Index Assignment for Beamforming with Limited-Rate Imperfect Feedback

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***Abstract*— Beamforming in most multiple-antenna systems often requires channel state information (CSI) at the transmitter through feedback. In practice, CSI must be quantized into a finite set of vectors and feedback only sends the index representing the desired vector. In addition to quantization error of the channel coefficients, feedback errors, which lead to incorrect beamforming vectors to be applied at the transmitter, also degrade beamforming performance. We present an index-assignment algorithm that minimizes the impact of feedback errors. The proposed algorithm requires exhaustive search to find the best mapping. When the codebook size is large, the complexity of the algorithm becomes prohibitive. We thus propose a group-based index assignment (GIA) that has a low computational load while still performing better than random index assignments.**

***Index Terms*— Beamforming, index assignment, limited-rate feedback.**

I. INTRODUCTION

WIRELESS systems with multiple transmit antennas can use beamforming for reliable communications. Beamforming requires channel state information (CSI) at the transmitter, which is typically obtained through feedback from the receiver. Due to the finite rate of the feedback, CSI must be quantized into a finite set. Beamforming with limited-rate feedback has been studied extensively [1]–[4]. The Grassmannian line packing technique in beamforming vector quantization has been shown to have excellent performance [1], [2]. In these schemes, the receiver sends the index that represents a predefined set of quantized channel coefficients, rather than the actual coefficients.

Existing work on Grassmannian beamforming assumes an error-free feedback channel. In this case, the indexes can be arbitrarily assigned to the sets of coefficients. In practice, the indexes could be corrupted by feedback errors, causing the transmitter to apply the undesired set of channel coefficients. This letter proposes algorithms to optimize the index assignment so that when feedback is not error-free, the performance degradation of beamforming is minimized. We will compare the performance of the proposed scheme with that of the scheme used in [5].

II. SYSTEM MODEL

Consider a wireless system with M_t transmit antennas and M_r receive antennas. We assume that one data stream is transmitted using beamforming and the receiver uses maximum ratio combining (MRC). The received signal is expressed as

$$y = \boldsymbol{c}^H \boldsymbol{H}\boldsymbol{w}x + \boldsymbol{c}^H \boldsymbol{n} \tag{1}$$

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where $x \in \mathcal{C}$ is the transmitted symbol with average symbol energy $E_s = E[|x|^2]$, y is the observation after MRC, $\boldsymbol{c}$ is the $M_r \times 1$ MRC weight vector, $\boldsymbol{n}$ is the $M_r \times 1$ noise vector whose elements are independent and identically distributed (i.i.d.) complex Gaussian noise samples with mean zero and variance N_0, $\boldsymbol{w}$ is the $M_t \times 1$ beamforming vector, and $\boldsymbol{H}$ is the $M_r \times M_t$ channel matrix whose entries, h_{ij}, $1 \le i \le M_r$, $1 \le j \le M_t$, are complex Gaussian random variables with mean zero and unit variance. The beamforming vector $\boldsymbol{w}$ satisfies $\|\boldsymbol{w}\| = 1$ ($\|\cdot\|$ denotes L2 norm) to ensure that the total signal power allocated among M_t transmit antennas is normalized. The MRC vector $\boldsymbol{c}$ has the form

$$\boldsymbol{c} = a\boldsymbol{H}\boldsymbol{w} \tag{2}$$

to maximize the received signal-to-noise ratio (SNR), where a is a scalar that is typically chosen as $a = 1/\|\boldsymbol{H}\boldsymbol{w}\|$, so that $\|\boldsymbol{c}\| = 1$. After MRC, the instantaneous received SNR is $\gamma = (E_s/N_0)\|\boldsymbol{H}\boldsymbol{w}\|^2$.

The optimal beamforming vector $\boldsymbol{w}$ that maximizes the instantaneous SNR is expressed as

$$\boldsymbol{w}^* = \arg\max_{\|\boldsymbol{w}\|=1} \|\boldsymbol{H}\boldsymbol{w}\| \tag{3}$$

which turns out to be the right singular vector corresponding to the largest singular value of the channel matrix $\boldsymbol{H}$ [6]. When $\boldsymbol{H}$ has i.i.d. entries and the channel obeys the block-fading law [7], the optimal beamforming vector is uniformly distributed on the unit hypersphere Ω^{M_t} [1]. For systems with limited-rate feedback, it is impossible to send the optimal beamforming vector in (3) for each channel realization. A feasible solution is to partition the unit hypersphere Ω^{M_t} into N non-overlaying and exhaustive Voronoi regions, each of which is represented by a vector $\boldsymbol{w}_i$, $0 \le i \le N-1$. Thus there are N such vectors, which form a beamforming codebook $\boldsymbol{W} = \{\boldsymbol{w}_0, \boldsymbol{w}_1, \cdots, \boldsymbol{w}_{N-1}\}$ that is known to both the transmitter and receiver. Let B be the number of feedback bits required for each channel realization. Obviously $B = \lceil \log_2(N) \rceil$, where $\lceil x \rceil$ denotes the smallest integer that is not less than x. Each codeword is a B-bit index, which represents the beamforming vector in the codebook $\boldsymbol{W}$ for the corresponding Voronoi region. For each channel realization, the receiver chooses one vector from the codebook that maximizes the metric given in (3), and sends the corresponding index to the transmitter. The transmitter applies the beamforming vector indicated by the received index for beamforming.

III. INDEX ASSIGNMENT

A. *Near-optimum index assignment*

When feedback is error-free, the index-vector mapping can be chosen arbitrarily; otherwise, different index-vector mappings could result in different performance. In this letter, we use the term "good codebook" as the codebook that not only has the best beamforming vector set, but also the optimized index-vector mapping to minimize performance degradation due to feedback errors. While the construction of the best vector set of a codebook has been well studied [1], [2], [8], optimization of index assignment, the focus of this letter, has not been addressed. We focus on nontrivial cases where $N > M_t$ [1] and assume that $N = 2^B$ for simplicity.

The bit errors in the feedback index might cause the transmitter to apply the incorrect beamforming vector, thus reducing the instantaneous SNR at the receiver. For example, let $\boldsymbol{b}_i = [b_{i1}, \cdots, b_{iB}]$ be the index for vector $\boldsymbol{w}_i$, and $\boldsymbol{b}_j = [b_{j1}, \cdots, b_{jB}]$ for $\boldsymbol{w}_j$. If the receiver sends $\boldsymbol{b}_i$, but the transmitter receives $\boldsymbol{b}_j$, the performance will degrade as a result of incorrectly applying the weight vector $\boldsymbol{w}_j$. However, assuming that the index bits have an equal probability to be in error, the probabilities of receiving index $\boldsymbol{b}_j$ given the desired index $\boldsymbol{b}_i$, $P(\boldsymbol{b}_j|\boldsymbol{b}_i)$, will be different for different values of j when the error rate is not unrealistically high (e.g., not greater than 10^{-2}). For example, $P([1, \cdots, 1]|[0, \cdots, 0]) \ll P([0, \cdots, 0, 1]|[0, \cdots, 0])$. Thus SNR degradation will be minimized if indexes with higher transition probabilities are assigned to beamforming vectors that have a larger square magnitude of mutual inner products. This ensures that the degradation in the instantaneous received SNR due to occasional index-bit errors is minimized.

We line up the beamforming vectors in a codebook in an arbitrary but fixed sequence in the form $[\boldsymbol{w}_0, \boldsymbol{w}_1, ...\boldsymbol{w}_{N-1}]$ without assigning indexes. We then pick an index sequence from the $N!$ possible permutations in the form $[I_0, I_1, ..., I_{N-1}]$ to match $[\boldsymbol{w}_0, \boldsymbol{w}_1, ...\boldsymbol{w}_{N-1}]$, that is, I_k represents $\boldsymbol{w}_k$, where each I_k is a B-bit index. The index-vector mapping that maximizes the average received SNR maximizes the following target function:

$$C = \sum_{i=0}^{N-1} P(I_i) \sum_{\substack{j=0 \\ j \neq i}}^{N-1} P_{I_i I_j} |\boldsymbol{w}_i^H \boldsymbol{w}_j|^2 \tag{4}$$

where $P(I_i)$ is the probability that the receiver sends I_i for feedback and $P_{I_i I_j}$ is the transition probability from I_i to I_j.

There exists a class of codebooks, named equiangular frame (EF) codebooks [9]–[11], which have the property that the absolute values of the inner product of any pair of distinct vectors in the codebook are identical. For EF codebooks, (4) becomes constant; thus there is no need to optimize index-vector mapping. However, it is shown in [10] that equiangular vector sets do not exist for $N > M_t^2$. Additionally, EF codebooks work well only when the channel is modeled as quasi-static [7] and $\boldsymbol{H}$ has i.i.d. entries. When the channel is correlated in spatial and/or temporal domains, EF codebooks are far from being optimal [3], [12], [13].

We consider the most commonly accepted scenario that $\boldsymbol{H}$ has i.i.d. entries and the channel fading can be modeled as quasi-static in this letter. Under these conditions, we have $P(I_0) = P(I_1) = \cdots = P(I_{N-1}) = 1/N$. Additionally, under normal operation conditions the bit error rate (BER) of the feedback channel p_e would not be too high (e.g., not higher than 10^{-2}). Hence the probability that two or more index bits are in error will be much smaller than the probability that one index bit is in error. We can thus ignore the $O(p_e^2)$ terms in (4). Applying the symmetry property between $P_{I_i I_j}|\boldsymbol{w}_i^H \boldsymbol{w}_j|^2$ and $P_{I_j I_i}|\boldsymbol{w}_j^H \boldsymbol{w}_i|^2$ and after some mathematical manipulations, we rewrite (4) as

$$C = \sum_{i=0}^{N-1} \sum_{j>i}^{N-1} I(d_{I_i I_j}) |\boldsymbol{w}_i^H \boldsymbol{w}_j|^2 \tag{5}$$

where $d_{I_i I_j}$ is the Hamming distance between index I_i and index I_j, $I(d_{I_i I_j}) = 1$ if $d_{I_i I_j} = 1$ and $I(d_{I_i I_j}) = 0$ otherwise. With (5), we can search for the optimized index numerically.

B. Group index assignment scheme

When N is large, the computational load of exhaustive search becomes prohibitive. We propose a group-based index assignment (GIA) method that has a reduced search load and still outperforms random index assignment. The GIA method starts with a smaller-size good codebook of size N_p ($N_p < N$ but is made as large as possible) with optimized index-vector mapping, named the parent codebook expressed as $\boldsymbol{C}_p = [\boldsymbol{c}_0, \boldsymbol{c}_1, \cdots, \boldsymbol{c}_{N_p-1}]$. Any larger codebook with size N, named a child codebook, is partitioned into N_p non-overlapping groups. The partitioning procedure is given as

1) Initialize $i = 0$.
2) With $j = [i]_{N_P}$, where $[\cdot]_{N_P}$ denotes modulo-N_P operation, select a beamforming vector from the child codebook that has the largest magnitude of inner product with vector $\boldsymbol{c}_j$ and add it in the j-th group. A new child codebook with a reduced-size is then formed by removing this vector from the current child codebook.
3) Let $i = i + 1$. If $i < N$, repeat step 2); otherwise the partition process ends.

We assume that $N = 2^B$ and $N_p = 2^{B'}$, where B' and B are positive integers and $B' < B$. When B and/or B' are not integers, the extension is straightforward. After the partition, there are $2^{B'}$ groups, each of which has $2^{B-B'}$ beamforming vectors. For the j-th group, $0 \leq j \leq N_P - 1$, we copy the index of $\boldsymbol{c}_j$ as the B' most significant bits of the index of each beamforming vector within this group. For the rest $B - B'$ unassigned index bits, we perform random index-vector mapping for simplicity; additional performance improvement due to further optimization within the group is negligible. Thus the resulting index for each beamforming vector in the child codebook has B' most significant bits to indicate which group it belongs to, and $B-B'$ least significant bits for index mapping within the group.

IV. SIMULATION RESULTS

We simulate the performance of two codebooks from [5] with different index assignment strategies. The simulations assume binary phase-shift keying modulation and perfect knowledge of $\boldsymbol{H}$ to the receiver. We use BER to denote the average bit error rate of information bits and FBER to denote the feedback channel BER. BER is averaged over 2×10^4 channel realizations. For each channel realization BER is calculated by considering all possible received indexes and the corresponding probabilities of receiving them.

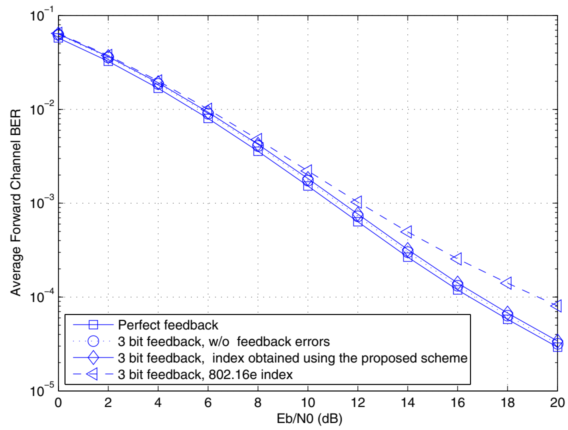

Fig. 1. BER performance of various schemes (3-bit feedback, FBER=10^{-2}, and (M_t, M_r)=$(2, 1)$).

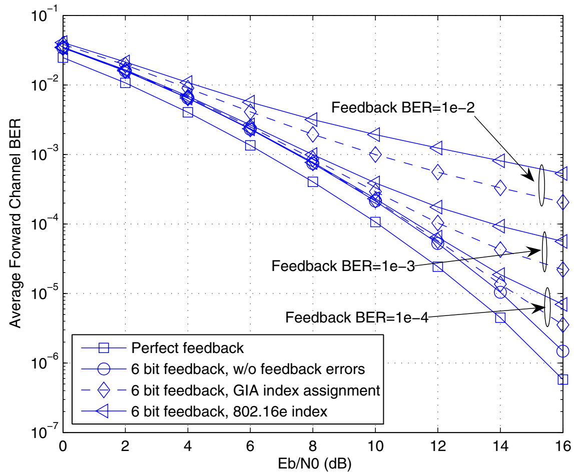

Fig. 2. BER of various schemes (6-bit feedback, $(M_t, M_r) = (3, 1)$).

Simulation 1: The system has two transmit antennas and one receive antenna and applies codebook $V(2, 1, 3)$ from Table $298m$ in [5] with 3-bit feedback. Fig. 1 shows the BER performance with different index assignments. The curve labeled 'perfect feedback' is obtained assuming that the transmitter has perfect knowledge of the channel coefficients. The case of '3-bit feedback without feedback error' means that the indexes are always received correctly by the transmitter. The optimized index assignment is obtained by applying the proposed method to maximize the target function in (5). The proposed scheme is found to achieve nearly the same performance as the case of no feedback errors when FBER is at 10^{-2}. At a BER of 10^{-4}, the proposed scheme achieves a gain of about 2 dB over the scheme in [5], and more at BER values below 10^{-4}.

Simulation 2: The system has three transmit antennas and one receive antenna, and applies codebook $V(3, 1, 6)$ from Table $298u$ in [5] with 6-bit feedback. Since the computational load to determine the optimal index sequences for 6-bit codebook is prohibitively high, we use the proposed GIA method. We adopt the 3-bit codebook from [1] as the parent codebook. It is easy to verify that $|\boldsymbol{w}_i^H \boldsymbol{w}_j| = 0.5$ for $0 \leq i \neq j \leq 7$. It is an EF codebook, and thus we can randomly select an index sequence to maximize (5) and then follow the procedure given in Section III-B to complete the index assignment. Fig. 2 shows the performance of various index assignment schemes at different FBERs for this scenario. Comparison with the scheme adopted in [5]: at FBER of 10^{-2}, the proposed GIA scheme achieves a gain of about 3 dB at BER of 10^{-3}; at FBER 10^{-3}, the gain is about 1.8 dB at BER of 10^{-4}; at FBER=10^{-4}, the gain is about 0.75 dB at BER of 10^{-5}.

Note that although the improvement in performance decreases as the FBER reduces, there is no penalty implementing the codebook optimized using the proposed exhaustive-search based and GIA methods for any scenarios.

V. Conclusion

We have presented two index-assignment methods, an exhaustive-search based algorithm and the GIA scheme, to improve beamforming performance in the presence of feedback errors. When the complexity of exhaustive search becomes prohibitively high, the GIA scheme can be applied. The improved codebooks outperform the codebooks adopted in [5]. The gain over the schemes in [5] decreases when the feedback channel BER decreases; however, once the optimization is complete, there is no penalty implementing the codebook obtained using the proposed methods.

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