For example, for a system with a bit rate of 1Mbps and a hopping rate of 2000hops/s, the penalty in spectral efficiency is only 0.2% but the error floor caused by frequency hopping can be lowered considerably.

IV. NUMERICAL RESULTS

For all simulations, the Rayleigh fading channel coefficients are generated using the method described in [14]. Specifically, $c_h(t) = \sum_{n=1}^{N_h} c_{h,n} \cos(2\pi f_{h,n}t + \theta_{h,n})$ (h represents either i or q) at frequency f_1 , where N_h is the number of sinusoids. In order to ensure that $c_i(t)$ and $c_q(t)$ are uncorrelated, $N_i \neq N_q$ ($N_i = 20$ and $N_q = 21$ used in simulations) is chosen. Other parameters are given by $c_{h,n} = \sigma_h \sqrt{2/N_h}$ where $\sigma_h^2 = 0.5$, resulting in normalized channel power (i.e., $E\{|c(t)|^2\} = 1$), $f_{h,n} = f_d \sin\left[\frac{\pi}{2N_h}\left(n - \frac{1}{2}\right)\right]$, $\theta_{h,n} = 2\pi f_1 \phi_{h,n}$, and $\phi_{h,n} = \psi \ln\left(\frac{1}{1 - (n - 0.5)/N_h}\right)$, where $\psi = 108.6\text{ns}$ is the delay spread. The system bit rate is $r_b = 1\text{Mbps}$ with a hopping rate of 1600hops/s, which yields 625 bits per hop. The total spectrum that the system can hop to is 79MHz. The receive pre-detection filter $h_r(t)$ is assumed to be a 4th-order Butterworth filter resulting in $\beta = 1.0262$ and has a time-bandwidth product $B_r T = 1^2$.

Fig. 2 illustrates the performance of the system for three values of normalized Doppler shifts $f_d T$. The time-bandwidth product $BT = 0.5$ and the modulation index $h = 0.32$ are fixed. Simulation results are plotted on the same figure. Fig. 3 shows the analytical and simulated error performance of the system for two values of the modulation index h . $BT = 0.5$ and $f_d T = 2 \times 10^{-5}$ (equivalent to $f_d = 20\text{Hz}$ at a bit rate of

²It should be pointed out that when the pre-modulation Gaussian filter BT is large (e.g., greater than $B_r T$), the assumption made in Section II-C that distortion to signal component caused by $h_r(t)$ is negligible will be violated.

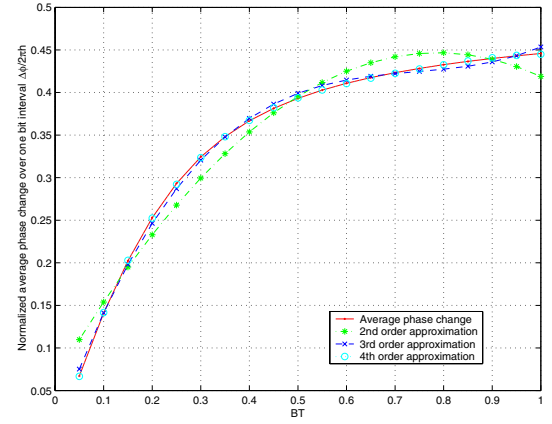


Fig. 1. Approximation to average phase change (normalized by $2\pi h$) over one bit interval as a function of BT

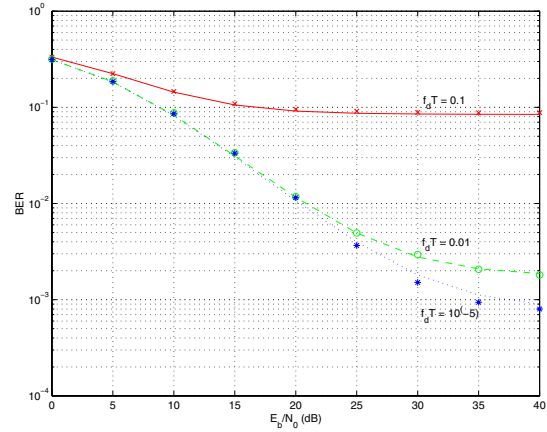


Fig. 2. Analytical and simulated (with marks) BER curves ($BT = 0.5$, $h = 0.32$)

1Mbps) are fixed. Fig. 4 gives the analytical error performance of the system for three values of BT . The normalized Doppler and the modulation index are fixed at $f_d T = 2 \times 10^{-5}$ and $h = 0.32$, respectively.

Fig. 4 clearly shows an error floor of about 10^{-3} even with extremely small Doppler shift. This error floor is caused by the significant phase change from one hop to the other resulting in a very high error probability for the first bit in each hop. In order to lower the error floor due the frequency hopping, a pilot symbol assisted scheme as described in Section III-B is analyzed and simulated. Fig. 5 shows the performance of the system for the case with pilot symbols. The BT product was fixed at 0.5, the modulation index was fixed at $h = 0.32$, and the normalized Doppler $f_d T = 2 \times 10^{-5}$ is chosen. Performance was compared with the case without pilot symbols for the same set of parameters. For the pilot symbol assisted scheme, an error floor did not appear for an E_b/N_0 value as high as 40dB with other parameters chosen.

V. CONCLUSION

The BER analysis for a slow frequency hopped GFSK system in time-varying Rayleigh fading channels corrupted

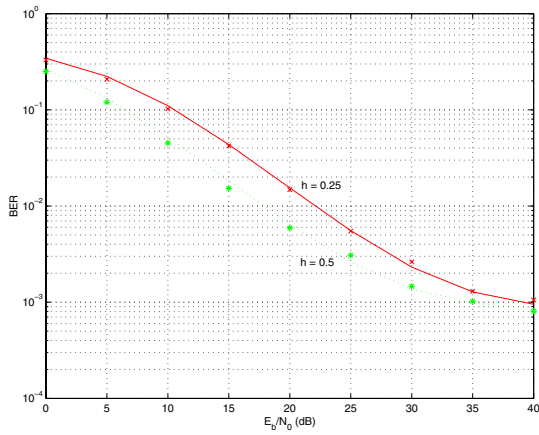


Fig. 3. Analytical and simulated (with marks) BER curves ($BT = 0.5$, $f_d T = 2 \times 10^{-5}$)

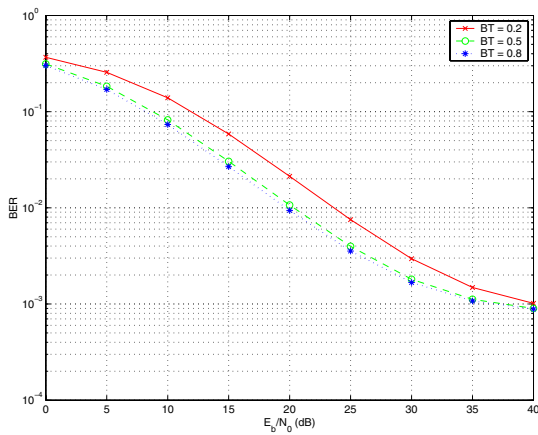


Fig. 4. Impacts of BT on BER performance (analytical with $h = 0.32$, $f_d T = 2 \times 10^{-5}$)

by additive white Gaussian noise was performed. Closed-form expressions for the BER as a function of E_b/N_0 , the normalized fading rapidity $f_d T$, the time-bandwidth product BT , and modulation index h of such a system were derived. The performance improvement of the pilot symbol assisted differential detection scheme has been presented. Simulation results validated all analytical performance formulas. Compared with existing performance analysis results for GMSK systems, the result in this paper is more comprehensive covering the general case of GFSK with frequency hopping and the analytical BER expressions are more tractable. These expressions are relatively simple and provide accurate error performance predictions with any set of parameter combinations.

REFERENCES

- [1] C. H. Park, Y. H. You, J. H. Paik, M. C. Jou, and J. W. Cho, "Channel estimation and DC-offset compensation schemes for frequency-hopped Bluetooth networks," *IEEE Communications Letters*, vol. 5, pp. 4-6, Jan. 2001.
- [2] K. V. S. Sairam, N. Gunasekaran, and S. R. Reddy, "Bluetooth in wireless communication," *IEEE Communications Magazine*, pp. 90-96, June 2002.
- [3] M. K. Simon and C. C. Wang, "Differential detection of Gaussian MSK in a mobile radio environment," *IEEE Trans. on Vehicular Technology*, vol. VT-33, pp. 307-319, Nov. 1984.

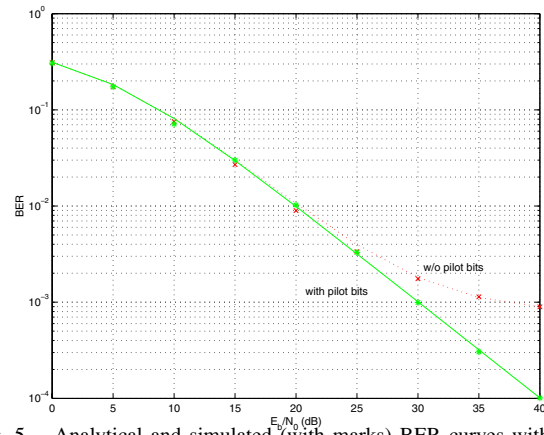


Fig. 5. Analytical and simulated (with marks) BER curves with pilot bits ($BT = 0.5$, $h = 0.32$, $f_d T = 2 \times 10^{-5}$)

- [4] S. M. Elnoubi, "Analysis of GMSK with discriminator detection in mobile radio channels," *IEEE Trans. on Vehicular Technology*, vol. 35, pp. 71-76, May. 1986.
- [5] N. Benvenuto, A. Salloum, and L. Tomba, "Further results on differential detection of GMSK signals," *IEEE Trans. on Communications*, vol. 45, pp. 761-764, Jul. 1997.
- [6] M. K. Simon and M. S. Alouini, "A unified approach to the probability of error for noncoherent and differentially coherent modulations over generalized fading channels," *IEEE Trans. on Communications*, vol. 46, pp. 1625-1638, Dec. 1998.
- [7] P. Varshnet and S. Kumar, "Performance of GMSK in a land mobile radio channel," *IEEE Trans. on Vehicular Technology*, vol. 40, pp. 607-614, Aug. 1984.
- [8] W. S. Smith and P. H. Wittke, "Differential detection of GMSK in Rician fading," *IEEE Trans. on Communications*, vol. 42, pp. 216-220, Feb./Mar./Apr. 1994.
- [9] A. Yongacoglu, D. Makrakis, and K. Feher, "Differential detection of GMSK using decision feedback," *IEEE Trans. on Communications*, vol. 36, pp. 641-649, Jun. 1994.
- [10] H. Mathis, "Differential detection of GMSK signals with low $B_f T$ using the SOVA," *IEEE Trans. on Communications*, vol. 46, pp. 428-430, Apr. 1994.
- [11] S. S. Shin and P. T. Mathiopoulos, "Differentially detected GMSK signals in CCI channels for mobile cellular telecommunications systems," *IEEE Trans. on Vehicular Technology*, vol. 42, pp. 289-293, Aug. 1993.
- [12] Y. T. Su, W. C. Kao, and J. S. Li, "The effects of Rician fading and multiple CCI on differentially detected GMSK signals," *Proc. of the 8th IEEE International Symposium on Personal, Indoor and Mobile Radio Communication*, vol. 3, pp. 959-963, 1997.
- [13] M. A. Wickert and J. M. Jacobsmeyer, "GMSK receiver performance with narrowband FM interference on a cellular radio channel," *Proc. of the 46th IEEE Vehicular Technology Conference*, vol. 2, pp. 756-760, 1996.
- [14] M. Pätzold, F. Laue, and U. Killat, "A frequency hopping Rayleigh fading channel simulator with given correlation properties," *Proc. of the 1997 IEEE International Workshop on Intelligent Signal Processing and Communication System*, pp. S8.1.1-S8.1.6, Malaysia 1997.
- [15] W. C. Jakes, *Microwave Mobile Communications*. New York, NY: John Wiley and Sons, Inc., 1974.
- [16] M. Barrett, "Error probability for optimal and suboptimal quadratic receivers in rapid Rayleigh fading channels," *IEEE Journal on Selected Areas in Communications*, vol. SAC-5, no. 2, pp. 302-304, Feb. 1987.
- [17] R. B. Lee, M. B. El-Arini, and N. L. Broome, "Impact of large Doppler on error performance of FH/MFSK with RS or BCH coding in Rayleigh and Rician fading," *IEEE Trans. on Communications*, vol. 42, pp. 2361-2365, Jul. 1994.
- [18] B. Kim and K. S. Kwak, "Performance of TDMA system with SFH and 2-bit differentially detected GMSK over Rayleigh fading channel," *Proc. of the 46th IEEE Vehicular Technology Conference*, vol. 2, pp. 789-794, 1996.