

On Angle Feedback and Antenna Shuffling in Double Space-Time Transmit Diversity Systems

Tao Xu and Huaping Liu, *Senior Member, IEEE*

Abstract—In this letter, we compare angle feedback and transmit antenna shuffling schemes for double space-time transmit diversity (DSTTD) systems with four transmit antennas and at least two receive antennas. We show that a DSTTD system with one bit of angle feedback does not provide better performance than the same system with one bit of transmit antenna shuffling. We also present a simplified general result about the selection of antenna shuffling matrix from six permutation matrices to facilitate our arguments. In simulation, we observe that antenna shuffling outperforms angle feedback in i.i.d. Rayleigh fading channels.

Index Terms—Double space-time transmit diversity, antenna shuffling, angle rotation, limited rate feedback.

I. INTRODUCTION

DOUBLE space-time transmit diversity (DSTTD) systems with four transmit antennas have been studied for high data rate transmission [1]–[5]. Due to feedback, the performance of closed-loop DSTTD systems is much better than that of open-loop DSTTD systems. An angle feedback scheme is proposed in [3] to suppress interference caused by the non-orthogonal structure of the effective channel. In this scheme, a rotation factor $c = e^{j\theta}$ is applied to the symbols being transmitted on the second antenna. In [5], transmit antenna shuffling is adopted to improve system performance. Both schemes are simple to implement.

It is of interest to understand the relative performance between angle feedback [3] and transmit antenna shuffling [5]. In this letter, we compare these two schemes and show that the latter outperforms the former.

II. ANGLE FEEDBACK VERSUS TRANSMIT ANTENNA SHUFFLING

A. Angle feedback

Consider a four-transmit-antenna system that employs angle feedback. We focus on the general case with four rotation factors. The effective channel matrix from the four transmit antennas to the j th receive antenna, \mathbf{H}_j , is a modified version of \mathbf{H}_j given in [3, eq. (4)]

$$\mathbf{H}_j = \begin{bmatrix} c_1 h_{1j} & c_2 h_{2j} & c_3 g_{1j} & c_4 g_{2j} \\ -c_2^* h_{2j}^* & c_1^* h_{1j}^* & -c_4^* g_{2j}^* & c_3^* g_{1j}^* \end{bmatrix}^T, \quad (1)$$

where $c_i = e^{j\theta_i}$, $1 \leq i \leq 4$. Let $(\cdot)^H$, $(\cdot)^T$, and $(\cdot)^*$ represent, respectively, Hermitian, transpose, and conjugate. With the

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The authors are with the School of Electrical Engineering and Computer Science, Oregon State University, Corvallis, Oregon 97331 USA (e-mail: {xuta, hliu}@eecs.oregonstate.edu).

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notation $\mathbf{c} = [c_1, c_2, c_3, c_4]$, the quadratic channel product in [3, eq. (5)] becomes

$$\mathbf{H}^H \mathbf{H} = \begin{bmatrix} \rho \mathbf{I}_2 & \mathbf{U} \\ \mathbf{U}^H & \mu \mathbf{I}_2 \end{bmatrix} \quad (2)$$

where \mathbf{I}_N is the $N \times N$ identity matrix and

$$\mathbf{H} = [\mathbf{H}_1 \ \mathbf{H}_2 \ \cdots \ \mathbf{H}_{N_r}]^T, \quad (3)$$

$$\rho = \sum_{i=1}^2 \sum_{j=1}^{N_r} |h_{ij}|^2, \quad \mu = \sum_{i=1}^2 \sum_{j=1}^{N_r} |g_{ij}|^2, \quad (4)$$

$$\mathbf{U} = \begin{bmatrix} \delta_1(\mathbf{c}) & \delta_2(\mathbf{c}) \\ -\delta_2^*(\mathbf{c}) & \delta_1^*(\mathbf{c}) \end{bmatrix}, \quad (5)$$

$$\delta_1(\mathbf{c}) = c_1^* c_3 \sum_{j=1}^{N_r} h_{1j}^* g_{1j} + c_2^* c_4 \sum_{j=1}^{N_r} h_{2j} g_{2j}^*, \quad (6)$$

$$\delta_2(\mathbf{c}) = c_1^* c_4 \sum_{j=1}^{N_r} h_{1j}^* g_{2j} - c_2^* c_3 \sum_{j=1}^{N_r} h_{2j} g_{1j}^*. \quad (7)$$

Let $\eta = |\delta_1(\mathbf{c})|^2 + |\delta_2(\mathbf{c})|^2$. In [3], the interference suppression criterion is to minimize η .

When a minimum mean-square error (MMSE) equalizer is applied to the received signal given by [3, eq. (2)], the estimate of the transmitted symbol vector $\mathbf{x} = [x_1, x_2, x_3, x_4]^T$ is expressed as $\hat{\mathbf{x}}_{\text{MMSE}} = \left(\mathbf{H}^H \mathbf{H} + \frac{1}{\zeta} \mathbf{I}_4 \right)^{-1} \mathbf{H}^H \mathbf{r}$, where $\zeta \triangleq E_s / \sigma^2$ is defined as the signal-to-noise ratio (SNR), E_s is the average transmit symbol energy, and σ^2 is the variance of the complex Gaussian noise samples at each receive antenna. The MSE of the MMSE equalizer is given by [4]

$$\begin{aligned} \mathbb{E} \left[\|\hat{\mathbf{x}}_{\text{MMSE}} - \mathbf{x}\|^2 \right] &= \sigma^2 \text{tr} \left(\left[\mathbf{H}^H \mathbf{H} + \frac{1}{\zeta} \mathbf{I}_4 \right]^{-1} \right) \\ &= \frac{\sigma^2 [2(\rho + \mu) + 4/\zeta]}{1/\zeta^2 + (\rho + \mu)/\zeta + \rho\mu - \eta}. \end{aligned} \quad (8)$$

For a zero-forcing (ZF) equalizer, we have $\hat{\mathbf{x}}_{\text{ZF}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{r}$. The MSE of the ZF equalizer is expressed as

$$\mathbb{E} \left[\|\hat{\mathbf{x}}_{\text{ZF}} - \mathbf{x}\|^2 \right] = \sigma^2 \text{tr} \left(\left[\mathbf{H}^H \mathbf{H} \right]^{-1} \right) = \frac{2(\rho + \mu)\sigma^2}{\rho\mu - \eta}. \quad (9)$$

Since ζ , σ^2 , ρ , and μ are independent of \mathbf{c} , we have $\arg \min_{\mathbf{c}} \mathbb{E} \left[\|\hat{\mathbf{x}} - \mathbf{x}\|^2 \right] = \arg \min_{\mathbf{c}} \eta$ for both MMSE and ZF equalizers. Thus, minimizing η is optimal in the MMSE sense.

Note that η is a function of \mathbf{c} and can be written as

$$\eta = \delta_1(\mathbf{c})\delta_1(\mathbf{c})^* + \delta_2(\mathbf{c})\delta_2(\mathbf{c})^* = \alpha \cdot \mathbf{c} + \alpha^* \cdot \mathbf{c}^* + \xi$$

$$|\Gamma_m(c=1)| = \left| \det \begin{pmatrix} \tilde{h}_{1\ell_m(1)} & \tilde{h}_{2\ell_m(1)} \\ \tilde{h}_{1\ell_m(2)} & \tilde{h}_{2\ell_m(2)} \end{pmatrix} + \det \begin{pmatrix} \tilde{h}_{3\ell_m(1)} & \tilde{h}_{4\ell_m(1)} \\ \tilde{h}_{3\ell_m(2)} & \tilde{h}_{4\ell_m(2)} \end{pmatrix} \right|. \quad (12a)$$

$$|\Gamma_m(c=-1)| = \left| \det \begin{pmatrix} \tilde{h}_{1\ell_m(1)} & \tilde{h}_{2\ell_m(1)} \\ \tilde{h}_{1\ell_m(2)} & \tilde{h}_{2\ell_m(2)} \end{pmatrix} - \det \begin{pmatrix} \tilde{h}_{3\ell_m(1)} & \tilde{h}_{4\ell_m(1)} \\ \tilde{h}_{3\ell_m(2)} & \tilde{h}_{4\ell_m(2)} \end{pmatrix} \right|. \quad (12b)$$

where α and ξ are derived in [3, eq. (11)] and $c = c_1 c_2 c_3^* c_4^* = e^{j(\theta_1 + \theta_2 - \theta_3 - \theta_4)}$. The solution for c that minimizes η is expressed as $c = e^{j(\pi - \psi)}$ [3], where ψ is the phase of α . It can be shown that to suppress interference, it is sufficient to perform rotation on only one transmit antenna rather than on four transmit antennas. When the i th transmit antenna ($1 \leq i \leq 4$) is selected for rotation, the optimum rotation angle is $\theta_i = (-1)^{\lceil i/2 \rceil}(\psi - \pi)$ and $\forall j, 1 \leq j \neq i \leq 4, \theta_j = 0$. $\lceil \cdot \rceil$ represents the ceiling function. Without loss of generality, we rotate the data for the second transmit antenna as in [3] for performance comparison. The quantized angle feedback and its selection criterion are presented in [3].

B. Transmit antenna shuffling

Antenna shuffling has been adopted in WiMAX [5]. In this scheme, there is no angle rotation, that is, $c_i = 1$ for $1 \leq i \leq 4$. For simplicity of notation, we treat h_{ij} and g_{ij} in (1), $i = 1, 2$ and $1 \leq j \leq N_r$, as logical channels, and denote \tilde{h}_{kj} as the physical channel between the k th ($1 \leq k \leq 4$) transmit antenna and the j th receive antenna. The mapping between physical channels and logical channels is [5]

$$[h_{1j} \ h_{2j} \ g_{1j} \ g_{2j}] = [\tilde{h}_{1j} \ \tilde{h}_{2j} \ \tilde{h}_{3j} \ \tilde{h}_{4j}] \mathbf{W} \quad (10)$$

where \mathbf{W} is the permutation matrix:

$$\mathbf{W} \in S_W = \left\{ \begin{array}{l} [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3, \mathbf{i}_4], [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_2, \mathbf{i}_4], \\ [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_4, \mathbf{i}_2], [\mathbf{i}_1, \mathbf{i}_4, \mathbf{i}_2, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_4, \mathbf{i}_3, \mathbf{i}_2] \end{array} \right\}$$

and \mathbf{i}_k is the k th column of \mathbf{I}_4 .

We can verify that permutation might change the values of ρ and μ but does not change the sum of ρ and μ . In order to minimize the MSE of MMSE and ZF equalizers in (8) and (9), we must maximize $\rho\mu - \eta$ using one of the six permutation matrices in S_W . A simple permutation matrix selection criterion for $N_r = 2$ is given in [4]. We generalize the matrix selection criterion to cases of $N_r \geq 2$ through the next lemma. This helps facilitate permutation matrix selection.

Lemma 1: For ZF or MMSE receiver, the permutation matrix selection criterion to minimize MSE for $N_r \geq 2$ is expressed as

$$\arg \max_{\mathbf{W} \in S_W} (\rho\mu - \eta) = \arg \min_{\mathbf{W} \in S_W} \sum_{m=1}^M |\Gamma_m|^2 \quad (11)$$

where

$$\Gamma_m = \det \begin{pmatrix} h_{1\ell_m(1)} & h_{2\ell_m(1)} \\ h_{1\ell_m(2)} & h_{2\ell_m(2)} \end{pmatrix} + \det \begin{pmatrix} g_{1\ell_m(1)} & g_{2\ell_m(1)} \\ g_{1\ell_m(2)} & g_{2\ell_m(2)} \end{pmatrix}$$

and $M = \binom{N_r}{2} = \frac{N_r!}{2(N_r-2)!}$ is the number of sets of combination with each set containing two receive antenna indices. The 1×2 vector $\ell_m = [\ell_m(1) \ \ell_m(2)]$ denotes the m th set with indices $1 \leq \ell_m(1) < \ell_m(2) \leq N_r, 1 \leq m \leq M$.

Proof: With some math manipulations, we have $\rho\mu - \eta = \sum_{m=1}^M (\rho_m \mu_m - \eta_m)$, where $\rho_m = \sum_{i=1}^2 \sum_{k=1}^2 |h_{i\ell_m(k)}|^2$, $\mu_m = \sum_{i=1}^2 \sum_{k=1}^2 |g_{i\ell_m(k)}|^2$ and $\eta_m = |\delta_{1,m}|^2 + |\delta_{2,m}|^2$ with $\delta_{1,m} = \sum_{k=1}^2 (h_{1\ell_m(k)}^* g_{1\ell_m(k)} + h_{2\ell_m(k)} g_{2\ell_m(k)}^*)$ and $\delta_{2,m} = \sum_{k=1}^2 (h_{1\ell_m(k)}^* g_{2\ell_m(k)} - h_{2\ell_m(k)} g_{1\ell_m(k)}^*)$. Following the proof in [4, *Property 1*]¹, we can show that $\rho_m \mu_m - \eta_m = \Lambda_m - |\Gamma_m|^2$ with $\Lambda_m = \sum_{i=1}^4 \sum_{j=1, j \neq i}^4 |\tilde{h}_{i\ell_m(1)} \tilde{h}_{j\ell_m(2)}|^2 - 2\Re \left\{ \sum_{i=1}^4 (\tilde{h}_{i\ell_m(1)}^* \tilde{h}_{i\ell_m(2)}) \sum_{j=i+1}^4 \tilde{h}_{j\ell_m(1)} \tilde{h}_{j\ell_m(2)}^* \right\}$. Since Λ_m is independent of \mathbf{W} , we have (11). ■

When $N_r = 2$, the right-hand side of (11) has only one term, which can be written as $\arg \min_{\mathbf{W} \in S_W} |\Gamma_1|$. This has the same form as in [4, *Property 1*], a special case of lemma 1.

Now we show that 1-bit angle feedback is a special case of 1-bit transmit antenna shuffling. With 1-bit angle feedback $c = \pm 1$ applied to the second antenna [3], we have $[h_{1j} \ h_{2j} \ g_{1j} \ g_{2j}] = [\tilde{h}_{1j} \ c\tilde{h}_{2j} \ \tilde{h}_{3j} \ \tilde{h}_{4j}]$ and h_{2j} could be either \tilde{h}_{2j} or $-\tilde{h}_{2j}$. $|\Gamma_m(c)|$ is expressed at the top of this page. For antenna shuffling, we select shuffling matrices $\mathbf{W}_1 = [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3, \mathbf{i}_4]$ and $\mathbf{W}_2 = [\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3]$ for 1-bit feedback. We can verify

$$|\Gamma_m(\mathbf{W} = \mathbf{W}_1)| = |\Gamma_m(c=1)|, \quad (13a)$$

$$|\Gamma_m(\mathbf{W} = \mathbf{W}_2)| = |\Gamma_m(c=-1)|. \quad (13b)$$

From the proof of *Lemma 1*, we have $\rho\mu - \eta = \sum_{m=1}^M (\Lambda_m - |\Gamma_m|^2)$ and the value of Λ_m does not change under $c = \pm 1$ and $\mathbf{W} = \mathbf{W}_1/\mathbf{W}_2$. Combining (13a) and (13b), we have

$$(\rho\mu - \eta)|_{c=1} = (\rho\mu - \eta)|_{\mathbf{W}=\mathbf{W}_1}, \quad (14a)$$

$$(\rho\mu - \eta)|_{c=-1} = (\rho\mu - \eta)|_{\mathbf{W}=\mathbf{W}_2}. \quad (14b)$$

Furthermore, since the values of ρ and μ in (4) do not change either under $c = \pm 1$ and $\mathbf{W} = \mathbf{W}_1/\mathbf{W}_2$, we have 1) $\eta|_{c=1} = \eta|_{\mathbf{W}=\mathbf{W}_1}$ and $\eta|_{c=-1} = \eta|_{\mathbf{W}=\mathbf{W}_2}$; 2) $c = -1$ (1) in angle rotation and $\mathbf{W} = \mathbf{W}_2$ (\mathbf{W}_1) in transmit antenna shuffling have the same MSE from (8) and (9) and their selections must be in pair (one-to-one).

With one-to-one mapping selections for any channel realization \mathbf{H} , let us further consider the BER performance of both schemes with the above 1-bit feedback. We only need to consider angle rotation $c = -1$ and shuffling matrix $\mathbf{W} = \mathbf{W}_2$ because angle feedback system with $c = 1$ and transmit antenna shuffling system with $\mathbf{W} = \mathbf{W}_1$ are exactly the same and have the same BER with a ZF or MMSE receiver. We define $\eta|_{c=-1} = \eta|_{\mathbf{W}=\mathbf{W}_2} = \bar{\eta}$ and $\mathbf{\Pi} = \mathbf{H}^H \mathbf{H}$. The post-processing SNR of the k th ($1 \leq k \leq 4$) data stream of

¹We believe that κ in $\arg \max_{\mathbf{W} \in S_W} (c_2 - c_3 - \kappa)$ in [4] should be κ^2 .

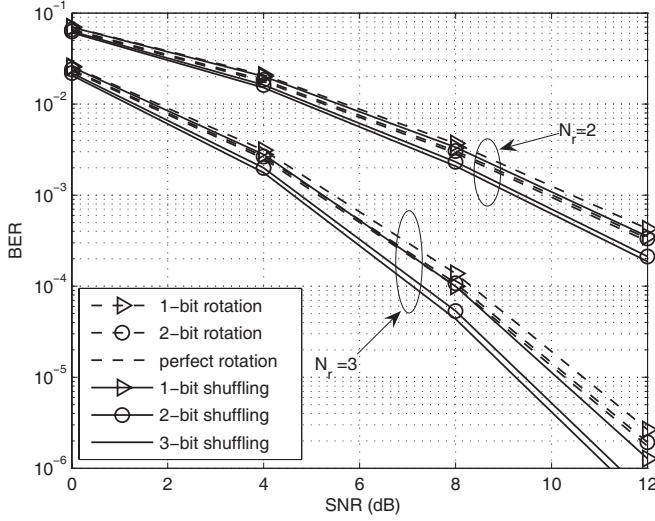


Fig. 1. Performance of antenna shuffling scheme and angle feedback scheme.

ZF and MMSE equalizers can be expressed as [6]

$$\gamma_k^{ZF} = \frac{\zeta}{\mathbf{\Pi}_{k,k}^{-1}},$$

$$\gamma_k^{MMSE} = \frac{\zeta}{[\mathbf{\Pi} + (\frac{1}{\zeta})\mathbf{I}_4]_{k,k}^{-1}} - 1.$$

Using matrix inversion lemma for partitioned matrices, we can verify for any channel realization \mathbf{H} that results in $c = -1$ and $\mathbf{W} = \mathbf{W}_2$, the following equalities

$$\mathbf{\Pi}_{k,k}^{-1}|_{c=-1} = \mathbf{\Pi}_{k,k}^{-1}|_{\mathbf{w}=\mathbf{w}_2} = (\rho - \bar{\eta}/\mu)^{-1}, \quad k = 1, 2$$

$$\mathbf{\Pi}_{k,k}^{-1}|_{c=-1} = \mathbf{\Pi}_{k,k}^{-1}|_{\mathbf{w}=\mathbf{w}_2} = (\mu - \bar{\eta}/\rho)^{-1}, \quad k = 3, 4$$

$$[\mathbf{\Pi} + (\frac{1}{\zeta})\mathbf{I}_4]_{k,k}^{-1}|_{c=-1} = [\mathbf{\Pi} + (\frac{1}{\zeta})\mathbf{I}_4]_{k,k}^{-1}|_{\mathbf{w}=\mathbf{w}_2}$$

$$= \begin{cases} [\rho + 1/\zeta - \bar{\eta}/(\mu + 1/\zeta)]^{-1}, & k = 1, 2 \\ [\mu + 1/\zeta - \bar{\eta}/(\rho + 1/\zeta)]^{-1}, & k = 3, 4 \end{cases}$$

always hold. Therefore, we have

$$\gamma_k^{ZF}|_{c=-1} = \gamma_k^{ZF}|_{\mathbf{w}=\mathbf{w}_2},$$

$$\gamma_k^{MMSE}|_{c=-1} = \gamma_k^{MMSE}|_{\mathbf{w}=\mathbf{w}_2}.$$

By combining the scenarios of $c = 1$ and $\mathbf{W} = \mathbf{W}_1$, we conclude that both schemes, i.e., angle rotation with $c = \pm 1$ and antenna shuffling with $\mathbf{W} = \mathbf{W}_1/\mathbf{W}_2$, have the same BER performance with a ZF or MMSE receiver.

Because any pair of shuffling matrices in S_W could be selected for 1-bit antenna shuffling, we have the following lemma:

Lemma 2: With a MMSE or ZF receiver, under MSE selection criterion, the 1-bit angle feedback scheme proposed in [3] is a special case of antenna shuffling with shuffling matrices $[\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3, \mathbf{i}_4]$ and $[\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3]$, which requires 1 bit of feedback. Furthermore, it does not perform better than the 1-bit transmit antenna shuffling scheme with a pair of properly selected shuffling matrices.

Lemma 2 gives the relative performance between angle feedback and antenna shuffling schemes with 1-bit feedback.

With two or more feedback bits, it is difficult to compare the performance of the two schemes analytically. We thus resort to simulation, through which we observe that antenna shuffling with only two bits of feedback performs better than angle feedback with infinite number of feedback bits.

III. SIMULATION RESULTS

We simulate the bit-error rate (BER) performance of an uncoded DSTTD system with 4 transmit antennas, and 2 and 3 receive antennas. The system employs quadrature phase-shift keying modulation and a MMSE receiver. Channel coefficients for different transmit-receive links are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. We adopt a quasi-static fading model for the channel. For each realization, 4000 QPSK symbols are transmitted, and BER is averaged over 10^6 independent channel realizations. 1-bit antenna shuffling selects shuffling matrix from the set $\{[\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_2, \mathbf{i}_4]\}$; 2-bit shuffling matrix set is $\{[\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_4, \mathbf{i}_3], [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_2, \mathbf{i}_4], [\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_4, \mathbf{i}_2], [\mathbf{i}_1, \mathbf{i}_4, \mathbf{i}_2, \mathbf{i}_3]\}$; 3-bit antenna shuffling matrix set is S_W . The angle rotation scheme uses $c = \pm 1$ for 1-bit angle rotation and $c = \pm 1, \pm j$ for 2-bit angle rotation on the second antenna. The results are shown in Fig. 1. We observe that the 1-bit antenna shuffling scheme has a better performance than the 1-bit angle feedback scheme; 2-bit antenna shuffling performs better than angle feedback scheme with infinite number of feedback bits. This shows that antenna shuffling scheme is more effective than the angle rotation scheme in term of feedback gain.

IV. CONCLUSION

A comprehensive comparison of the performance of angle feedback and antenna shuffling schemes for DSTTD systems is provided in this letter. We have shown that antenna shuffling outperforms angle feedback. The reason is that antenna shuffling effectively boosts the minimum post-processing SNR among all data streams by maximizing the quantity $\rho\mu - \eta$ defined in the paper, whereas angle feedback only minimizes η ; thus the potential gain in the post-processing SNR achievable by balancing ρ and μ is not exploited with angle feedback.

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