Spatial-Correlation-Based Antenna Grouping for MIMO Systems

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***Abstract*—We investigate antenna-grouping algorithms, which are hybrids of beamforming and spatial multiplexing. We partition transmit antennas into several groups and use beamforming in a group and spatial multiplexing between groups. With antenna grouping, we can achieve diversity gain through beamforming and spectral efficiency through spatial multiplexing. In this paper, we review existing criteria and present several new criteria in multiple-input–multiple-output (MIMO) antenna-grouping systems where the number of transmit antennas is larger than that of receive antennas. We then propose a novel low-complexity antenna-grouping algorithm, which reduces the computational complexity with little degradation of the bit error rate (BER). Comparing the BER and complexity with existing criteria, we demonstrate the benefits of the proposed algorithm. We also derive the BER lower bound of the proposed algorithm in independent and identically distributed (i.i.d.) channels by using the probability density function of the largest eigenvalue of a Wishart matrix.**

***Index Terms*—Antenna grouping, beamforming, multiple-input–multiple-output (MIMO), spatial correlation.**

I. Introduction

MULTIPLE-INPUT–multiple-output (MIMO) systems have been drawing much attention because of their high spectral efficiency and reliable transmission of data [1], [2]. In MIMO systems, spatial multiplexing is the mode in which all transmit antennas send independent data streams. With spatial multiplexing, we can achieve high throughput, but the bit-error-rate (BER) performance could be poor, particularly when there is channel correlation. On the other hand, all transmit antennas can send the same data stream so that we can get good BER performance, which is called the diversity mode. However, the amount of data that can be transmitted in one transmission is limited. Generally speaking, good spectral efficiency results in worse BER performance in a spatial multiplexing system, whereas good BER performance leads to lower spectral efficiency in a diversity system. The tradeoff between diversity and multiplexing in MIMO systems is thoroughly discussed in [3].

It is well known that the channel information known at the transmitter improves system performance [4], and to obtain channel-state information (CSI), channel feedback is required. It is called a closed-loop (CL) system, and beamforming is a type of a CL system. It involves singular value decomposition (SVD) of a channel matrix H and uses the singular vector with the largest singular value as the weighting vector at the transmit side. Through this weighting vector, we can increase the received SNR and improve the BER performance. In highly correlated channels, beamforming is known to be the best transmission strategy in terms of BER. However, the throughput or the spectral efficiency may be low since only one data stream is transmitted from the transmit antennas. In weakly correlated channels, beamforming alone may result in poor overall performance. When higher spectral efficiency is needed, a hybrid scheme of beamforming and spatial multiplexing is desired. When CSI is available at the transmitter, eigenmode transmission using SVD with water-filling is known to be optimal in terms of Shannon capacity. Error usually occurs in the worst eigenmode (stream), and conventional spatial multiplexing with water-filling cannot compensate for the worst eigenmode (stream). It may not be an optimal strategy in terms of BER performance.

When the number of transmit antennas N_t is larger than the number of receive antennas N_r, the BER performance can be improved by an antenna-grouping technique, which is a combination of beamforming and spatial multiplexing [5]. In [5], N_t transmit antennas are partitioned into N_r groups. The antennas in each group are used for beamforming, and an independent data stream is transmitted in each group. The criterion used in [5] is to maximize the sum capacity of subchannels. For a given channel, a receiver considers all possible cases of grouping, calculates the beamforming vector for each case, and adds up the subchannel capacities. The receiver sends the best beamforming matrix that maximizes the sum capacity of subchannels to the transmitter, which uses the matrix for transmission. The computational complexity of the algorithm is high because the SVD needs to be calculated for every grouping case. In addition, the criterion in [5] is based on Shannon capacity, which is not optimal in terms of BER performance.

An optimal precoding (optimized eigenmode-based spatial multiplexing) matrix in terms of BER was derived in [6]–[8]. With convex optimization, [6] uses the singular vector with the largest singular value of the SVD, a diagonal matrix for power allocation, and a unitary matrix. Linear transmit and receive matrices are also similarly derived in [7] and [8].

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However, those algorithms are complex; therefore, they may not be practical. In addition to that, the algorithms in [7] and [8] are developed for linear transceivers. They cannot be directly applied to a maximum-likelihood (ML) detector (a nonlinear detector), which is used in this paper. Moreover, the number of feedback bits in [6]–[8] is larger than that of antenna grouping. When N_r streams are transmitted, the receiver needs to feed back first N_r singular vectors in the optimized (spatial) multiplexing transmission, while the receiver feeds back only one vector and their grouping information in the proposed antenna-grouping technique.

In this paper, we consider the impact of various grouping criteria on the BER, and some of them are related with antenna selection [9]. We also propose a novel correlation-based grouping technique, which has low complexity, and analyze the BER performance. The grouping criteria are compared in terms of the BER performance. It is shown that the proposed algorithm reduces the computational complexity by a factor of 6–3700 with similar BER performance when it is compared with the Euclidean distance-based antenna-grouping algorithm. Compared with the existing algorithm in [5], it also reduces the complexity by more than 50% with about 0.5-dB SNR gain (at the BER of 10^{-3}) for an independent and identically distributed (i.i.d.) channel. A BER lower bound of the proposed algorithm is obtained by computing the BER of a decomposed beamforming system, which always has a better BER than the proposed algorithm. Its performance can be analyzed by the distribution of the largest eigenvalue of a Wishart matrix. This paper is organized as follows. In Section II, we present the system model. Section III summarizes various antenna-grouping criteria that we consider. In Section IV, we propose a low-complexity grouping algorithm and analyze the BER lower bound for an i.i.d. channel. In Section V, we provide simulation results, and conclusions are made in Section VI.

II. System Model

We assume that the receiver and the transmitter know the CSI. The number of transmit antennas is assumed to be larger than the number of receiver antennas $(N_t \geq N_r)$. We partition N_t transmit antennas into N_r groups, and each group transmits an independent data stream by beamforming. Let $\mathbf{H}$ be an $N_r \times N_t$ matrix, where the (i,j)th element $h_{i,j}$ is the path gain from the jth transmit antenna to the ith receive antenna. Independence condition of $h_{i,j}$ and $h_{k,l}$ ($i \neq k$ or $j \neq l$) is not necessary. A correlated channel matrix can be modeled by

$$\mathbf{H} = \mathbf{R}^{\frac{1}{2}}\mathbf{H}_w\mathbf{T}^{\frac{1}{2}} \tag{1}$$

where $\mathbf{R}$ and $\mathbf{T}$ are the receiver and transmitter antenna correlation matrices, respectively. The (i,j)th element of $\mathbf{H}_w$, i.e., $h_{w_{i,j}}$, is modeled as an i.i.d. complex Gaussian random variable with zero mean and unit variance. When the channel experiences i.i.d. Rayleigh fading, $\mathbf{R}$ and $\mathbf{T}$ in (1) are identity matrices.

The channel matrix $\mathbf{H}$ is written as

$$\mathbf{H} = [\,\mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_{N_t}\,]\,. \tag{2}$$

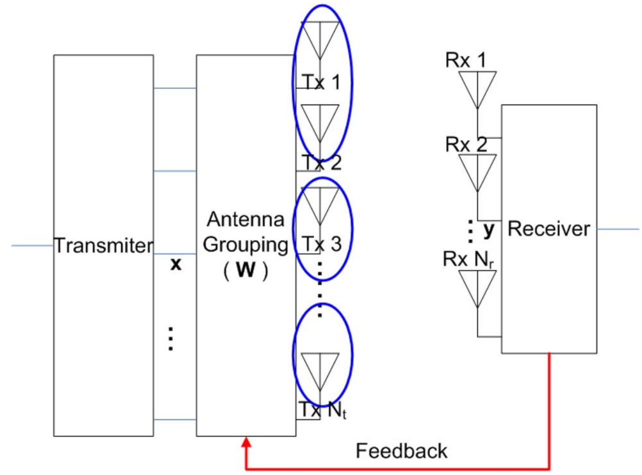

Fig. 1. Block diagram for an antenna grouping system.

We partition the integer set $\{1, 2, \ldots, N_t\}$ into N_r disjoint groups (subsets), which are denoted by

$$S_1, S_2, \ldots, S_{N_r}.$$

Let $|S_i| = n_i$ (where $|S_i|$ is the number of elements in S_i for $i = 1, \ldots, N_r$), which satisfies

$$n_1 + \cdots + n_{N_r} = N_t. \tag{3}$$

We can define an $N_r \times n_i$ subchannel matrix $\mathbf{H}_i$ as

$$\mathbf{H}_i = \left[\mathbf{h}_{s_{i1}}, \ldots, \mathbf{h}_{s_{in_i}}\right] \tag{4}$$

where s_{ij} is the jth element of set S_i.

We can obtain the beamforming vector of each subchannel $\mathbf{w}_i$ as the right singular vector corresponding to the largest singular value in the SVD of $\mathbf{H}_i$. The received signal can be modeled as

$$\mathbf{y} = \mathbf{HWx} + \mathbf{n} \tag{5}$$

where $\mathbf{x}$ is the $N_r \times 1$ transmitted signal vector, and $E[\mathbf{x}^H\mathbf{x}] = 1$. $\mathbf{W}$ is the $N_t \times N_r$ matrix; the mth element of the subchannel beamforming vector $\mathbf{w}_i$ corresponds to $[\mathbf{W}]_{s_{im}i}$, and the other elements of the ith column of $\mathbf{W}$ are 0 ($\|\mathbf{w}_i\|$ is normalized to 1). The noise $\mathbf{n}$ is additive white Gaussian noise (AWGN) with variance of σ_n^2. In this system, the BER performance depends on the antenna grouping, and the total number of groups is equal to the number of cases where the set of integers from 1 to N_t is partitioned into N_r groups. The system block diagram is shown in Fig. 1. In this paper, it is assumed that an ML detector is used at the receiver, and no channel coding is used; therefore, an uncoded BER is dealt with.

III. Antenna-Grouping Criteria

Here, we introduce the existing algorithm (A) presented in [5]. We also propose several new algorithms (B, C, and D), which were motivated by antenna-selection techniques [9].

A. Maximization of the Sum Capacity of Subchannels (Algorithm A)

This is an existing algorithm in [5]. The sum capacity of subchannels is approximated by

$$C_{S_1,S_2,\ldots,S_{N_r}}^{w_1,w_2,\ldots,w_{w_{N_r}}} \cong \log\left(1+\frac{\rho}{N_r}\Sigma_{i=1}^{N_r}\mathbf{w}_i^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{w}_i\right) \quad (6)$$

where ρ is the SNR. To maximize (6), we need to search for the subchannel group that maximizes

$$\sum_{i=1}^{N_r}\mathbf{w}_i^H\mathbf{H}_i^H\mathbf{H}_i\mathbf{w}_i. \quad (7)$$

Since (6) is an approximation, the algorithm is not optimal, even in terms of capacity. In this algorithm, we consider every possible grouping, compute the corresponding beamforming vectors $\mathbf{w}_i$'s by the SVD of $\mathbf{H}_i$s, and calculate (7). To reduce the complexity, [5] searches adjacent grouping only, which makes this algorithm even more suboptimal in terms of both the BER and the sum capacity.

B. Maximization of the Minimum Euclidean Distance of Receive Constellations (Algorithm B)

The minimum Euclidean distance of receive constellation is calculated [9] as

$$d_{\min}^2 := \min_{\mathbf{x}_i,\mathbf{x}_j\in X,\mathbf{x}_i\neq\mathbf{x}_j}\frac{\|\mathbf{HW}(\mathbf{x}_i-\mathbf{x}_j)\|^2}{N_r} \quad (8)$$

where X is the set of all possible transmitted vectors $\mathbf{x}$. In the first step of this algorithm, we consider all possible subchannel groups and calculate the corresponding beamforming vectors by SVD. We then compute the effective channel $\mathbf{HW}$ in (5), calculate the minimum Euclidean distance of receive constellation for every possible $\mathbf{HW}$, and find the best subchannels $\mathbf{H}_i$'s and $\mathbf{W}$ that maximize (8). If the number of antennas is large or the modulation order is high, this method is very complex because it needs to calculate the distance of every possible pairwise combination of constellation, which is not practical. However, as will be shown in Section V, this criterion gives the best BER performance at a high SNR; therefore, it will be used as a benchmark.

C. Maximization of the Minimum Singular Value of an Effective Channel (Algorithm C)

A MIMO channel can be decomposed into multiple single-input–single-output channels by SVD [1], and the received SNR is proportional to the squared singular value of a channel. The BER performance is, thus, dominated by the minimum singular value of $\mathbf{H}_i$'s. As in Algorithm B, we consider all possible groups and subchannel weight vectors and compute the corresponding effective channel $\mathbf{HW}$. We then find the minimum singular value of each $\mathbf{HW}$ and pick the best $\mathbf{W}$ that maximizes the minimum singular value.

D. Maximization of the Capacity (Algorithm D)

Unlike Algorithm A, this algorithm does not consider the sum capacity of subchannels but the overall channel capacity itself. As in other algorithms, for every possible effective channel $\mathbf{HW}$, we calculate the channel capacity

$$C = \log\det\left(\mathbf{I}_{N_r}+\frac{\rho}{N_r}\mathbf{w}^H\mathbf{H}^H\mathbf{HW}\right) \quad (9)$$

where $\rho = 1/\sigma_n^2$ is the SNR. We then select the transmit antenna grouping and the beamforming vectors that maximize (9).

IV. Low-Complexity Antenna Grouping

A. Correlation-Based Grouping

Here, we propose a low-complexity technique using a normalized instantaneous channel correlation matrix (NICCM) and derive the BER lower bound of the system. In this algorithm, transmit antennas that are highly correlated with each other are grouped together to transmit a data stream with beamforming. Transmit antennas that are less correlated transmit a different data stream. Let us define an NICCM as

$$\mathbf{M} = \begin{bmatrix} 1 & m_{1,2} & \cdots & m_{1,N_t} \\ m_{1,2}^* & 1 & \cdots & m_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1,N_t}^* & m_{2,N_t}^* & \cdots & 1 \end{bmatrix} \quad (10)$$

$$[\mathbf{M}]_{ij} = \frac{1}{\|\mathbf{h}_i\|\cdot\|\mathbf{h}_j\|}\cdot[\mathbf{H}^H\mathbf{H}]_{ij} \quad (11)$$

where $\mathbf{h}_i$ is the ith column vector of $\mathbf{H}$ (a^* denotes the complex conjugate of a).

In (10), if $|m_{ij}|$ is large, it means that the correlation between the ith transmit antenna and the jth transmit antenna is large at that time instant. Using this concept, we can devise a simple antenna-grouping algorithm. For simplicity, we assume $N_t = 4$ and $N_r = 2$. In a 4×2 system, $\mathbf{M}$ is written as

$$\mathbf{M}_{4\times 2} = \begin{bmatrix} 1 & m_{1,2} & m_{1,3} & m_{1,4} \\ m_{1,2}^* & 1 & m_{2,3} & m_{2,4} \\ m_{1,3}^* & m_{2,3}^* & 1 & m_{3,4} \\ m_{1,4}^* & m_{2,4}^* & m_{3,4}^* & 1 \end{bmatrix}. \quad (12)$$

We consider only antenna grouping with an equal group size of two in this case. The possible antenna-grouping cases are $\{\{1,2\},\{3,4\}\}$, $\{\{1,3\},\{2,4\}\}$, and $\{\{1,4\},\{2,3\}\}$. We then compare $(|m_{1,2}|+|m_{3,4}|)$, $(|m_{1,3}|+|m_{2,4}|)$, and $(|m_{1,4}|+|m_{2,3}|)$. If $(|m_{1,2}|+|m_{3,4}|)$ is the largest, it means that the correlation between transmit antennas 1 and 2 and the correlation between transmit antennas 3 and 4 are larger than the others. We group $\{1,2\}$ and $\{3,4\}$ together, which are denoted by $\{\{1,2\},\{3,4\}\}$. Likewise, if $(|m_{13}|+|m_{24}|)$ is the largest, we use the grouping of $\{\{1,3\},\{2,4\}\}$. If $(|m_{14}|+|m_{23}|)$ is the maximum, we use the grouping of $\{\{1,4\},\{2,3\}\}$. The advantage of this algorithm is that it significantly reduces the search complexity. In other grouping criteria of Section III, we need to compute the SVD to obtain the beamforming vector for every possible grouping (subchannel) before we select the optimal grouping (subchannel). However, in this algorithm, we select the grouping first by the correlation-based approach and then compute the SVD just once for each subchannel of the selected grouping.

The algorithm can be applied to systems with a different number of antennas such as 6×3 or 6×2 systems. Suppose that the NICCM is given by

$$\mathbf{M}_{6\times(3\,\mathrm{or}\,2)} = \begin{bmatrix} 1 & m_{1,2} & m_{1,3} & m_{1,4} & m_{1,5} & m_{1,6} \\ m_{1,2}^* & 1 & m_{2,3} & m_{2,4} & m_{2,5} & m_{2,6} \\ m_{1,3}^* & m_{2,3}^* & 1 & m_{3,4} & m_{3,5} & m_{3,6} \\ m_{1,4}^* & m_{2,4}^* & m_{3,4}^* & 1 & m_{4,5} & m_{4,6} \\ m_{1,5}^* & m_{2,5}^* & m_{3,5}^* & m_{4,5}^* & 1 & m_{5,6} \\ m_{1,6}^* & m_{2,6}^* & m_{3,6}^* & m_{4,6}^* & m_{5,6}^* & 1 \end{bmatrix}. \tag{13}$$

The grouping algorithm for the 6×3 system is straightforward. The 6×2 system is a little different from the 6×3 system in that $N_t \neq 2 \times N_r$. However, a simple modification makes it possible to use the proposed grouping idea. In the 6×2 system, we partition six transmit antennas into two groups of three antennas each. Between the three transmit antennas, there are three pairwise correlations. For example, there are $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ pairwise correlations in the $\{1, 2, 3\}$ group. Because there are two groups of three antennas, we should check six pairwise correlations. The proposed antenna-grouping algorithm can be extended to any MIMO system where N_t is larger than N_r $(N_t \geq N_r)$ with a similar modification. The proposed algorithm only considers groups of equal size or near-equal size. Algorithms B, C, and D can have better BER performance than the proposed algorithm. However, the advantage of the proposed algorithm is low computational complexity with negligible performance degradation. The BER performance of the proposed algorithm is compared with other criteria in Section V.

B. BER Lower Bound for an I.I.D. Channel

For simplicity's sake, we only consider the case where the number of transmit antennas is an integer multiple of the number of receive antennas. Let us consider the example of a 4×2 system. Assume that the instantaneous channel is given by

$$\mathbf{H} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_5 & h_6 & h_7 & h_8 \end{bmatrix} \tag{14}$$

and assume that the optimal antenna grouping is $\{\{1, 4\}, \{2, 3\}\}$. Then, the beamforming matrix is given by

$$\mathbf{w} = \begin{bmatrix} a & 0 \\ 0 & b \\ 0 & c \\ d & 0 \end{bmatrix} \tag{15}$$

where $\begin{bmatrix} a \\ d \end{bmatrix}$ and $\begin{bmatrix} b \\ c \end{bmatrix}$ are the singular vectors with the largest singular value of $\begin{bmatrix} h_1 & h_4 \\ h_5 & h_8 \end{bmatrix}$ and $\begin{bmatrix} h_2 & h_3 \\ h_6 & h_7 \end{bmatrix}$. From (5), the received signal for the given antenna grouping is

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1 a + h_4 d & h_2 b + h_3 c \\ h_5 a + h_8 d & h_6 b + h_7 c \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n1 \\ n_2 \end{bmatrix}. \tag{16}$$

Let us consider two 2×2 beamforming systems with one-half transmit power when the channels are $\begin{bmatrix} h_1 & h_4 \\ h_5 & h_8 \end{bmatrix}$ and $\begin{bmatrix} h_2 & h_3 \\ h_6 & h_7 \end{bmatrix}$. The received vectors of the two 2×2 beamforming systems are

$$\mathbf{z}_1 = \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} = \begin{bmatrix} h_1 a + h_4 d \\ h_5 a + h_8 d \end{bmatrix} \cdot x_1 + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \tag{17}$$

$$\mathbf{z}_2 = \begin{bmatrix} z_{21} \\ z_{22} \end{bmatrix} = \begin{bmatrix} h_2 b + h_3 c \\ h_6 b + h_7 c \end{bmatrix} \cdot x_2 + \begin{bmatrix} l_3 \\ l_4 \end{bmatrix} \tag{18}$$

where l_i with $i = 1, \ldots, 4$ is the independent noise with one half of the variance of n_1 and n_2. The sum of $\mathbf{z}_1$ and $\mathbf{z}_2$ is exactly the same as $\mathbf{y}$ in (16). We can think of (16) as simultaneous transmission of two beamformed signals. Thus, the antenna-grouping system can be modeled as the sum of two separated beamforming systems. However, it is clear that the BER of the separated systems described by (17) and (18) is lower than that of the original system due to the reduced (actually zero) interference. The BER lower bound of the proposed antenna-grouping system can then be obtained by analyzing the separated beamforming systems. The 4×2 system with antenna grouping can be divided into the two 2×2 beamforming systems with one-half transmit power. Similarly, the 6×3 system with antenna grouping can be divided into three 2×3 beamforming systems with one-third transmit power. In general, the $mN_r \times N_r$ system with antenna grouping can be divided into N_r of $m \times N_r$ beamforming systems with $1/N_r$ transmit power.

In an M-ary rectangular quadratic-amplitude modulation (QAM) $(M = I \cdot J)$ system with Gray code, the BER P_b in the AWGN channel can be derived exactly [10]. In [10], the probability that the kth bit of I in-phase components and the lth bit of J in quadrature components are in error in terms of the SNR is expressed as (19) and (20), shown at the bottom of the next page, where ε denotes the SNR per bit, and $\lfloor x \rfloor$ denotes the largest integer that is smaller than x. The average bit-error probability of M-ary QAM is

$$P_b(\varepsilon) = \frac{1}{\log_2(I \cdot J)} \left(\sum_{k=1}^{\log_2 I} P_I(k, \varepsilon) + \sum_{l=1}^{\log_2 J} P_J(l, \varepsilon) \right). \tag{21}$$

A beamforming system uses only the largest eigenmode, and it is modeled by

$$y = \sqrt{\lambda_{\max}} x + n \tag{22}$$

where $\lambda_{\max}$ is the maximum eigenvalue of $H^H H$. In the case of an i.i.d. fading channel, the lower bound can be averaged using the properties of $\lambda_{\max}$. In this case, $\mathbf{H}^H \mathbf{H}$ is a Wishart matrix,

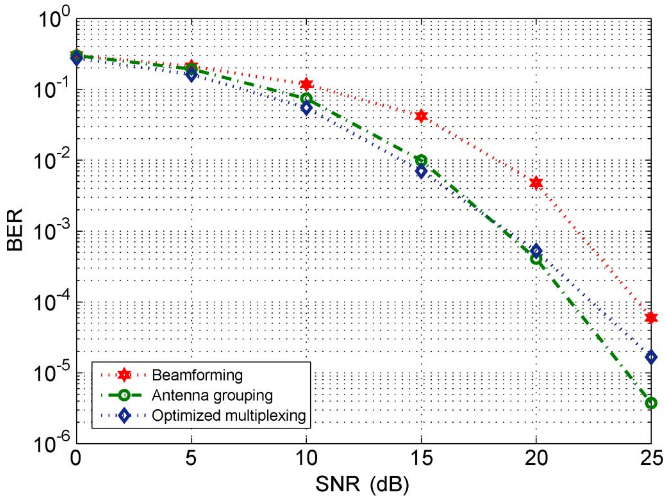

Fig. 2. BER of various closed-loop transmit strategies in a 4×2 i.i.d. channel with BPCU $= 8$.

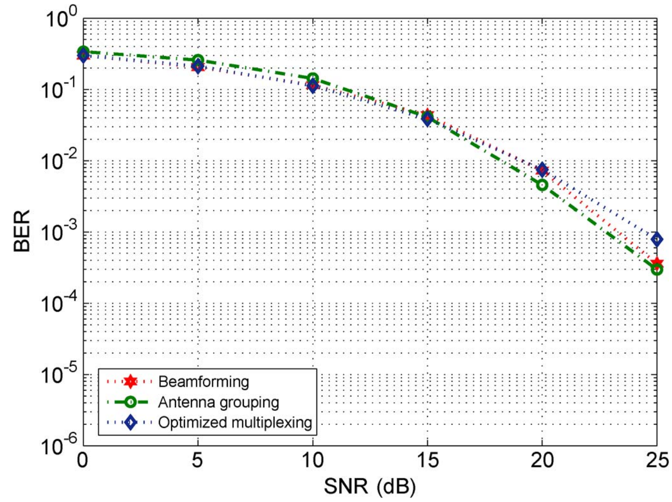

Fig. 3. BER of various closed-loop transmit strategies in a 4×2 correlated channel with BPCU $= 8$.

and the probability density function of its largest eigenvalue is given [11] by

$$f_{\lambda_{\max}}(\lambda_{\max}) = \frac{|\Psi(\lambda_{\max})| \, tr\left(\Psi^{-1}(\lambda_{\max})\Phi(\lambda_{\max})\right)}{\prod_{k=1}^{s}\Gamma(t-k+1)\Gamma(s-k+1)} \tag{23}$$

where $t = \max(N_t, N_r)$, $s = \min(N_t, N_r)$, $\{\Psi(\lambda_{\max})\}_{i,j} = \gamma(t+s-i-j+1, x)$, $\{\Phi(\lambda_{\max})\}_{i,j} = \lambda_{\max}^{t+s-i-j} e^{-\lambda_{\max}}$, and $\gamma(\cdot,\cdot)$ is the incomplete gamma function [12] $(i, j = 1, \ldots, s)$. Thus, the average BER of the beamforming system is

$$P_{\mathrm{err}} = \int P_b(\varepsilon) \cdot f_{\lambda_{\max}}(\lambda_{\max}) \, d\lambda_{\max} \tag{24}$$

where ε in (19) and (20) is $\varepsilon = \lambda_{\max}/(\sigma_n^2 \cdot \log_2 M)$. As we mentioned earlier, the BER of the proposed antenna-grouping system is lower bounded by the BER of two separated beamforming systems. Thus, the BER performance of the proposed antenna-grouping system is lower bounded by P_{err} in an i.i.d. Rayleigh fading channel. It is difficult to get a closed-form expression for the integral, and the integral is calculated numerically instead. The numerical results will be shown in Section V.

V. Simulations

We start with BER comparison of different transmission strategies. For all the simulations here, we use ML detection, and no channel coding is used. In Figs. 2 and 3, the BER results of beamforming, optimized spatial multiplexing (by [6]), and the proposed antenna grouping for a 4×2 MIMO system with bit per channel use (BPCU) of 8 are compared for an i.i.d. channel and a correlated channel. For the correlated channel, a channel model (the TGn model [15], [16]) from the IEEE 802.11n standard is used with an angle of departure (AOD) of 45° and an angular spread (AS) of 15°. Since the TGn model is used for wireless local area network standard, it is used for indoor channels that are suitable for MIMO communication systems due to the rich scattering environments. A base station can afford more antennas than a mobile station. Although the proposed algorithm can be applied to both downlink and uplink,

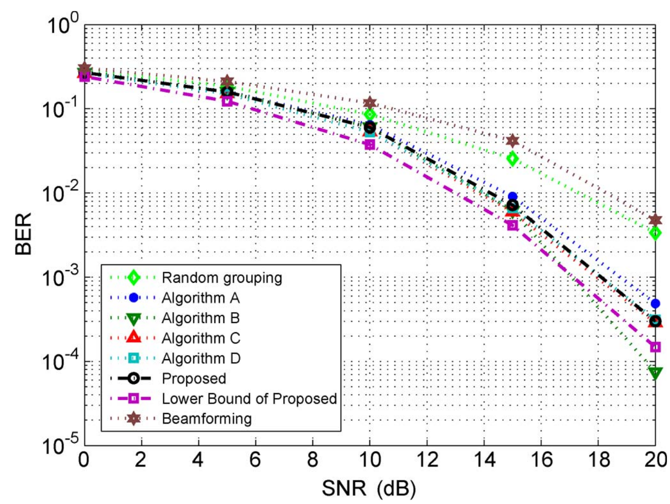

Fig. 4. BER comparison for a 4×2 i.i.d. channel with BPCU $= 8$.

$$P_I(k,\varepsilon) = \frac{1}{I}\sum_{i=0}^{(1-2^{-k})I-1}\left\{(-1)^{\left\lfloor \frac{i\cdot 2^{k-1}}{I}\right\rfloor}\left[2^{k-1} - \left\lfloor \frac{i\cdot 2^{k-1}}{I} + \frac{1}{2}\right\rfloor\right] \mathrm{erfc}\left((2i+1)\sqrt{\frac{3\log_2(I\cdot J)\cdot\varepsilon}{I^2+J^2-2}}\right)\right\} \tag{19}$$

$$P_J(l,\varepsilon) = \frac{1}{J}\sum_{j=0}^{(1-2^{-l})J-1}\left\{(-1)^{\left\lfloor \frac{j\cdot 2^{l-1}}{J}\right\rfloor}\left[2^{l-1} - \left\lfloor \frac{j\cdot 2^{l-1}}{J} + \frac{1}{2}\right\rfloor\right] \mathrm{erfc}\left((2j+1)\sqrt{\frac{3\log_2(I\cdot J)\cdot\varepsilon}{I^2+J^2-2}}\right)\right\} \tag{20}$$

TABLE I
COMPLEXITY COMPARISON IN TERMS OF THE NUMBER OF GROUPS, THE NUMBER OF SVDs (SEARCH COMPLEXITY), THE NUMBER OF ARITHMETIC OPERATIONS FOR SVDs, AND THE NORMALIZED COMPLEXITY COMPARED WITH THE PROPOSED ALGORITHM

System	Algorithm	# of groups	# of SVD's	# of arithmetic operations for SVD	Normalized complexity
4×2	Proposed	3	2	16	1
	A (adjacent only)	3	6	44	2.75
	B	7	14	104	6.5
	C	7	21	160	10
	D	7	14	104	6.5
6×2	Proposed	10	2	54	1
	A (adjacent only)	5	10	116	2.15
	B	31	62	732	13.56
	C	31	93	980	18.15
	D	31	62	732	13.56
6×3	Proposed	15	3	36	1
	A (adjacent only)	10	30	414	11.5
	B	90	270	3690	102.5
	C	90	360	6120	170
	D	90	270	3690	102.5
8×2	Proposed	35	2	32	1
	A (adjacent only)	7	14	220	6.88
	B	127	254	4048	126.5
	C	127	381	5064	158.25
	D	127	254	4048	126.5
8×4	Proposed	105	4	64	1
	A (adjacent only)	35	140	2832	44.25
	B	1701	6804	131712	2058
	C	1701	8505	240576	3759
	D	1701	6804	131712	2058

the proposed grouping scenario, where we assume $N_t > N_r$ is more suitable for downlink systems. We consider only the correlation between transmit antennas because it would directly affect the grouping algorithm; therefore, the receive correlation matrix R in (1) is assumed to be an identity matrix.

It is observed that the BER curve of the proposed antenna grouping has a diversity order that is similar to that of the beamforming method. The antenna-grouping scheme has better BER performance than the beamforming scheme in an i.i.d. channel, and they are similar in a correlated channel. In terms of the BER performance, it should be noted that the antenna-grouping scheme is a little worse than the optimized spatial multiplexing scheme for a low SNR, but the former is slightly better than the latter in the high SNR region.

We also compare various algorithms of Section III and the proposed algorithm in terms of the BER and computational complexity. We assume equal power allocation, fixed BPCU, and identical modulation order for each stream. As a reference, we also simulated random grouping. Fig. 4 shows the BER performance of a 4×2 system in an i.i.d. channel. We use 16-QAM in the antenna-grouping algorithms and 256-QAM in the beamforming method to have the same BPCU of 8. Algorithm B has the best BER performance among all the antenna-grouping algorithms at a high SNR (above 20 dB). The reason why Algorithm B does not perform the best for a low SNR regime (below 15 dB) may be explained by the following two factors. One is that Algorithm B focuses on the signal portion only as in (8) and does not consider noise. The other is that it only minimizes vector symbol error as in (8), which may not exactly correspond to bit-error minimization. As mentioned before, the proposed algorithm has little performance loss compared with Algorithms B, C, and D. However, it is interesting to note that the proposed algorithm has better BER performance than Algorithm A, even with lower complexity. The proposed algorithm has about 0.5-dB SNR gain at the BER of 10^{-3}. Algorithms B, C, and D have better BER performance than the proposed algorithm since the proposed algorithm considers equal-size grouping only. On the other hand, the proposed algorithm has the lowest complexity, as shown in Table I, and its performance loss is minimal.

Fig. 5 shows the BER performance of a 4×2 system in a correlated channel with BPCU $= 8$. Here, the correlated channel model also uses an AOD of 45° and an AS of 15°. In Fig. 5, the BER of antenna grouping in a correlated channel is slightly worse than in an i.i.d. channel. Except for the low SNR region, antenna grouping is better than beamforming in terms of the BER. The complexity of the proposed algorithm is significantly low compared with the other grouping algorithms, and it has robust performance in a correlated channel, as well as in the i.i.d. channel.

Fig. 6 shows the BER performance of a 6×2 system in an i.i.d. channel. In Fig. 6, we use 8-QAM for each stream so that the BPCU is fixed to 6. In this case, the beamforming scheme uses 64-QAM. As in the 4×2 system, Algorithm B shows

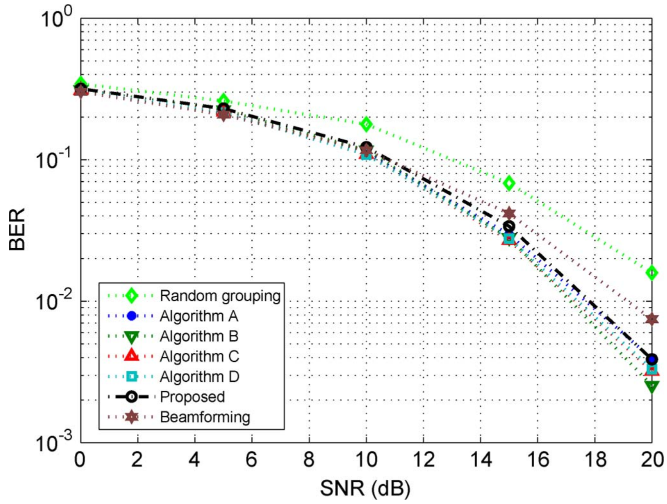

Fig. 5. BER comparison for a 4 × 2 correlated channel with BPCU = 8.

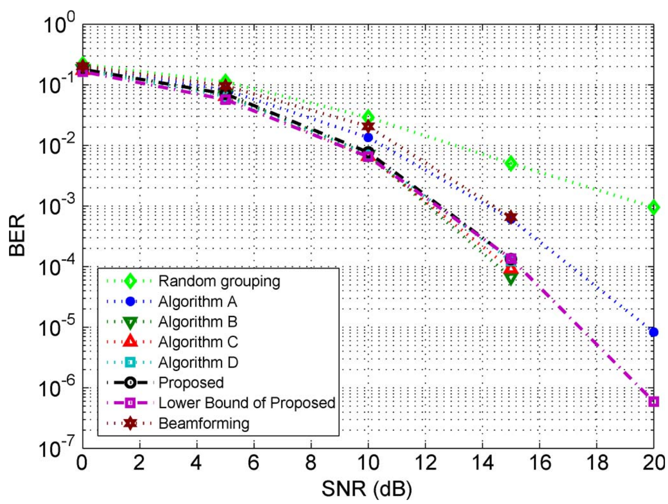

Fig. 6. BER comparison for a 6 × 2 i.i.d. channel with BPCU = 6.

the best performance at a high SNR (above 15 dB) among the antenna-grouping algorithms, and the proposed algorithm has performance that is close to those of Algorithms C and D. The proposed algorithm performs better than Algorithm A, and the SNR gain is about 1.5 dB at the BER of 10^{-3}.

When N_t and N_r are large, Algorithms B, C, and D may not be practical due to high computational complexity. Table I shows the number of groups, the number of SVDs, and the number of arithmetic operations for an SVD. It can be shown that the SVD of an $m \times n$ matrix requires $\min(m^2 n, mn^2)$ arithmetic operations [13]. Different algorithms use different requirements for the group size and the adjacency of antennas in a group, which is summarized as follows. The proposed algorithm uses nonadjacent grouping with an identical group size. Algorithm A uses adjacent grouping, where the group size can be different. Algorithms B, C, and D use nonadjacent grouping with a different group size, which is the most computationally complex case. The number of groups for the proposed algorithm is given by $N_t!/(m!)^{N_r} N_r!$ when $N_t = mN_r$. The number of groups for Algorithm A is given by $\binom{N_t-1}{N_r-1}$. The number of groups for Algorithms B, C, or D is not given in a closed form, but it can be computed recursively. As shown here, even if only the computational complexity of the SVD is considered, Algorithms A, B, C, and D are much more complex than the proposed algorithm by a factor of 10–1000. Algorithm B requires fewer SVDs than Algorithm C, but Algorithm B needs to calculate all the distance pairs of received constellation. It is the most computationally complex and has the best BER performance. We also estimated the CPU runtime of all the algorithms for a 4 × 2 system, although it is not exactly proportional to computational complexity. The proposed algorithm takes about less than 70% of the runtime of Algorithm A, which uses only adjacent grouping, less than 20% of Algorithm D, less than 10% of Algorithm C, and less than 1% of Algorithm B with 16-QAM in a 4 × 2 system.

VI. Conclusion

In a MIMO system that has more transmit antennas than receive antennas, we can improve the BER performance by antenna grouping, which is a hybrid of beamforming and spatial multiplexing. In this paper, we have introduced various antenna-grouping criteria and have proposed a novel low-complexity algorithm based on an NICCM. It improves the computational complexity without significantly sacrificing the BER performance. The proposed algorithm has more than 0.5-dB SNR gain compared with the existing low-complexity algorithm (Algorithm A) in an i.i.d. channel. In a correlated channel, the BER performance is degraded slightly, but it is still close to the performance of the other algorithms. Compared with the Euclidean distance-based algorithm (Algorithm B), it reduces the computational complexity by a factor of more than 100 with a slightly degraded BER. Compared with the existing low-complexity algorithm (Algorithm A), it reduces the computational complexity by about 30% with an improved BER. The proposed algorithm appears to be a promising practical technique that combines beamforming and spatial multiplexing. The performance of the proposed algorithm may also be improved by adequate power allocation for each subchannel.

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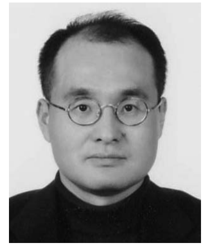

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