Abstract—We propose an efficient space-time coded cooperative relay communications scheme that employs linear precoding and transmission-pattern selection. If effectively exploited, distributed spatial diversity and the cooperative nature of multihop communications could significantly increase the coverage area of a wireless network that consists of a number of power-limited relay nodes. We build upon an existing block linear precoding technique for conventional multiple-input multiple-output systems studied in to improve the diversity performance of a multihop relay network. We propose a distributed-relay-selection algorithm to maximize the signal-to-noise ratio at the receiver. We show, via simulation, that the proposed pattern-selection scheme with precoding outperforms the conventional space-time coded relay system using quasi-orthogonal space-time block codes by over 2 dB at a bit error rate of $10^{-3}$.

Index Terms—Multihop relay, cooperative communications, space-time block codes, linear precoding, distributed relay terminal selection.

I. INTRODUCTION

Space-time wireless systems exploit multiple co-located spatial elements at the transmitter and/or receiver to overcome multipath fading or to increase transmission rates. When multiple transmit antennas are available, this is typically combined with an appropriate signal design such as space-time coding [1]–[9] to achieve spatial diversity and/or spatial multiplexing. However, implementing multiple antennas on small-size terminals becomes difficult. Given a fixed transmission power, there exists a fundamental tradeoff between the achievable data rates and transmission distance of a transmitter-receiver pair: a higher data rate will be possible over a shorter communications distance. In networks with a number of distributed terminals that are either mobile or at fixed locations, the network coverage area can be significantly extended by exploiting cooperative diversity. This is achieved by allowing one or multiple terminals to relay the data of an adjacent transmitter toward the more distant destination, forming multihop communications [10]–[12]. In order to effectively exploit distributed diversity, cooperative diversity techniques where single-antenna terminals cooperate to exploit virtual multiple-input multiple-output (MIMO) benefits have been studied [12], [13]. One challenge is how to effectively achieve the maximum achievable diversity order with low complexity when the available spatial elements are on terminals at different locations. The Alamouti scheme [2] is a simple and effective orthogonal space-time block code (STBC) of rate one for systems with two transmit antennas. It is well known that complex orthogonal design with transmission rate one does not exist for more than two transmit antennas. The reduced-complexity space-time code, proposed by Nir and Helard [1] applies block linear precoding to the Alamouti code applied on blocks of two antennas. This scheme leads to a diversity order that increases with the size of the precoding matrix at the expense of a linear increase in complexity [1].

In this paper, we propose a new space-time block coded cooperative relay communications scheme with transmission pattern selection and precoding. We provide a set of candidate distributed transmission patterns. The pattern is selected to maximize the signal-to-noise ratio (SNR) based on the channel conditions. With the proposed patterns, at any time instant, only half of the chosen relays are actively transmitting for a particular source-to-destination pair. Simulation results show that the proposed scheme outperforms the conventional approach using quasi-orthogonal space-time block codes (QOSTBC) introduced in [10]. Since transmission patterns are predetermined and the number of patterns is finite (e.g., 4), the overhead required to feedback the pattern selected is minimal.

In Section II, we briefly review the reduced-complexity space-time code described in [1] and then provide candidate transmission patterns that are needed for distributed antenna selection. In Section III, we discuss the proposed pattern-selection scheme for wireless non-regenerative relay networks with block linear precoding. We also derive the expression of the interference level as a function of the number of transmission patterns that the system can select from. In Section IV, we simulate the bit-error-rate performance of the proposed scheme and compare it with that of existing relay schemes. Concluding remarks are given in Section V.

II. PRECODED STBC AND PATTERN SELECTION

For convenience of discussion in the rest of the paper, this section briefly reviews the reduced-complexity STBC described in [1]. Then, we propose candidate transmission patterns to allow the system to choose the one that results in the highest SNR at the receiver based on the specific channel condition.

A. Reduced-Complexity STBC [1]

Nir and Helard [1] proposed a space-time block code that applies linear precoding to the extended orthogonal STBC to improve the space-time diversity performance. The diversity of this scheme increases with the size of the precoding matrix. For systems with four transmit antennas, the basic idea of [1] is to apply the $2 \times 2$ Alamouti STBC alternatively to transmit antenna pair 1 and 2, and then to transmit antenna pair 3 and 4. The equivalent channel matrix is written as [1]

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{12} & 0 \\ 0 & \mathbf{H}_{34} \end{bmatrix},$$

(1)

where

$$\mathbf{H}_{ij} = \begin{bmatrix} h_i^* & h_j \\ -h_j^* & h_i \end{bmatrix}$$

(2)

is the equivalent channel matrix for two successive symbol durations of each pair of antennas. The elements of $\mathbf{H}_{ij}$, $h_i$, and $h_j$, are the channel responses from transmit antennas $i$ and $j$ to the receiver, respectively. Using the common reception of...
where \((\cdot)^H\) denotes Hermitian transpose, the diversity order is only two. A precoding matrix can be applied at the transmitter to increase the diversity order. For example, when the number of transmit antenna \(L\) equals an integer power of 2, the \(L \times L\) linear precoding matrix can be calculated recursively from the \(2 \times 2\) precoding matrix as

\[
\Theta_L = \begin{bmatrix}
\Theta_{L/2} & \Theta_{L/2} \\
\Theta_{L/2} & -\Theta_{L/2}
\end{bmatrix},
\]

where \(\Theta_2\) is expressed as

\[
\Theta_2 = \begin{bmatrix}
e^{j\theta_1} \cos(\eta) & e^{j\theta_2} \sin(\eta) \\
-e^{-j\theta_2} \sin(\eta) & e^{-j\theta_1} \cos(\eta)
\end{bmatrix},
\]

where \(\eta, \theta_1, \) and \(\theta_2\) are parameters to be optimized. For \(L = 4\), \(\Theta_2\) is optimized as

\[
\Theta_2 = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
-1 & 1
\end{bmatrix}.
\]

The overall system is described by matrix

\[
\mathbf{A}_4 = \Theta_4 \cdot \mathbf{A}_4 \cdot \Theta_4^H
\]

\[
= \frac{1}{2} \begin{bmatrix}
\lambda_{12} + \lambda_{34} & 0 & \lambda_{12} - \lambda_{34} & 0 \\
0 & \lambda_{12} + \lambda_{34} & 0 & \lambda_{12} - \lambda_{34} \\
\lambda_{12} - \lambda_{34} & 0 & \lambda_{12} + \lambda_{34} & 0 \\
0 & \lambda_{12} - \lambda_{34} & 0 & \lambda_{12} + \lambda_{34}
\end{bmatrix},
\]

where \(\lambda_{ij} = |h_i|^2 + |h_j|^2\) and \(\mathbf{A}_4 = \mathbf{H}_4 \cdot \mathbf{H}_4^H\). The diagonal elements of \(\mathbf{A}_4\) are equal to \((1/2) \sum_{l=1}^L |h_l|^2\). The precoding effectively increases the diversity order to four.

This scheme can be applied to other STBCs and several antenna configurations. Moreover, the greater the value of \(L\), the smaller the interference terms [1].

**B. The Proposed Transmission Patterns and Pattern Selection**

In designing the possible transmission patterns, we consider a network with one source node, one destination node, and four relay nodes \((L = 4)\) between the source and the destination. This represents a typical scenario, and extension to other configurations such as \(L = 2,\ L = 6,\) or \(L = 8\) is straightforward.

There exists quasi-orthogonal STBCs that can be applied in systems with four antennas. These codes provide partial diversity but transmission rate one [8]. There are schemes to achieve full diversity or to improve the performance of quasi-orthogonal codes such as constellation rotation and transmit antenna shuffling [9]. For different sets of fading coefficients, the scheme proposed in [1] does not always result in the highest SNR at the receiver. In the case of four transmit antennas, we propose six alternative transmission matrices that conveniently exploit the basic Alamouti code as \((\mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4, \mathbf{H}_5,\) and \(\mathbf{H}_6)\):

\[
\mathbf{H}_1 = \begin{bmatrix}
h_1 & h_2 \\
-h_2^* & h_1^*
\end{bmatrix},
\]

\[
\mathbf{H}_2 = \begin{bmatrix}
h_1 & 0 & h_3 \\
0 & h_2 & h_4 \\
0 & -h_4^* & h_2^*
\end{bmatrix},
\]

\[
\mathbf{H}_3 = \begin{bmatrix}
h_1 & 0 & h_3 \\
0 & h_2 & h_4 \\
0 & -h_4^* & h_2^*
\end{bmatrix},
\]

\[
\mathbf{H}_4 = \begin{bmatrix}
h_1 & 0 \\
0 & h_2 \\
0 & -h_2^* \\
h_1^* & 0
\end{bmatrix},
\]

\[
\mathbf{H}_5 = \begin{bmatrix}
h_1 & 0 \\
0 & h_2 \\
0 & -h_2^* \\
h_1^* & 0
\end{bmatrix},
\]

\[
\mathbf{H}_6 = \begin{bmatrix}
h_1 & h_2 \\
-h_2^* & h_1^*
\end{bmatrix}.
\]

Note that \(\mathbf{H}_1\), which is the scheme given in [1] without precoding, is also listed above for convenience of description in the sequel. Precoding as described in Section II-A for the case of \(\mathbf{H}_1\) could be applied to the above matrices to achieve full diversity.

Now there are six different transmission patterns for the relays. These patterns will have different performances depending on the specific set of channel coefficients. We assume a linear minimum mean-square error (MMSE) receiver. The SNR with the each of the transmission patterns given in (7) is expressed as [14]

\[
\text{SNR}_k = \frac{1}{(I_4 + \rho \mathbf{H}_k^H \mathbf{H}_k)^{-1}} - 1, \ k = 1, \ldots, 6
\]

where \(I_4\) is the \(4 \times 4\) identity matrix and \(\rho = E_b/N_0\) with \(E_b\) being the energy per bit and \(N_0\) being the single-sided power spectral density of the additive Gaussian noise. In the case of a zero-forcing (ZF) receiver, the SNR is calculated as [14]

\[
\text{SNR}_k = \frac{1}{(\rho \mathbf{H}_k^H \mathbf{H}_k)^{-1}}, \ k = 1, \ldots, 6.
\]

For each set of channel coefficients, the six SNR values for the six transmission patterns \(\mathbf{H}_k\) \((k = 1, \ldots, 6)\) can be calculated using Eqs. (8) or (9). In order to achieve the optimum performance, the system chooses the pattern that results in the maximum SNR value.

The receiver needs to send this information to the relays through a feedback channel, which requires 3 feedback bits for the case with 6 patterns. Once a specific transmission pattern is chosen, the same precoding procedure as described in [1] is applied.

**III. THE PROPOSED COOPERATIVE RELAY COMMUNICATION SCHEME**

In this section, we apply linear precoding, the proposed transmission patterns, and the pattern-selection algorithm described...
in Section II for non-regenerative multihop relay communication systems, where each relay is equipped with only one antenna. Multihop relay exploits the intermediate relay stations to communicate with the more distant receiver. This scheme could result in significantly increased network coverage areas.

A. Proposed Distributed Space-Time Encoding for Cooperative Relay Communications

We focus on a single-hop scenario as shown in Fig. 1, where the network consists of a source (transmitter), a destination (receiver), and the relay group with $L$ relay nodes, either mobile or at a fixed location. The objective is to efficiently use the distributed antennas on the relays to maximize the diversity order for the signal from the source to the destination. We again explain the principle using the specific scenario of $L = 4$. For systems with many relay nodes around the transmitter and the receiver, a group of only four nearby relays is selected.

![Fig. 1. Single-hop relay communication system model.](image)

We assume a slowly fading flat Rayleigh channel and the received signal is further distorted by additive white Gaussian noise (AWGN). The transmitter directly precodes complex symbols $s_1, s_2, s_3, s_4$ with $\Theta_4$ as

$$\left[ \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \right]^T = \Theta_4 \left[ \begin{array}{cccc} 1 & s_2 & s_3 & s_4 \end{array} \right]^T$$

(10)

where $(.)^T$ denotes transpose, and then sends them to the relays. Each relay group then applies a space-time block code chosen out of the six transmission patterns. The transmitted signals reach the receiver through the relay group.

We are interested in optimizing the signal design, that is, to optimize the signal transmission format of the relay nodes for each time slot. Existing STBCs can be applied for the distributed-antenna scenario by simply viewing each relay as an independent transmit antenna. This requires clock synchronization of the whole network, which is beyond the scope of this paper. Note that the source-to-relay and relay-to-destination distances do not have to be identical as long as the propagation delay caused by the relative distance is insignificant compared with a symbol interval. For example, assume that the maximum relative distance is 5 meters and each node operates at a rate of 1 Mbps. The maximum relative arrival time of the signals from the source to the relays or from the relays to the destination is only 1.65% of a symbol interval. Therefore, if transmissions of all relays are synchronized, the signals from the relays will arrive at the destination approximately at the same time.

The distributed relay transmission can be achieved by having the relay group encode the received symbols with a space-time code, denoted by symbol blocks $C_i (i = 1, \ldots, 6)$. The corresponding transmission matrices are expressed as

$$C_1 = \left[ \begin{array}{cccc} c_1 & c_2 & 0 & 0 \\ c_2^* & -c_1^* & 0 & 0 \\ 0 & 0 & c_3 & c_4 \\ 0 & 0 & c_4^* & -c_3^* \end{array} \right]$$

(11a)

$$C_2 = \left[ \begin{array}{cccc} c_1 & 0 & c_3 & 0 \\ c_2^* & 0 & -c_1^* & 0 \\ 0 & c_2 & 0 & c_4 \\ 0 & c_4^* & 0 & -c_2^* \end{array} \right]$$

(11b)

$$C_3 = \left[ \begin{array}{cccc} c_1 & 0 & c_4 & 0 \\ 0 & c_1 & 0 & -c_4^* \\ 0 & c_2 & 0 & c_4 \\ 0 & c_4^* & 0 & -c_1^* \end{array} \right]$$

(11c)

$$C_4 = \left[ \begin{array}{cccc} c_1 & 0 & c_3 & 0 \\ 0 & c_2 & 0 & c_4 \\ 0 & c_3 & 0 & -c_2^* \\ 0 & c_4 & 0 & -c_3^* \end{array} \right]$$

(11d)

$$C_5 = \left[ \begin{array}{cccc} c_1 & 0 & c_4 & 0 \\ 0 & c_2 & 0 & c_3 \\ 0 & c_3 & 0 & -c_1^* \\ 0 & c_4 & 0 & -c_2^* \end{array} \right]$$

(11e)

$$C_6 = \left[ \begin{array}{cccc} c_1 & 0 & c_3 & 0 \\ 0 & c_2 & 0 & c_4 \\ 0 & c_3 & 0 & -c_2^* \\ c_2^* & -c_1^* & 0 & 0 \end{array} \right]$$

(11f)

Given the transmitted precoded symbols $S_i, i = 1, \ldots, 4$, to the $j$-th relay, through the channel with fading coefficient $h_{sj}, j = 1, \ldots, 4$ (see Fig. 1), the received signals at the relays are expressed as $r_{ij} = h_{sj} S_i + n_{ij}$, where $r_{ij}$ and $n_{ij}$ are the received symbol and AWGN at the $j$-th terminal, respectively. The $j$-th relay encodes a block of four received symbols with the code associated with the $j$-th row of the symbol block $C_i$ based on the corresponding transmission pattern chosen that yields the maximum SNR value given by Eq. (8) for MMSE receiver or Eq. (9) for zero-forcing receiver. Of course, it is assumed that this information has already been sent to the relay group from the receiver via feedback for space-time encoding, and receiver knows the channel state information between source and relay, and between relay and destination. After encoding the transmitted symbols, the relays transmit the encoded data to the destination.

For example, when encoding is done with $C_1$, the received signals over four consecutive time slots are expressed as

$$\left[ \begin{array}{c} r_1 \\ r_2 \\ r_3 \\ r_4 \end{array} \right] = \left[ \begin{array}{cccc} r_{11} & r_{22} & 0 & 0 \\ r_{21} & -r_{12} & 0 & 0 \\ 0 & 0 & r_{33} & r_{44} \\ 0 & 0 & r_{34} & -r_{43} \end{array} \right] \left[ \begin{array}{c} h_{1d} \\ h_{2d} \\ h_{3d} \\ h_{4d} \end{array} \right] + \left[ \begin{array}{c} n_1 \\ n_2 \\ n_3 \\ n_4 \end{array} \right]$$

(12)

where $h_{id}, i = 1, \ldots, 4$, are channel fading coefficients as shown in Fig. 1, and $n_i$ is the AWGN.

Following a similar analysis to the one given in [10], we can derive the expression of the received signals for each of the transmission patterns as

$$\left[ \begin{array}{c} r_1 \\ r_2 \\ r_3 \\ r_4 \end{array} \right] = \mathcal{H}_i \Theta_4 \left[ \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} \right] + \left[ \begin{array}{c} N_1 \\ N_2 \\ N_3 \\ N_4 \end{array} \right]$$

(13)

where $\mathcal{H}_i (i = 1, \ldots, 6)$ was given in Section II-B, whose elements are $h_1 = h_{s1} h_{1d}, h_2 = h_{s2} h_{2d}, h_3 = h_{s3} h_{3d},$ and
Note that $f_W(w)$ in (17a) is a raised-cosine function with a roll-off factor 0.25.

The impact of the channel-dependent interference in (14) can be estimated by calculating the statistical average of the absolute value of $W$ as a function of the number of transmission patterns $n$. With pdf and cdf of $W$ given in (17), and following the analysis given in [7], [9], we have

$$E[|W(n)|] = \int_0^{1.25} n[1 - F_W(w)]^{n-1} f_W(w) dw$$

The values of (18) as a function of $n$ is listed in Table I. As seen from this table, the average interference decreases as $n$ increases. With $n = 2$ patterns, the average interference is reduced by about 51 percent relative to the case with $n = 1$ (only one pattern). Note that for the case of $n = 2$, one feedback bit is needed; for the case of $n = 1$, no feedback is required.

### Table I

<table>
<thead>
<tr>
<th>$n$</th>
<th>0.5</th>
<th>0.2571</th>
<th>0.1250</th>
<th>0.0625</th>
<th>0.0313</th>
<th>0.0156</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[</td>
<td>W(n)</td>
<td>]$</td>
<td>0.5</td>
<td>0.2571</td>
<td>0.1250</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

**IV. SIMULATION RESULTS**

We simulate the bit-error rate (BER) performance of the proposed scheme and compare four transmission scenarios: (a) transmission using a $4 \times 4$ quasi-orthogonal STBC [5], [10]; (b) the proposed scheme with the precoded STBC given in [1] (no feedback is needed); (c) the proposed scheme with precoding where transmission is chosen out of arbitrarily selected two patterns (1-bit feedback); and (d) the proposed scheme with precoding where transmission is chosen out of arbitrarily selected four patterns (2-bit feedback). As in Section III-A, we assume that a group of four relay has been selected and all the scenarios above use the same relay group.

The system uses QPSK modulation. Since our focus is on non-regenerative relay, no channel encoding/decoding is employed in all simulations. We assume flat fading and the channel coefficients obey a quasi-static model, which is acceptable for slowly fading channels. It is also assumed that the channels from the transmitter to the relays, and from all relays to the receiver
are independent, and perfect channel estimates are available at the relays and the receiver. Since fading is assumed to be quasi-static, the delay of the feedback to select the space-time code pattern that yields the largest SNR can be neglected.

Fig. 3 compares the performances of four cases assuming an MMSE receiver. It is observed that the relay scheme using the quasi-orthogonal STBC outperforms the scheme using the code given in [1] in the high-SNR region. If we increase the number of transmission patterns, the performance of the proposed scheme with only two relays is found to outperform that of the scheme with quasi-orthogonal STBC at all SNR values. The slight expense paid is that it requires 2 bits of feedback information. At a BER of $10^{-3}$, the proposed scheme with 1 bit of feedback (two transmission patterns) achieves about 2 dB gain over the scheme using quasi-orthogonal codes with four relays.

As seen from simulation results, no matter if an MMSE or a ZF receiver is employed, the BER performance gap between the cases with two and four patterns is much smaller than that between one and two transmission patterns, that is, the performance gap between 1-bit and 2-bit feedbacks is less than 1 dB. Therefore, the system can achieve a good performance with only one bit of feedback.

V. CONCLUSION

We have proposed a new cooperative relay communication scheme with relay-selection that efficiently exploits distributed spatial diversity to improve performance. This scheme maximizes the SNR at the receiver by selecting the transmission pattern out of several possible choices, depending on the channel fading coefficients. We have provided the system model for single-hop communications scenario. Performance comparison is made between the proposed scheme with different number of possible transmission patterns and relay using an existing quasi-orthogonal STBC. Simulation results show that the proposed scheme with only 1 bit of feedback achieves over 2 dB gain at a BER of $10^{-3}$ over the relay scheme using the quasi-orthogonal STBC. The number of patterns does not need to be high to realize the potential performance gain; with only any two patterns (1-bit feedback) to choose from, the performance already approaches that with four or more patterns (more than 2-bit feedback).

REFERENCES