Frequency-Domain Correlative Coding for MIMO-OFDM Systems Over Fast Fading Channels

Yu Zhang, *Student Member, IEEE* and Huaping Liu, *Member, IEEE*

***Abstract*— Multiple-input multiple-output (MIMO) antennas combined with orthogonal frequency division multiplexing (OFDM) are very attractive for high-data-rate communications. However, MIMO-OFDM systems are very vulnerable to time-selective fading as channel time-variation destroys the orthogonality among subchannels, causing inter-carrier interference (ICI). In this letter, we apply frequency-domain correlative coding in MIMO-OFDM systems over frequency-selective, fast-fading channels to mitigate ICI. We derive the analytical expression of the carrier-to-interference ratio (CIR) to quantify the impact of time-selective fading and demonstrate the effectiveness of correlative coding in mitigating ICI in MIMO-OFDM systems.**

***Index Terms*— Time-selective fading, inter-carrier interference, correlative coding, orthogonal frequency division multiplexing.**

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM), though effective in avoiding intersymbol interference due to multipath delay, is sensitive to time-selective fading, which destroys the orthogonality among subcarriers in one OFDM symbol and thus causes inter-carrier interference (ICI) [1], [2]. If not compensated for, ICI will result in an error floor, which increases as Doppler shift and symbol duration increase. To combat ICI in single-antenna OFDM systems, various methods such as frequency-domain correlative coding [3], ICI self-cancellation [4], [5], and partial response coding [6] have been studied. The scheme in [3] can be viewed as a special type of frequency-domain partial response coding with a correlation polynomial $F(D) = 1 - D$.

Multiple-input multiple-output (MIMO) antennas can be combined with OFDM to improve spectral efficiency through spatial multiplexing [7]. Support of high mobility in MIMO-OFDM systems is critical for many applications (e.g., IEEE 802.16e). Similar to single-antenna OFDM, performance of MIMO-OFDM is also sensitive to time-selective fading.

In this letter, we apply frequency-domain correlative coding originally proposed in [3] for single-antenna OFDM systems to MIMO-OFDM to improve system robustness to time-selective fading. While the analysis in [3] considered a simple case in which ICI is caused by a single parameter – the frequency offset normalized to the subcarrier separation, we consider a more comprehensive and realistic scenario which includes not only the spatial elements, but also the time-varying and frequency-selective aspects of the channel. We focus on deriving, via an analytical approach, a tractable, closed-form expression of the carrier-to-interference ratio (CIR) as a function of channel Doppler shift, number of subcarriers, OFDM symbol duration, and the power-delay profile of the multipath fading channel. With the CIR expression derived, we can quantify the impact of time-selective fading and the improvement due to correlative coding in MIMO-OFDM.

Manuscript received January 9, 2006. The associate editor coordinating the review of this letter and approving it for publication was Dr. Rohit Nabar.

The authors are with the School of Electrical Engineering and Computer Science, Oregon State University, Corvallis, OR 97331 USA (e-mail: hliu@eecs.oregonstate.edu).

Digital Object Identifier 10.1109/LCOMM.2006.05011.

II. SYSTEM MODEL

The following notation will be adopted. Column vectors/matrices are denoted by boldface lower/upper case letters; superscripts $(\cdot)^*$ and $(\cdot)^H$ denote complex conjugate and complex conjugate transpose, respectively; $E[\cdot]$ and $var(\cdot)$ stand for expectation and variance, respectively; $\boldsymbol{I}_N$ represents the $N \times N$ identity matrix; $\otimes$ denotes Kronecker product; $\{\boldsymbol{A}\}_{ij}$ denotes the (i, j)-th element of matrix $\boldsymbol{A}$.

Consider a MIMO-OFDM system with N_t transmit antennas, N_r receive antennas, and N_s subcarriers which employs binary phase shift keying (BPSK) modulation. Input symbols $a_i \in \{1, -1\}$ are assumed to be independent and identically distributed with normalized power. The correlative coding to encode a_i is achieved through the frequency-domain polynomial $F(D) = 1 - D$ [3], which generates a new sequence $b_i = a_i - a_{i-1}$ with $E[b_i] = 0$ and

$$E[b_i b_j^*] = \begin{cases} 2E[a_i^2] = 2, & i = j \\ -E[a_i^2] = -1, & |i-j| = 1 \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

It is well known that the general form of MIMO-OFDM over slowly fading channels (i.e., the channel is time-invariant over several OFDM symbol periods) can be expressed as [7]

$$\boldsymbol{y}_k = \boldsymbol{\Lambda}_k \boldsymbol{x}_k + \boldsymbol{n}_k \quad (2)$$

where $\boldsymbol{x}_k$ and $\boldsymbol{y}_k$ represent, respectively, the transmitted and received signals for all antennas on subcarrier k, $\boldsymbol{\Lambda}_k$ is an $N_r \times N_t$ matrix with $\{\boldsymbol{\Lambda}_k\}_{ij}$ being the channel frequency response between transmit antenna j and receive antenna i, and $\boldsymbol{n}_k$ is an $N_r \times 1$ vector denoting the zero-mean AWGN with covariance $\sigma_n^2 \boldsymbol{I}_{N_r}$ for all antennas on subcarrier k.

III. EFFECTS OF TIME-SELECTIVE FADING

In a time-selective channel, the $N_s N_r \times N_s N_t$ channel matrix $\boldsymbol{H}$ in one OFDM symbol period is expressed as

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_0(0) & \cdots & \boldsymbol{H}_{L-1}(0) & \cdots & \boldsymbol{H}_1(0) \\ \vdots & & \ddots & & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{H}_{L-1}(N_s-1) & \cdots & \boldsymbol{H}_0(N_s-1) \end{bmatrix} \quad (3)$$

where L is the number of resolvable paths and $\boldsymbol{0}$ is an $N_r \times N_t$ zero matrix. Each non-zero block of $\boldsymbol{H}$ contains the $N_r \times N_t$ channel matrix $\boldsymbol{H}_l(n)$ for path l at time nT_s (T_s is the data symbol period).

Assuming a wide sense stationary uncorrelated scattering channel, all elements of $\boldsymbol{H}_l(n)$ are modeled as independent complex Gaussian random variables with zero mean and equal variance. The channel is assumed to have an exponential

$$C_{corr}^{(k)} = \frac{E\left[\{\boldsymbol{G}_{kk}\}_{ij} b_k b_k^* \{\boldsymbol{G}_{kk}\}_{ij}^*\right]}{\sum_{\substack{k'=0 \\ k' \neq k}}^{N_s-1} \sum_{\substack{k''=0 \\ k'' \neq k}}^{N_s-1} E\left[\{\boldsymbol{G}_{kk'}\}_{ij} b_{k'} b_{k''}^* \{\boldsymbol{G}_{kk''}\}_{ij}^*\right]} = \frac{2\gamma_0}{\sum_{k'=1}^{N_s-1} 2\gamma_{k'} - \sum_{\substack{k'=0 \\ k' \neq k, k-1}}^{N_s-2} E\left[\{\boldsymbol{G}_{kk'}\}_{ij}\{\boldsymbol{G}_{k,k'+1}\}_{ij}^* + \{\boldsymbol{G}_{k,k'+1}\}_{ij}\{\boldsymbol{G}_{kk'}\}_{ij}^*\right]}$$
$$= \frac{\frac{2}{N_s^2} \sum_{l=0}^{L-1} \left\{ N_s + 2 \sum_{i=1}^{N_s-1} (N_s - i)\, J_0(2\pi i f_d T_s) \right\} e^{-\frac{\tau_l}{\tau_{rms}}}}{\frac{2}{N_s^2} \sum_{k'=1}^{N_s-1} \sum_{l=0}^{L-1} \left\{ N_s + 2 \sum_{i=1}^{N_s-1} (N_s - i) J_0(2\pi i f_d T_s) \cos\left(\frac{2\pi}{N_s} k' i\right) \right\} e^{-\frac{\tau_l}{\tau_{rms}}} - \sum_{k'=0, k' \neq k, k-1}^{N_s-2} \Omega_{k'}}. \quad (9)$$

power-delay profile $\theta(\tau_l) = e^{-\tau_l/\tau_{rms}}$ [8], where τ_l is the delay of the l-th path and τ_{rms} is the root-mean square (rms) delay spread. Since the channel is time-variant, the relationship between the channel coefficients for path l at times nT_s and $(n+m)T_s$ can be described as [9], [10]

$$\{\boldsymbol{H}_l(n+m)\}_{ij} = \alpha_m \{\boldsymbol{H}_l(n)\}_{ij} + \beta_{l,ij}(n+m) \quad (4)$$

where

$$\alpha_m = \frac{E\left[\{\boldsymbol{H}_l(n)\}_{ij}\{\boldsymbol{H}_l(n+m)\}_{ij}^*\right]}{e^{-\tau_l/\tau_{rms}}} = J_0(2\pi m f_d T_s) \quad (5)$$

f_d is the maximum Doppler shift, $J_0(\cdot)$ is the zero-th order Bessel function of the first kind, and $\beta_{l,ij}(n)$ are independent complex Gaussian random variables with zero mean and variance $e^{-\frac{\tau_l}{\tau_{rms}}}(1 - \alpha_m^2)$. It is observed that the channel matrix $\boldsymbol{H}$ in (3) is no longer a block-circulant matrix as the case of slowly fading channels. Consequently, $\boldsymbol{G} = (\boldsymbol{U} \otimes \boldsymbol{I}_{N_r})\boldsymbol{H}(\boldsymbol{U} \otimes \boldsymbol{I}_{N_t})^H$ is no longer a block diagonal matrix, where $\boldsymbol{U}$ is the unitary discrete Fourier transform (DFT) matrix with $\{\boldsymbol{U}\}_{ij} = 1/\sqrt{N_s} e^{(-2\pi\sqrt{-1}/N_s)ij}$, $0 \leq i, j \leq N_s - 1$. This shows that time-selective fading causes ICI, which is represented by the off-diagonal blocks of $\boldsymbol{G}$. Let $\boldsymbol{G}_{ij}$ denote the (i,j)-th block of $\boldsymbol{G}$. Eq. (2) can be re-written as

$$\boldsymbol{y}_k = \boldsymbol{G}_{kk}\boldsymbol{x}_k + \sum_{\substack{k'=0 \\ k' \neq k}}^{N_s-1} \boldsymbol{G}_{kk'}\boldsymbol{x}_{k'} + \boldsymbol{n}_k, \; k = 0, \cdots, N_s - 1. \quad (6)$$

Let $\boldsymbol{\Upsilon}_{ij}$ be an $N_s \times N_s$ matrix given by

$$\boldsymbol{\Upsilon}_{ij} = \begin{bmatrix} var(\{\boldsymbol{G}_{00}\}_{ij}) & \cdots & var(\{\boldsymbol{G}_{0,N_s-1}\}_{ij}) \\ \vdots & \ddots & \vdots \\ var(\{\boldsymbol{G}_{N_s-1,0}\}_{ij}) & \cdots & var(\{\boldsymbol{G}_{N_s-1,N_s-1}\}_{ij}) \end{bmatrix},$$
$$1 \leq i \leq N_r, 1 \leq j \leq N_t. \quad (7)$$

As shown in the appendix of [11], $\boldsymbol{\Upsilon}_{ij}$ has a circulant structure, i.e.,

$$\{\boldsymbol{\Upsilon}_{ij}\}_{i'j'} = \gamma_{[j'-i']} = \frac{1}{N_s^2} \sum_{l=0}^{L-1} \left\{ N_s + 2 \sum_{i=1}^{N_s-1} (N_s - i)\, J_0(2\pi \times i f_d T_s) \cos\left(\frac{2\pi}{N_s}[j' - i']i\right) \right\} e^{-\frac{\tau_l}{\tau_{rms}}}, \; 1 \leq i', j' \leq N_s \quad (8)$$

where $[n]$ denotes n modulo N_s. The CIR of the k-th subcarrier for MIMO-OFDM systems over time-selective fading channels is given by (9) at the top of this page. As shown in the appendix, $\Omega_{k'}$ in (9) is given as

$$\Omega_{k'} = \frac{1}{N_s^2} \sum_{l=0}^{L-1} \sum_{r=0}^{N_s-1} \sum_{s=0}^{N_s-1} J_0(2\pi|r-s| f_d T_s) e^{-\frac{\tau_l}{\tau_{rms}}} \times \left(e^{-(2\pi\sqrt{-1}/N_s)t_{kk'rs}^{(1)}} + e^{-(2\pi\sqrt{-1}/N_s)t_{kk'rs}^{(-1)}} \right). \quad (10)$$

Without correlative coding, the CIR expression given by (9) simplifies to

$$C = \frac{E\left[\{\boldsymbol{G}_{kk}\}_{ij} a_k a_k^* \{\boldsymbol{G}_{kk}\}_{ij}^*\right]}{\sum_{\substack{k'=0 \\ k' \neq k}}^{N_s-1} \sum_{\substack{k''=0 \\ k'' \neq k}}^{N_s-1} E\left[\{\boldsymbol{G}_{kk'}\}_{ij} a_{k'} a_{k''}^* \{\boldsymbol{G}_{kk''}\}_{ij}^*\right]}$$
$$= \frac{N_s + 2 \sum_{i=1}^{N_s-1} (N_s - i) J_0(2\pi i f_d T_s)}{\sum_{k'=1}^{N_s-1} \left\{ N_s + 2 \sum_{i=1}^{N_s-1} (N_s - i) J_0(2\pi i f_d T_s) \cos\left(\frac{2\pi}{N_s} k' i\right) \right\}}. \quad (11)$$

Note that in this case CIR is the same for all subcarriers and is independent of the channel power-delay profile as well as the number of resolvable paths. Obviously, $C_{corr}^{(k)} \geq C, \forall k$. Therefore, correlative coding effectively increases CIR. It is observed from (9) that although $C_{corr}^{(k)}$ is different for different subcarriers, the difference diminishes as N_s increases. As indicated in [3], when frequency-domain correlative coding with $F(D) = 1 - D$ is used, the signals modulated on subcarriers are identical with alternate mark inversion code and $\{a_i\}$ can be recovered by using a maximum likelihood (ML) sequence detector [12].

IV. Numerical Results and Discussion

In obtaining the numerical results, we consider a system with two transmit antennas and two receive antennas which employs BPSK modulation and adopt the "SUI-5" channel model [13]. The time-selective Rayleigh fading channel is assumed to have three resolvable multipath components occurring at 0, 5, and $10\mu s$. These paths are modeled as independent complex Gaussian random variables and the rms delay spread of the channel is $3.05\mu s$. The maximum Doppler shift is calculated based on a carrier frequency of $f_c = 2$GHz.

CIR levels versus T_s calculated using Eqs. (9) and (11) are plotted in Fig. 1, where the vehicle speed applied is $v_s = 100$Km/h. CIR curves of the MIMO-OFDM system with different number of subcarriers in one OFDM symbol

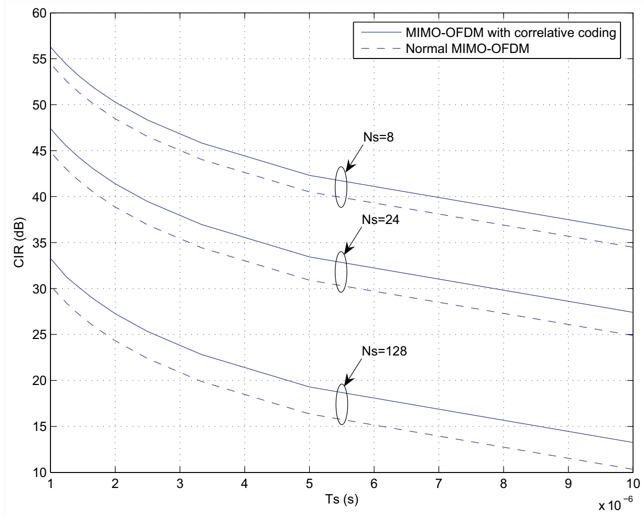

Fig. 1. CIR curves of MIMO-OFDM systems with and without frequency-domain correlative coding.

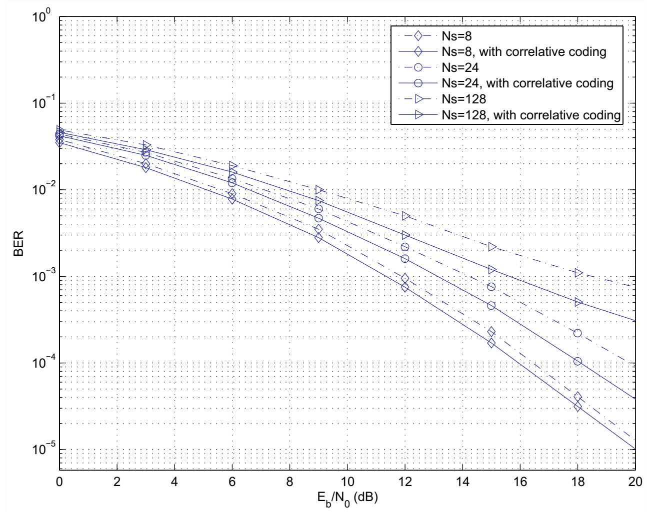

Fig. 2. BER versus ${}_b\ \ {}_0$ for MIMO-OFDM systems with and without frequency-domain correlative coding.

($N_s = 8,\ 24,$ and 128) are compared. As shown in Fig. 1, frequency-domain correlative coding incorporated in this letter can effectively increase CIR and the improvement is proportional to the number of subcarriers. With $N_s = 128$, the improvement is observed to be as high as 3.0dB.

The bit-error-rate (BER) performances of MIMO-OFDM systems with and without frequency-domain correlative coding are compared in Fig. 2, where $T_s = 5 \times 10^{-7}s$ and $v_s = 100$Km/h are applied. The ML detection scheme [7] is used when correlative coding is applied. The improvement in the BER performance is also found proportional to the number of subcarriers.

V. Conclusion

We have applied frequency-domain correlative coding to mitigate the effect of time-selective fading to the performance of MIMO-OFDM systems. We derived the analytical expression of CIR as a function of the maximum Doppler shift and power-delay profile of the channel, the number of subcarriers, and the OFDM symbol duration. The CIR expression can be used to quantify the amount of ICI caused by channel time-variations. Numerical results indicate that a simple correlative coding scheme with correlation polynomial $F(D) = 1 - D$ can effectively increase the CIR of a 128-subcarrier MIMO-OFDM system by as much as 3.0dB, and the improvement further increases as the number of subcarriers increases.

Appendix
Derivation of (10)

Following Eq. (45) in the appendix of [11], we define

$$\varpi_l^{(1)} = \sum_{r=0}^{N_s-1}\sum_{s=0}^{N_s-1} \eta_{ijr}\chi(r,s)\eta_{i(j+1)s}^{*} = \frac{1}{N_s^2}\sum_{r=0}^{N_s-1}\sum_{s=0}^{N_s-1} J_0(2\pi|r-s|f_dT_s)e^{-(2\pi\sqrt{-1}/N_s)t_{ijrs}^{(1)}}e^{-\frac{\tau_l}{\tau_{rms}}} \quad (12)$$

where $t_{ijrs}^{(1)} = ir - j[r-l] - is + (j+1)[s-l]$. Similar to (12), we have

$$\varpi_l^{(-1)} = \frac{1}{N_s^2}\sum_{r=0}^{N_s-1}\sum_{s=0}^{N_s-1} J_0(2\pi|r-s|f_dT_s) \times e^{-(2\pi\sqrt{-1}/N_s)t_{ijrs}^{(-1)}}e^{-\frac{\tau_l}{\tau_{rms}}} \quad (13)$$

where $t_{ijrs}^{(-1)} = ir - (j+1)[r-l] - is + j[s-l]$. Finally, we have

$$\Omega_{k'} = \sum_{l=0}^{L-1}\varpi_l^{(1)} + \varpi_l^{(-1)} = \frac{1}{N_s^2}\sum_{l=0}^{L-1}\sum_{r=0}^{N_s-1}\sum_{s=0}^{N_s-1} J_0(2\pi|r-s| \times f_dT_s)e^{-\frac{\tau_l}{\tau_{rms}}}\left(e^{-(2\pi\sqrt{-1}/N_s)t_{ijrs}^{(1)}} + e^{-(2\pi\sqrt{-1}/N_s)t_{ijrs}^{(-1)}}\right). \quad (14)$$

References

[1] M. Russell and G. J. Stüber, "Interchannel interference analysis of OFDM in a mobile environment," in *Proc. IEEE VTC 1995*, pp. 820–824.

[2] J. Li and M. Kavehrad, "Effects of time selective multipath fading on OFDM systems for broadband mobile applications," *IEEE Commun. Lett.*, vol. 3, pp. 332–334, Dec. 1999.

[3] Y. Zhao and S. G. Häggman, "Intercarrier interference compression in OFDM communication systems by using correlative coding," *IEEE Commun. Lett.*, vol. 2, pp. 214–216, Aug. 1998.

[4] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, pp. 365–369, Mar. 1999.

[5] Y. Zhao and S. G. Häggman, "Intercarrier interference self-cancellation scheme for OFDM mobile communication systems," *IEEE Trans. Commun.*, vol. 49, pp. 1185–1191, July 2001.

[6] H. Zhang and Y. Li, "Optimum frequency-domain partial response encoding in OFDM system," *IEEE Trans. Commun.*, vol. 51, pp. 1064–1068, July 2003.

[7] G. J. Stüber, J. R. Barry, S. W. Mclaughlin, Y. Li, M. A. Ingram, and T. G. Pratt, "Broadband MIMO-OFDM wireless communications," *Proc. of the IEEE*, vol. 92, pp. 271–294, Feb. 2004.

[8] O. Edfors, M. Sandell, J. -J. v. d. Beek, S. K. Wilson, and P. O. Börjesson, "OFDM channel estimation by singular value decomposition," *IEEE Trans. Commun.*, vol. 46, pp. 931–939, July 1998.

[9] F. C. Zheng and A. G. Burr, "Signal detection for non-orthogonal space-time block coding over time-selective fading channels," *IEEE Commun. Lett.*, vol. 8, pp. 491–493, Aug. 2004.

[10] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell Syst. Tech. J.*, vol. 47, pp. 957–1000, July 1968.

[11] Y. Zhang and H. Liu, "Impact of time selective fading on the performance of quasi-orthogonal space-time coded OFDM systems," *IEEE Trans. Commun.*, vol. 54, pp. 251–260, Feb. 2006.

[12] G. D. Forney, Jr., "Maximum-likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. 18, pp. 363–378, May 1972.

[13] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, pp. 58–66, Apr. 2002.