The Impact of Fading Correlation on the Error Performance of MIMO Systems Over Rayleigh Fading Channels

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***Abstract*—This paper analyzes the impact of receive fading correlation on the error performance of a multiple-input multiple-output (MIMO) system that employs a zero-forcing detection scheme over frequency-nonselective Rayleigh fading channels. Error rate expressions as a function of the eigenvalues of the fading correlation matrix and the number of transmit and receive antennas are derived. Numerical results indicate that MIMO systems are resistant to receive fading correlation.**

***Index Terms*—Fading correlation, multiple-input multiple-output (MIMO) systems, performance analysis.**

I. Introduction

MULTIPLE receiving antennas have been used in mobile communication systems to suppress or cancel interference [1], [2]. Multiple-input multiple-output (MIMO) systems apply multiple antennas at both ends of the wireless link, and have been shown to provide high spectral efficiencies [3], [4].

Existing research on MIMO techniques has focused on the theoretical capacity limit of MIMO systems (e.g., [4]–[8]) over Rayleigh fading channels. The diversity system in the presence of multiple cochannel interferers studied in [1] can be considered as an MIMO system. Based on the error-rate upper bound derived, it was concluded in [1] that with M receive antennas for N $(N \leq M)$ simultaneously transmitted data streams through independent flat Rayleigh channels, the N streams of data can be separated by using a zero-forcing (ZF) scheme, and $d = M - N + 1$ path diversity can be achieved by each of the N streams. In most practical MIMO systems, independent channels are difficult to achieve. The effect of fading correlation on the capacity of MIMO systems was studied in [7]–[9]. In [10], error-probability upper bounds for space–time coded systems over correlated and independent fading channels were given. The authors of [11] addressed the loss in diversity and coding gain of space–time codes due to spatial fading correlation, while the work in [12] examined the effect of the channel autocorrelation matrix on the performance a space–time code. In [13], a transmit selection algorithm that maximizes average throughput and minimizes the average probability of error was derived for spatial multiplexing systems with transmit spatial correlation. More recent work in [14] gave a framework for the analysis of spatial multiplexing in correlated MIMO channels and provided upper bounds of the pairwise error probability.

In this paper, we apply an analytical approach to study the error performance of spatial multiplexing MIMO systems that employ a ZF receiver in the presence of receive fading correlation[1] over a slowly fading frequency-nonselective channel. We show that in the presence of receive correlation, the $(M - N + 1)$-order diversity for a system with N transmit and M receive antennas still holds, but the relative branch signal-to-noise ratio (SNR) is determined by the eigenvalues of the fading correlation matrix.

The rest of this paper is organized as follows. Channel and system models are given in Section II. Error performance of the receiver is analyzed in Section III. In Section IV, we provide numerical results, followed by concluding remarks in Section V.

II. System Model

Consider a single-user system with N transmit and M $(M \geq N)$ receive antennas. We focus on the baseband model of the system that employs pulse amplitude modulation (PAM) with zero intersymbol interference (ISI) design. Input data are serial-to-parallel converted into N streams without space–time encoding and sent to the N transmit antennas for simultaneous transmission. The nth transmitted data stream is expressed as $x_n(t) = \sqrt{\mathcal{E}_s} \sum_{k=-\infty}^{\infty} s_n(k) p(t - kT)$, where $s_n(k)$ is the kth symbol of the nth data stream, $\mathcal{E}_s$ is the energy per symbol, $p(t)$ is the transmitted Nyquist pulse with $\int_{-\infty}^{\infty} p^2(t) \mathrm{d}t = 1$, and T is the symbol interval.

The received signal of the mth antenna is expressed as $r_m(t) = \sum_{n=1}^{N} h_{mn}(t) x_n(t) + \nu_m(t)$, where $h_{mn}(t)$ represents the channel fading process for the signal from the nth transmit antenna received by the mth receive antenna and $\nu_m(t)$ is a complex zero-mean white-Gaussian-noise process with a power spectral density N_0. The received signal $r_m(t)$ is filtered by a matched filter matched to $p(t)$, and then sampled at the symbol rate of each data stream. The received spatial signal vector at the matched filter output, $\boldsymbol{r}$ $(M \times 1)$, is represented by

$$\boldsymbol{r} = [\, r_1 \quad r_2 \quad \ldots \quad r_M \,]^{\mathrm{T}} \tag{1}$$

where $(\cdot)^{\mathrm{T}}$ denotes transpose. The $N \times 1$ data vector (over one symbol interval across N transmit antennas) is expressed as

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[1] Correlation among transmit antennas is not considered.

$\boldsymbol{s} = [\, s_1 \quad s_2 \quad \ldots \quad s_N \,]^{\mathrm{T}}$. The channel matrix $\boldsymbol{\mathcal{H}}$ $(M \times N)$ is expressed as

$$\boldsymbol{\mathcal{H}} = [\, \boldsymbol{h}_1 \quad \boldsymbol{h}_2 \quad \ldots \quad \boldsymbol{h}_N \,] \tag{2}$$

where $M \times 1$ vectors $\boldsymbol{h}_n = [\, h_{1n} \quad h_{2n} \quad \ldots \quad h_{Mn} \,]^{\mathrm{T}}$, $n = 1, \ldots, N$, represent the channel-fading coefficients from the nth transmit antenna to all M receive antennas. Each element of $\boldsymbol{\mathcal{H}}$ is a zero-mean complex Gaussian random variable (RV) of unit variance and is modeled as quasi-static [13], allowing it to be constant over a block of symbols (for each of the N parallel streams) and change independently to a new realization.

Receive correlation coefficients for multielement antenna systems were derived in [9]. It was shown that the normalized mutual correlation coefficient between any pair of antennas is $\rho_{ij} = E\{h_{in}h_{jn}^*\} = J_0((2\pi/l)d^{\mathrm{R}}(i,j))$ [9, eq. (6), case 1], where $(\cdot)^*$ denotes complex conjugate, $E\{\cdot\}$ denotes statistical expectation, $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind, l is the wavelength, and $d^{\mathrm{R}}(i,j)$ is the displacement between receive antennas i and j. For any physical arrangements of the receive antennas, the fading correlation matrix can be obtained as

$$\boldsymbol{R}_{h_n} = E\left\{\boldsymbol{h}_n\boldsymbol{h}_n^{\mathrm{H}}\right\} \tag{3}$$

where $(\cdot)^{\mathrm{H}}$ denotes conjugate transpose.

III. Receiver Analysis

A. Zero Forcing

The received signal vector $\boldsymbol{r}$ given in (1) can be written in matrix form as

$$\boldsymbol{r} = \sqrt{\mathcal{E}_s}\boldsymbol{\mathcal{H}}\boldsymbol{s} + \boldsymbol{\nu} \tag{4}$$

where $\boldsymbol{\nu} = [\, \nu_1 \quad \nu_2 \quad \ldots \quad \nu_M \,]^{\mathrm{T}}$ is the zero-mean complex Gaussian-noise vector with a covariance matrix $E\{\boldsymbol{\nu}\boldsymbol{\nu}^{\mathrm{H}}\} = N_0\boldsymbol{I}_M$ ($\boldsymbol{I}_M$ is the $M \times M$ identity matrix) and is independent of channel-fading processes. Although elements of $\boldsymbol{\mathcal{H}}$ might be statistically correlated, rows and columns of $\boldsymbol{\mathcal{H}}$ are assumed to be linearly independent [1], so that $\mathrm{rank}(\boldsymbol{\mathcal{H}}) = N$ and $\boldsymbol{\mathcal{H}}^H\boldsymbol{\mathcal{H}}$ is a full-rank matrix. In a practical spatial multiplexing system with $N \geq M$, the linear independence among columns of $\boldsymbol{\mathcal{H}}$ will be satisfied, except for the unrealistic scenario when the receive antennas are completely correlated.

Performance of the ZF receiver approaches that of the minimum mean-square error (MMSE) receiver at high SNR [1], [13], and it is easier to analyze the ZF structure. For these reasons, we apply a ZF receiver for which the received signal $\boldsymbol{r}$ is premultiplied by $\boldsymbol{\mathcal{H}}^+$, where $(\cdot)^+$ denotes the pseudoinverse. This results in the zero-forced spatial signal vector $\boldsymbol{y}$ $(N \times 1)$ as

$$\begin{aligned} \boldsymbol{y} &= [\, y_1 \quad y_2 \quad \ldots \quad y_N \,]^{\mathrm{T}} \\ &= \boldsymbol{\mathcal{H}}^+\boldsymbol{r} = \sqrt{\mathcal{E}_s}\boldsymbol{s} + \boldsymbol{\xi} \end{aligned} \tag{5}$$

where $\boldsymbol{\xi} = \boldsymbol{\mathcal{H}}^+\boldsymbol{\nu}$ is the noise vector.

Under the assumption of a quasi-static fading model, the instantaneous noise power of the nth data stream is calculated to be $[E\{\boldsymbol{\xi}\boldsymbol{\xi}^{\mathrm{H}}\}]_{nn} = N_0[\boldsymbol{\mathcal{H}}^+\boldsymbol{\mathcal{H}}^{+\mathrm{H}}]_{nn}$, where $[\cdot]_{nn}$ denotes the (nn)th element. Since $\boldsymbol{\mathcal{H}}^{\mathrm{H}}\boldsymbol{\mathcal{H}}$ is a full-rank matrix, it is easy to show [13] that $\boldsymbol{R}_\xi = E\{\boldsymbol{\xi}\boldsymbol{\xi}^{\mathrm{H}}\} = N_0(\boldsymbol{\mathcal{H}}^{\mathrm{H}}\boldsymbol{\mathcal{H}})^{-1}$. To simplify notations, we introduce a new variable (a positive real scalar) $\kappa_n = \{[(\boldsymbol{\mathcal{H}}^{\mathrm{H}}\boldsymbol{\mathcal{H}})^{-1}]_{nn}\}^{-1}$. In [1], it was shown that κ_n can be expressed in quadratic form as

$$\kappa_n = \boldsymbol{h}_n^{\mathrm{H}}\boldsymbol{G}\boldsymbol{h}_n \tag{6}$$

where $\boldsymbol{h}_n$ was defined in (2), $\boldsymbol{G}$ is an $M \times M$ Hermitian positive semidefinite matrix whose eigenvalues are either 1 or 0, and $d = M - N + 1$ of these eigenvalues are equal to 1. This conclusion still holds for the case being analyzed, because the proof given in [1] is not restricted to independent fading. Because $\boldsymbol{R}_{h_n}$ is a positive definite matrix and $\boldsymbol{G}$ is a positive semidefinite Hermitian matrix, the $M \times M$ matrix $\boldsymbol{R}_{h_n}\boldsymbol{G}$ is positive semidefinite, and has exactly d nonzero real eigenvalues (see [15, Th. 7.6.3]). Let the nonzero eigenvalues of $\boldsymbol{R}_{h_n}\boldsymbol{G}$ be $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_d$.

For the Hermitian quadratic variable in complex correlated Gaussian variates given in (6), there exists a linear transformation $\boldsymbol{\mathcal{T}}\boldsymbol{h}_n$ such that κ_n can be transformed into a diagonal Hermitian form with independent variates as [16, Appendix B, eqs. (B-2-3) and (B-3-5)]

$$\kappa_n = \sum_{i=1}^{M}\lambda_i|f_i|^2 = \sum_{i=1}^{d}\lambda_i|f_i|^2 = \boldsymbol{f}^{\mathrm{H}}\boldsymbol{\Lambda}\boldsymbol{f} \tag{7}$$

where $\boldsymbol{f} = [\, f_1 \quad \ldots \quad f_d \,]^{\mathrm{T}}$ is a zero-mean unit-variance complex Gaussian random vector with statistically independent elements (i.e., $E\{\boldsymbol{f}\boldsymbol{f}^{\mathrm{H}}\} = \boldsymbol{I}_d$) and

$$\boldsymbol{\Lambda} = \mathrm{diag}[\, \lambda_1 \quad \ldots \quad \lambda_d \,]. \tag{8}$$

Therefore, κ_n is a sum of chi-square distributed RVs, each scaled by the nonzero eigenvalues of $\boldsymbol{R}_{h_n}\boldsymbol{G}$.

The nth transmitted symbol s_n can be detected by passing the nth element of $\boldsymbol{y}$ through a decision device. Due to the correlation among the elements of $\boldsymbol{\xi}$ introduced by the ZF operation, the strategy to detect each component of $\boldsymbol{s}$ independently based on observation of $\boldsymbol{y}$ is not optimum. We still adopt this scheme because of its simplicity. To facilitate the error-performance analysis, we multiple y_n (the nth component of $\boldsymbol{y}$) with κ_n. Because κ_n is a positive real scalar, this process does not affect the decision. Let

$$z_n = \kappa_n y_n = \sqrt{\mathcal{E}_s}\kappa_n s_n + \zeta_n \tag{9}$$

where $\zeta_n = \kappa_n\xi_n$. The instantaneous noise component ζ_n conditioned on channel coefficients is a zero-mean Gaussian RV with $E\{\zeta_n\zeta_n^*\} = \kappa_n N_0$.

B. Error Performance

For simplicity, we focus on binary phase-shift keying (BPSK) for which $s_n \in (-1, 1)$ with $P(-1) = P(1) = 0.5$, and derive the bit-error-rate (BER) expression. The method and results can be easily extended to a general PAM scheme. We assume that the transmitted bit is $s_n = 1$ and determine the BER conditioned on $s_n = 1$.[2] The bit decision is obtained by taking the real part of z_n given in (9) and comparing it with the threshold "0."

The instantaneous SNR of z_n is obtained as

$$\alpha_n = \frac{\mathcal{E}_\mathrm{b}}{N_0}\kappa_n = \frac{\mathcal{E}_\mathrm{b}}{N_0}\sum_{i=1}^{d}\lambda_i |f_i|^2 \tag{10}$$

where we have replaced symbol energy $\mathcal{E}_s$ with bit energy $\mathcal{E}_\mathrm{b}$. For any given fading correlation matrix $\boldsymbol{R}_{h_n}$, the branch SNR is determined by the d nonzero eigenvalues of matrix $\boldsymbol{R}_{h_n}\boldsymbol{G}$. Generally, for two arbitrary matrices with known eigenvalues, it is difficult to relate the eigenvalues of their product matrix to that of the individual matrix. Let the eigenvalues of $\boldsymbol{R}_{h_n}$ be $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_M$. In the Appendix, we will show that the d nonzero eigenvalues of $\boldsymbol{R}_{h_n}\boldsymbol{G}$ are bounded by $\mu_1 \leq \lambda_i \leq \mu_M$, $i = 1, \cdots, d$. Therefore, BER upper and lower bounds[3] can be easily determined by finding, respectively, the minimum and maximum eigenvalues of the correlation matrix $\boldsymbol{R}_{h_n}$ and applying them in (8).

The variance of the zero-mean noise component of z_n is calculated to be $N_0 \sum_{i=1}^{d} \lambda_i |f_i|^2$. By examining (10), we found that (9) is equivalent to the maximal-ratio combined signal from d independent receiving antennas with unequal branch SNR determined by the nonzero eigenvalues of $\boldsymbol{R}_{h_n}\boldsymbol{G}$ over a frequency nonselective Rayleigh fading channel. If these nonzero eigenvalues were all equal or were all distinct, the error-rate expressions could have been easily obtained because α_n would have had a known distribution (see [17]). For the problem being addressed, these eigenvalues satisfy neither of these conditions, except for independent receive antennas, and the density function of α_n is difficult to obtain.

By expressing κ_n given in (7) as $\kappa_n = (\boldsymbol{\Lambda}^{1/2}\boldsymbol{f})^\mathrm{H}\boldsymbol{\Lambda}^{1/2}\boldsymbol{f}$, z_n given in (9) is modeled as

$$z_n = \left(\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{f}\right)^\mathrm{H}\left(\sqrt{\mathcal{E}_\mathrm{b}}s_n\boldsymbol{\Lambda}^{\frac{1}{2}}\boldsymbol{f} + \boldsymbol{\varepsilon}\right) \tag{11}$$

where $\boldsymbol{\varepsilon}$ is a $d \times 1$ noise vector with $E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^\mathrm{H}\} = N_0\boldsymbol{I}_d$ and $(\boldsymbol{\Lambda}^{1/2}\boldsymbol{f})^\mathrm{H}\boldsymbol{\varepsilon} = \zeta_n$. By introducing two complex Gaussian random vectors $\boldsymbol{w} = \boldsymbol{\Lambda}^{1/2}\boldsymbol{f}$ and $\boldsymbol{u} = \sqrt{\mathcal{E}_\mathrm{b}}s_n\boldsymbol{\Lambda}^{1/2}\boldsymbol{f} + \boldsymbol{\varepsilon}$, the decision variable can be written as

$$\phi_n = \Re\{z_n\} = \Re\{\boldsymbol{w}^\mathrm{H}\boldsymbol{u}\} = \boldsymbol{v}^\mathrm{H}\boldsymbol{Q}\boldsymbol{v} \tag{12}$$

where $\Re\{\cdot\}$ denotes the real part and

$$\boldsymbol{v} = \begin{bmatrix}\boldsymbol{w}\\ \boldsymbol{u}\end{bmatrix} \quad \boldsymbol{Q} = 0.5\begin{bmatrix}\boldsymbol{0}_d & \boldsymbol{I}_d\\ \boldsymbol{I}_d & \boldsymbol{0}_d\end{bmatrix}$$

where $\boldsymbol{0}_d$ is a $d \times d$ zero matrix. Conditioned on $s_n = 1$, the covariance matrix of the zero-mean vector $\boldsymbol{v}$ $(2d \times 1)$ is calculated as

$$\boldsymbol{V} = E\{\boldsymbol{v}\boldsymbol{v}^\mathrm{H}\} = \begin{bmatrix}\boldsymbol{\Lambda} & \sqrt{\mathcal{E}_\mathrm{b}}\boldsymbol{\Lambda}\\ \sqrt{\mathcal{E}_\mathrm{b}}\boldsymbol{\Lambda} & \mathcal{E}_\mathrm{b} + N_0\boldsymbol{I}_d\boldsymbol{\Lambda}\end{bmatrix} \tag{13}$$

where $\boldsymbol{\Lambda}$ was given in (8).

Let $\gamma_1, \ldots, \gamma_{2d}$ be the eigenvalues of the $2d \times 2d$ matrix $\boldsymbol{VQ}$ (product of $\boldsymbol{V}$ and $\boldsymbol{Q}$). Unlike $\boldsymbol{R}_{h_n}\boldsymbol{G}$, which has only d nonzero eigenvalues, $\boldsymbol{VQ}$ is a full-rank matrix with distinct eigenvalues, except for extreme values of $\mathcal{E}_\mathrm{b}/N_0$. The probability of error of the quadratic decision variable (conditioned on $s_n = 1$) in (12) is the one given in [18]

$$P_\mathrm{e} = P\{\phi_n < 0\} = \sum_{\gamma_i < 0}\prod_{\substack{j=1\\ j \neq i}}^{2d}\frac{\gamma_i}{\gamma_i - \gamma_j}. \tag{14}$$

IV. Numerical Results and Discussion

In all numerical examples, SNR per bit is defined as $\theta_\mathrm{b} = (M - N + 1)(\mathcal{E}_\mathrm{b}/N_0)$. Systems with $(M, N) = (4, 3)$, (3, 3), and (3, 2) that operate in the 1.9-GHz band are used to illustrate the impact of receive fading correlation. Receive antennas are assumed to be placed on the corners of an equilateral triangle (for $M = 3$) or a square (for $M = 4$) with a side length d^R. Based on the correlation model given in [9], the correlation matrix for a system with four receive antennas is expressed as

$$\boldsymbol{R}_{h_n} = E\left\{\boldsymbol{h}_n\boldsymbol{h}_n^\mathrm{H}\right\} = \begin{bmatrix}1 & \rho_1 & \rho_2 & \rho_1\\ \rho_1 & 1 & \rho_1 & \rho_2\\ \rho_2 & \rho_1 & 1 & \rho_1\\ \rho_1 & \rho_2 & \rho_1 & 1\end{bmatrix}$$

where $\rho_1 = J_0((2\pi/l)d^\mathrm{R})$ and $\rho_2 = J_0((2\pi/l)\sqrt{2}d^\mathrm{R})$. It should be pointed out that the analytical method can be applied for any given correlation matrix $\boldsymbol{R}_{h_n}$ and does not depend on the choices of the correlation model.

In simulations, the channel vector $\boldsymbol{h}_n$ with a correlation property defined by $\boldsymbol{R}_{h_n}$ is generated by premultiplying a zero-mean complex white Gaussian channel vector of unit variance $\boldsymbol{h}_{n,w}$ with $\boldsymbol{R}_{h_n}^{1/2}$. It is easy to verify that $\boldsymbol{h}_n = \boldsymbol{R}_{h_n}^{1/2}\boldsymbol{h}_{n,w}$ satisfies the desired correlation property. Detection of s_n follows exactly the processing given in (4), (5), and (9). In the analytical calculation of BER, we first determine the nonzero eigenvalues of matrix $\boldsymbol{R}_{h_n}\boldsymbol{G}$ (diagonal elements of $\boldsymbol{\Lambda}$). The eigenvalues of $\boldsymbol{VQ}$ can then be calculated for a specific value of $\mathcal{E}_\mathrm{b}/N_0$. Fig. 1 shows the BER curves with different values[4] of d^R. For all cases evaluated, the analytical and simulated BER curves (with markers) match very well. Compared with independent antennas, the performance degradation caused by receive fading correlation is found to be significant only when the distance between adjacent antennas d^R becomes very small (e.g., less than 5 cm).

Analytical BER curves versus d^R with SNR per bit fixed at $\theta_\mathrm{b} = 12$ dB are shown in Fig. 2. For the range of antenna-separation values considered, it is observed that BER

[2]With the assumption of equally probable information bits 1 and -1, the average P_e has the same expression as the conditional P_e.

[3]This is a much simpler way to estimate the impact of fading correlation on the system performance.

[4]The range of d^R ($3 \sim 50$ cm) chosen is appropriate for implementation of multiple antennas in handheld devices or small fixed terminals.

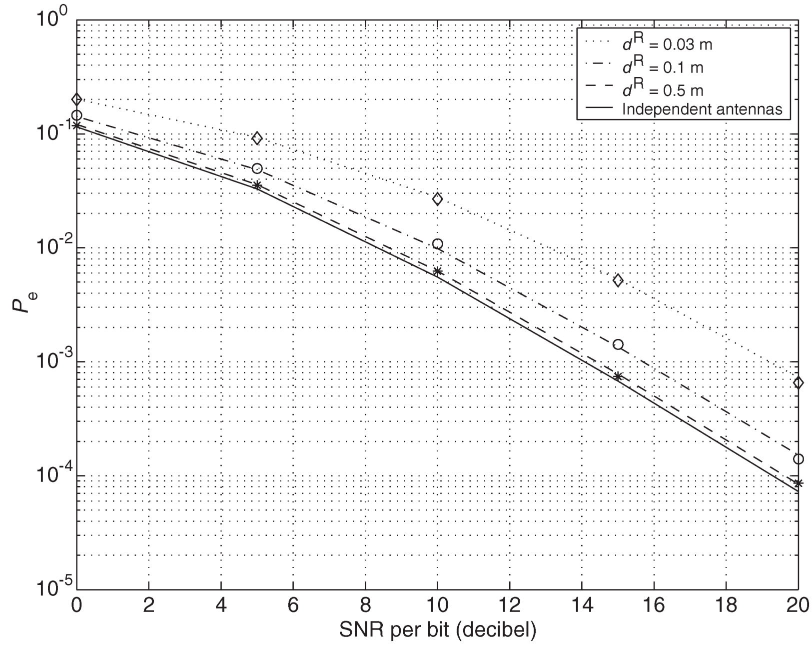

Fig. 1. Analytical and simulated (with markers) error-performance curves ($M = 4$, $N = 3$).

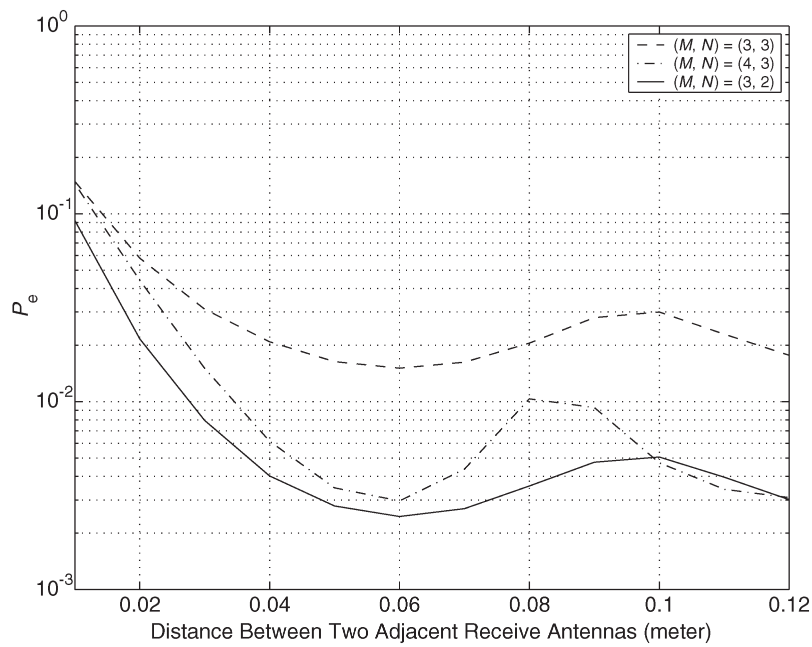

Fig. 2. Analytical BER versus d^{R} ($\theta_{\mathrm{b}} = 12$ dB).

performance does not improve monotonically as d^{R} increases. This is because the correlation coefficient $J_0((2\pi/l)d^{\mathrm{R}})$ is not a monotonically decreasing function of d^{R}. This indicates that there exists local optimal distances between antennas.

Figs. 3 and 4 show the BER upper and lower bounds calculated by using the minimum and maximum eigenvalues of correlation matrix $\boldsymbol{R}_{h_n}$ for a system with $(M, N) = (3, 2)$ and $d^{\mathrm{R}} = 0.1$ m and $d^{\mathrm{R}} = 0.2$ m. These bounds are quite tight for large antenna separations (e.g., $d^{\mathrm{R}} > 15$ cm or about one wavelength), and become weak for highly correlated receive antennas (e.g., d^{R} is a fraction of a wavelength).

In the presence of receive correlation, the diversity order $d = M - N + 1$ is still clearly seen in (10). For independent fading ($\lambda_1 = \cdots = \lambda_d = 1$), it results in d-order diversity with equal-branch SNR. When d^{R} becomes small, some of the diagonal elements of $\boldsymbol{\Lambda}$ become less than 1. This results in d-order diversity, but with unequal branch SNR.

The MMSE scheme provides the optimum balance between interantenna interference cancellation and noise enhancement. However, as mentioned in Section III, performance of the ZF receiver approaches that of the MMSE receiver at high SNR values. Therefore, the observations made in this paper are

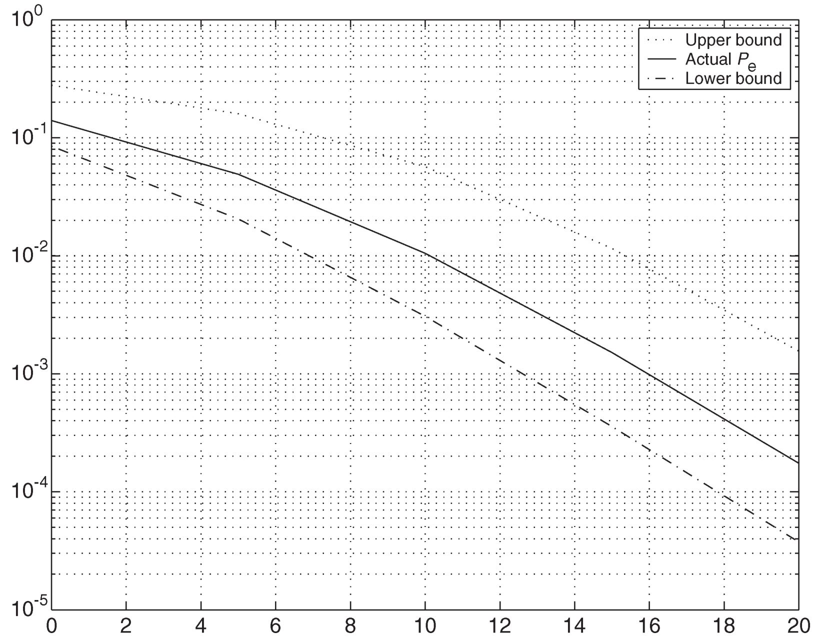

Fig. 3. Analytical performance bounds ($d^{\mathrm{R}} = 0.1$ m, $M = 3$, $N = 2$).

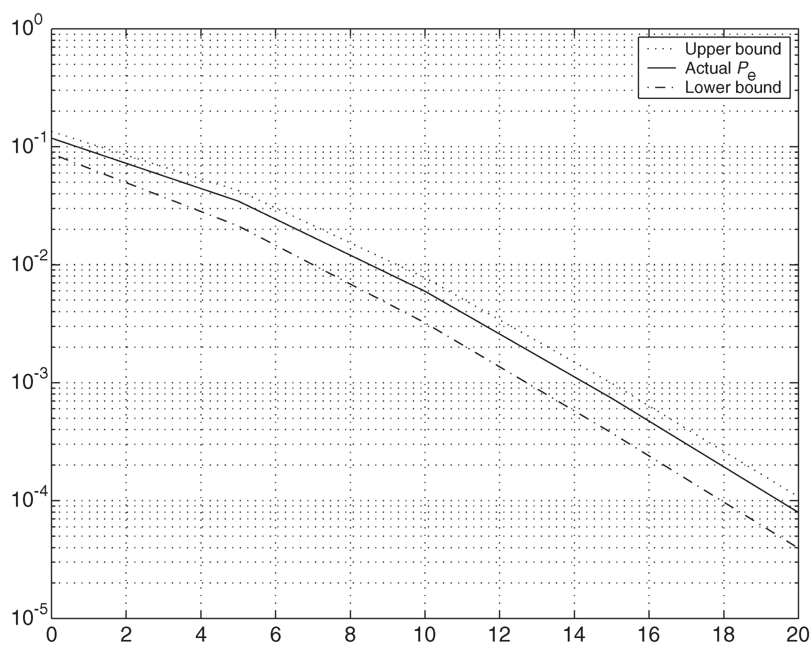

Fig. 4. Analytical performance bounds ($d^{\mathrm{R}} = 0.2$ m, $M = 3$, $N = 2$).

applicable to an MIMO system with an MMSE receiver at high SNR values.

V. Conclusion

The impact of receive fading correlation on the error performance of MIMO systems that employ a ZF receiver has been studied. With the set of system parameters chosen, performance degradation is about 2 dB compared to independent reception when antenna separation d^{R} is about half of a wavelength. BER performance does not monotonically improve as the antenna separation increases, which indicates that there exists local optimum distances. BER performance bounds can be obtained by using the maximum and minimum eigenvalues of the fading correlation matrix. These bounds are tight when antenna separations are comparable to or greater than one wavelength. The $(M - N + 1)$-order diversity, although with an unequal branch SNR, still holds for an (M, N) MIMO system in the presence of receive correlation.

Appendix

Let $\boldsymbol{A} \in \mathcal{C}^{M\times M}$ be a Hermitian matrix with eigenvalues $\beta_1 \le \beta_2 \le \cdots \le \beta_M$.

Lemma 1 [15, Th. 4.3.8: Interlacing Theorem]: Let $\boldsymbol{y} \in \mathcal{C}^M$ be a column vector and $a \in \Re$ be a real scalar. Let the eigenvalues of the $(M+1) \times (M+1)$ matrix $\hat{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{y} \\ \boldsymbol{y}^{\mathrm{H}} & a \end{bmatrix}$ be $\hat{\beta}_1 \leq \hat{\beta}_2 \leq \cdots \leq \hat{\beta}_{M+1}$. Then

$$\hat{\beta}_1 \leq \beta_1 \leq \hat{\beta}_2 \leq \beta_2 \leq \cdots \leq \hat{\beta}_M \leq \beta_M \leq \hat{\beta}_{M+1}.$$

Lemma 2: Let $\boldsymbol{A}_d \in \mathcal{C}^{d \times d}$ be a principal submatrix of $\boldsymbol{A}$ with eigenvalues $\beta_1^{(d)} \leq \beta_2^{(d)} \leq \cdots \leq \beta_d^{(d)}$. Then, it follows easily from Lemma 1 that

$$\beta_1 \leq \beta_1^{(d)} \leq \beta_2^{(d)} \leq \cdots \leq \beta_d^{(d)} \leq \beta_M.$$

Since $\boldsymbol{G}$ is a Hermitian and positive semidefinite matrix whose d nonzero eigenvalues are equal to 1, there exists a unitary matrix $\boldsymbol{P}$, such that $\boldsymbol{G} = \boldsymbol{P}^{\mathrm{H}} \boldsymbol{\Sigma} \boldsymbol{P}$ with

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{I}_d & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

Recall that the eigenvalues of positive-definite matrix $\boldsymbol{R}_{h_n}$ are $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_M$. Let $\boldsymbol{B} = \boldsymbol{R}_{h_n} \boldsymbol{G}$ and $\tilde{\boldsymbol{B}} = \boldsymbol{P} \boldsymbol{B} \boldsymbol{P}^{\mathrm{H}} = \boldsymbol{P} \boldsymbol{R}_{h_n} \boldsymbol{P}^{\mathrm{H}} \boldsymbol{\Sigma}$. Because $\boldsymbol{P}$ is a unitary matrix, the eigenvalues of $\tilde{\boldsymbol{B}}$ and $\boldsymbol{B}$ are identical. For the same reason, matrices $\boldsymbol{R}_{h_n}$ and $\boldsymbol{P} \boldsymbol{R}_{h_n} \boldsymbol{P}^{\mathrm{H}}$ also have identical eigenvalues. Let

$$\boldsymbol{P} \boldsymbol{R}_{h_n} \boldsymbol{P}^{\mathrm{H}} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} \end{bmatrix}$$

where the $d \times d$ matrix $\boldsymbol{R}_{11}$ is a principal submatrix of $\boldsymbol{P} \boldsymbol{R}_{h_n} \boldsymbol{P}^{\mathrm{H}}$. Because $\boldsymbol{R}_{h_n}$ is a real symmetric and positive definite matrix, $\boldsymbol{R}_{11}$ a Hermitian and positive-definite matrix. Matrix $\tilde{\boldsymbol{B}}$ is expressed as

$$\tilde{\boldsymbol{B}} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_d & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{0} \\ \boldsymbol{R}_{21} & \boldsymbol{0} \end{bmatrix}. \tag{15}$$

Because $\boldsymbol{R}_{11}$ a Hermitian and positive definite matrix, the eigenvalues of $\boldsymbol{R}_{11}$ are the nonzero eigenvalues of matrix $\begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{0} \\ \boldsymbol{R}_{21} & \boldsymbol{0} \end{bmatrix}$. Thus, from (15), it becomes clear that $\tilde{\boldsymbol{B}} = \boldsymbol{P} \boldsymbol{R}_{h_n} \boldsymbol{G} \boldsymbol{P}^{\mathrm{H}}$ and $\boldsymbol{R}_{11}$ have the same nonzero eigenvalues (all positive). Let us denote the eigenvalues of $\boldsymbol{R}_{11}$ (identical to the nonzero eigenvalues of $\boldsymbol{R}_{h_n} \boldsymbol{G}$) as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_d$. It follows, from Lemma 2, that $\mu_1 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_d \leq \mu_M$.

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