On the Optimum Linear Receiver for Impulse Radio Systems in the Presence of Pulse Overlapping

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***Abstract*— In impulse radio ultra-wideband systems, multipath delay may cause received pulses to overlap with each other. Pulse overlapping causes inter-pulse interference (IPI) which may, especially in dense multipath environments, severely limit the system performance. In this letter, we build a mathematical model with pulse overlapping considered and derive an optimum minimum mean-square error (MMSE) receiver. A simpler RAKE receiver is to take samples for each received pulse and perform maximal ratio combining (MRC) by ignoring the IPI. We then show, by an analytical approach, that the optimum linear MMSE receiver performs exactly the same as the simpler MRC receiver.**

***Index Terms*— Impulse radio, pulse overlapping, minimum mean-square error (MMSE), maximal ratio combining (MRC).**

I. Introduction

ULTRA-WIDEBAND (UWB) communications [1] could be achieved using orthogonal frequency division multiplexing [2] or impulse radio techniques [3], [4]. One of the advantages of impulse radio UWB communications is its ability to resolve individual multipath components. This requires a RAKE receiver to gain path diversity and to capture multipath energy.

Existing research on detection and performance analysis of pulse-based UWB systems has focused on the ideal case that received adjacent paths are separated in time by at least one pulse width. This assumption might be accurate enough for communications in line-of-sight (LOS) environments. In dense multipath non-LOS environments, however, there could be severe overlapping between adjacent received pulses, and pulse overlapping must be considered in performance analysis and receiver design. This is especially the case when a multi-band pulsed UWB [5], [6] is adopted. In a typical indoor environment, measurements [4], [7] have shown that the typical average multipath arrival rate is in the range of $0.5-2ns$. For an impulse radio UWB system using 500MHz-1GHz bandwidth (10-dB bandwidth), which is common for multi-band UWB systems, the typical pulse width is in the range of $2-4ns$ [8]. In this situation, apparently the received multipath components will experience severe time overlap and the assumption of ideal multipath resolution becomes inappropriate.

It is well known that a RAKE receiver with maximal ratio combining (MRC) is optimum when the desired signal is distorted by the additive white Gaussian noise (AWGN). In the presence of inter-pulse interference (IPI) caused by pulse overlapping, a minimum mean-square error (MMSE) scheme [9] is expected to improve the receiver performance over the MRC scheme. The major purpose of this letter is to derive the optimum MMSE scheme for pulsed UWB systems when pulse overlapping is considered and compare the performance of the MMSE receiver with that of a generic MRC receiver in an indoor lognormal fading environment.

II. System and Channel Model

In a commonly used binary pulse amplitude modulated (PAM) UWB system, the amplitude of short-duration pulses are modulated by information bits. These pulses are then transmitted over a frequency-selective lognormal fading channel [4], [7] with additive white Gaussian noise. The transmitted signal is expressed as

$$s(t) = \sum_{i=-\infty}^{\infty} s_i(t) = \sum_{i=-\infty}^{\infty} \sqrt{E_b}b(i)p(t-iT_b) \qquad (1)$$

where $p(t)$ is the short-duration UWB pulse shape of width T_m, E_b is the bit energy, T_b is the bit interval ($T_b \gg T_m$), and $b(i) \in \{1,-1\}$ is the i^{th} information bit. The energy of the basic pulse $p(t)$ is normalized to $E_p = \int_{-\infty}^{\infty} p^2(t)dt = 1$.

The channel for pulsed UWB systems exhibits highly frequency-selective fading, and can be modeled as a discrete linear filter [4] whose impulse response is expressed as

$$h(t) = \sum_{l=0}^{L-1} \alpha_l \delta(t-\tau_l) \qquad (2)$$

where L is the total number of multipath components, α_l is the channel fading coefficient for the l^{th} path, τ_l is the arrival time of the l^{th} path relative to the first path ($l=0$ and $\tau_0=0$ assumed), and $\delta(t)$ is the Dirac delta function. When $|\tau_j - \tau_i| < T_m$, $i,j \in \{0,1,\ldots,L-1\}$, the i^{th} and j^{th} received pulses overlap with each other and IPI occurs.

The channel gain α_l is modeled as $\alpha_l = \lambda_l\beta_l$, where $\lambda_l \in \{\pm 1\}$ with equal probability accounts for the random pulse inversion that could occur due to reflections [4]. The magnitude term β_l is modeled as having a lognormal distribution for indoor channels. The standard deviation of fading amplitudes is typically in the range of 3-5dB. The distribution of the path arrival time sequence τ_l and power delay profile [9] of the channel are chosen to follow the modified Saleh-Valenzuela (S-V) model suggested in [4]. Because multipath components tend to arrive in clusters [4], τ_l in (2) is expressed as $\tau_l = \mu_c + \nu_{m,c}$, where μ_c is the delay of the c^{th} cluster

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that the l^{th} path falls in, $\nu_{m,c}$ is the delay (relative to μ_c) of the m^{th} multipath component in the c^{th} cluster. The relative power of the l^{th} path to the first path can be expressed as $E\{|\alpha_l|^2\} = E\{|\alpha_0|^2\}e^{-\mu_c/\Gamma}e^{-\nu_m/\gamma}$, where $E\{\cdot\}$ denotes statistical expectation, Γ is the cluster decay factor, and γ is the ray decay factor. Fading is assumed to be quasistatic, allowing α_l and τ_l to be constant over a block of data and change independently from one block to another.

III. Optimum Detection in the Presence of Pulse Overlapping

We assume that the signaling rate is such that the received signal energy of a particular bit is contained within one pulse repetition interval (T_b) so that there is no inter-symbol interference (ISI). Thus, we can focus on a particular bit interval in the receiver modeling. After passing through the multipath fading channel described by (2), the transmitted signal $s_i(t)$ (for the i^{th} information bit) given in (1) is received in multiple independently faded copies as $r(t) = \sum_{l=0}^{L-1} \alpha_l s_i(t-\tau_l) + n(t)$, where $n(t)$ is the zero-mean white Gaussian noise process with a two-sided power spectral density (PSD) of $N_0/2$.

In a generic RAKE receiver, the received signal is filtered by a matched filter that is matched to $p(t)$, delayed and sampled according to the relative delays of different paths, and then combined. The first L_p ($L_p \leq L$) received paths ($l = 0, \cdots, L_p - 1$) are statistically the strongest among all multipath components. For simplicity, these L_p paths will be linearly combined when a generic RAKE receiver is employed.

The sampled signal for the l^{th} path of the i^{th} information bit is expressed as

$$r_l = \sum_{k=0}^{L_p-1} \left(\alpha_k \sqrt{E_b} b(i) \int_{-\infty}^{\infty} p(t-\tau_k) p(t-\tau_l) dt \right) + n_l \quad (3)$$

where the zero-mean noise component is $n_l = \int_{-\infty}^{\infty} n(t)p(t-\tau_l)dt$ with variance $\sigma_{n_l}^2 = N_0/2$. Note that due to the pulse overlapping, noise components $n_l, l = 0, \cdots, L_p$, may not be mutually independent.

By defining the partial correlation between $p(t-\tau_k)$ and $p(t-\tau_l)$ as $\rho_{l,k} = \int_{-\infty}^{\infty} p(t-\tau_l)p(t-\tau_k)dt = \rho_{k,l}$, we can simplify (3) as

$$r_l = \sum_{k=0}^{L_p-1} \rho_{l,k} \alpha_k \sqrt{E_b} b(i) + n_l, \quad l = 0, 1, \ldots, L_p - 1. \quad (4)$$

Since the energy of $p(t)$ is normalized to unity, $\rho_{l,k} = 1$ for $l = k$ and $0 < |\rho_{l,k}| < 1$ for $l \neq k$, with the exception that $\rho_{l,k} = 0$ if $p(t-\tau_k)$ and $p(t-\tau_l)$ do not overlap with each other.

The received L_p samples per bit interval can be written in vector form as $\boldsymbol{r} = [r_0,\ r_1,\ \ldots,\ r_{L_p-1}]^T$, where $(\cdot)^T$ denotes transpose. This received signal vector is expressed as

$$\boldsymbol{r} = \sqrt{E_b} b(i) \boldsymbol{R} \boldsymbol{\alpha} + \boldsymbol{n} \quad (5)$$

where $\boldsymbol{\alpha} = [\alpha_0,\ \alpha_1,\ \cdots,\ \alpha_{L_p-1}]^T$ is the channel fading coefficient vector, $\boldsymbol{n} = [n_0,\ n_1,\ \cdots,\ n_{L_p-1}]^T$ is the noise vector, and

$$\boldsymbol{R} = \begin{bmatrix} 1 & \rho_{0,1} & \cdots & \rho_{0,L_p-1} \\ \rho_{1,0} & 1 & \cdots & \rho_{1,L_p-1} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{L_p-1,0} & \rho_{L_p-1,1} & \cdots & 1 \end{bmatrix} \quad (6)$$

is the partial correlation matrix, which can be calculated using the relative multipath delays τ_l and the pulse shape $p(t)$. Note that in the received signal model described above, the effect due to the potential overlap from statistically weaker paths $L_p, \cdots, L$ are neglected.

The covariance matrix of the zero-mean noise vector $\boldsymbol{n}$ given in (5) is obtained to be $E\{\boldsymbol{n}\boldsymbol{n}^H\} = \boldsymbol{R}\frac{N_0}{2}$, where $(\cdot)^H$ denotes conjugate transpose. The decision variable is obtained by combining the L_p elements of $\boldsymbol{r}$ using certain tap weights. Because the noise components of $r_l, l = 0, \cdots, L_p$, are identically distributed, the tap weights can be set to match the conjugates of the channel fading coefficients $\boldsymbol{\alpha}^* = [\alpha_0,\ \alpha_1,\ \ldots,\ \alpha_{L_p-1}]^H$, which results in the generic RAKE receiver with MRC. Mathematically, this approach is not optimum because of the IPI caused by pulse overlapping and noise correlation.

It is of interest to derive the optimum linear combining scheme and compare its performance with that of the MRC scheme. For the optimum linear MMSE receiver, we need to determine the optimum tap weight vector $\boldsymbol{w} = [w_0,\ w_1,\ \cdots,\ w_{L_p-1}]^T$ to form the decision variable $\Delta = \boldsymbol{w}^T\boldsymbol{r}$. For the typical target desired signal $y = \sqrt{E_b}b(i)$, the weight vector is chosen to be such that the mean-square value of the error signal $e = y - \boldsymbol{w}^T\boldsymbol{r}$ is minimized. The mean-square error (MSE) is easily determined as

$$\begin{aligned} \varepsilon &= E\{ee^H\} \\ &= E\{yy^H\} + \boldsymbol{w}^T E\{\boldsymbol{r}\boldsymbol{r}^H\}\boldsymbol{w}^* - \\ &\quad E\{y^H\boldsymbol{r}^T\}\boldsymbol{w} - E\{y\boldsymbol{r}^H\}\boldsymbol{w}^*. \end{aligned} \quad (7)$$

By letting the derivative of ε with respective to $\boldsymbol{w}$, $d\varepsilon/d\boldsymbol{w} = E\{\boldsymbol{r}\boldsymbol{r}^H\}\boldsymbol{w}^* - E\{y^H\boldsymbol{r}\}$, equal zero, we obtain the conjugate of the optimum MMSE weight vector as

$$\boldsymbol{w}^* = \boldsymbol{P}^{-1}\boldsymbol{q} \quad (8)$$

where $\boldsymbol{P} = E\{\boldsymbol{r}\boldsymbol{r}^H\}$ and $\boldsymbol{q} = E\{y^H\boldsymbol{r}\}$.

With the quasistatic slowly fading channel model adopted, the channel fading coefficients and relative path delays are static over a block of data. The *instantaneous* matrix $\boldsymbol{P}$ and column vector $\boldsymbol{q}$ are obtained as

$$\boldsymbol{P} = E_b\boldsymbol{R}\boldsymbol{\alpha}\boldsymbol{\alpha}^H\boldsymbol{R}^H + \boldsymbol{R}\frac{N_0}{2} \quad (9a)$$

$$\boldsymbol{q} = E_b\boldsymbol{R}\boldsymbol{\alpha}. \quad (9b)$$

It will be very interesting to find out the difference between the MRC scheme with the tap weight $\boldsymbol{\alpha}^*$, for which noise correlation and IPI caused by pulse overlapping have been ignored, and the optimum MMSE scheme whose tap weight is given in (8). To analyze this, we re-write $\boldsymbol{w}^*$ as

$$\begin{aligned} \boldsymbol{w}^* &= E_b\left[E_b\boldsymbol{R}\left(\boldsymbol{\alpha}\boldsymbol{\alpha}^H\boldsymbol{R}^H + \frac{N_0}{2E_b}\boldsymbol{I}_{L_p}\right)\right]^{-1}\boldsymbol{R}\boldsymbol{\alpha} \\ &= \left(\boldsymbol{\alpha}\boldsymbol{\alpha}^H\boldsymbol{R}^H + \frac{N_0}{2E_b}\boldsymbol{I}_{L_p}\right)^{-1}\boldsymbol{\alpha} \end{aligned} \quad (10)$$

where $\boldsymbol{I}_{L_p}$ is the $L_p \times L_p$ identity matrix.

It is easy to obtain that $\left(\boldsymbol{\alpha}\boldsymbol{\alpha}^H\boldsymbol{R}^H + \frac{N_0}{2E_b}\boldsymbol{I}_{L_p}\right)\boldsymbol{\alpha} = \left(\zeta + \frac{N_0}{2E_b}\right)\boldsymbol{\alpha}$, where $\zeta = \boldsymbol{\alpha}^H\boldsymbol{R}^H\boldsymbol{\alpha}$ is a scalar. Thus, (10) transforms to[1]

$$\boldsymbol{w}^* = \frac{1}{\zeta + \frac{N_0}{2E_b}}\boldsymbol{\alpha}. \tag{11}$$

Eqs. (10) and (11) indicate that $\boldsymbol{\alpha}$ is an eigen vector of the matrix $\left[\boldsymbol{R}\left(\boldsymbol{\alpha}\boldsymbol{\alpha}^H\boldsymbol{R}^H + \frac{N_0}{2E_b}\boldsymbol{I}_{L_p}\right)\right]^{-1}\boldsymbol{R}$. This shows, interestingly, that the optimum MMSE tap weight $\boldsymbol{w}$ is essentially the same as the MRC tap weight $\boldsymbol{\alpha}^*$. We can therefore conclude that the generic MRC RAKE receiver performs the same as the linear optimum MMSE receiver when the unavoidable pulse overlapping in a pulsed UWB system is taken into consideration.

In a code-division multiple-access (CDMA) system, an MMSE receiver, when used as an equalizer in the presence of ISI and multiple-access interference (MAI) [10], should perform better than a generic RAKE receiver. In the pulsed UWB system, all L_p received paths given in (4) carry the same information bit, and the MMSE receiver derived in this letter is designed to optimally combine these components in the presence of pulse overlapping. In a CDMA system, however, MAI and ISI are from other users or bits, and the MMSE receiver separates desired signals from interference. When there are both ISI and IPI in the pulsed UWB system, an MMSE receiver is expected to perform better than an MRC RAKE receiver.

IV. Simulation Results and Discussion

In obtaining simulation results, a carrier-modulated, truncated Gaussian pulse is applied as the UWB pulse shape $p(t)$. This pulse has a width of $T_m = 2ns$ and a 10dB bandwidth of 1GHz. We adopted the CM3 channel model [4] with a root-mean-square (RMS) delay spread of $15ns$, an average cluster arrival rate of $0.0667/ns$, and an average path arrival rate of $2/ns$. The cluster decay factor applied is $\Gamma = 14ns$, and the ray decay factor applied is $\gamma = 7.9ns$. The standard deviation of the fading coefficients chosen is 3.4dB. The average power of the first path is normalized as $E\{|\alpha_0|^2\} = 1$. It is assumed that the transmission rate is such that inter-symbol interference caused by channel excess delay is negligible. Also, the receiver is assumed to have perfect knowledge of the channel coefficients and delays.

Fig. 1 shows the simulated performance curves of a RAKE receiver with the MRC scheme (with markers) and with the MMSE scheme (smooth curves) when different number of paths (L_p) are exploited. For all cases simulated, the generic MRC scheme performs exactly the same as the optimum linear MMSE scheme.

Although perfect channel knowledge is assumed in simulations, the above conclusion still holds with an imperfect knowledge of channel coefficients and delays. When $\boldsymbol{\alpha}$ and $\boldsymbol{R}$ in (10) and (11) are estimates (imperfect), the same conclusion can still be made from (11).

[1]This relationship is obtained as follows. Let $\boldsymbol{M}\boldsymbol{\alpha} = m\boldsymbol{\alpha}$, where $\boldsymbol{M}$ is a positive definite matrix, m is a scalar, and $\boldsymbol{\alpha}$ is a vector. We can write $\boldsymbol{M}^{-1}\boldsymbol{M}\boldsymbol{\alpha} = \boldsymbol{\alpha} = \boldsymbol{M}^{-1}m\boldsymbol{\alpha}$, from which $\boldsymbol{M}^{-1}\boldsymbol{\alpha} = \frac{1}{m}\boldsymbol{\alpha}$ follows.

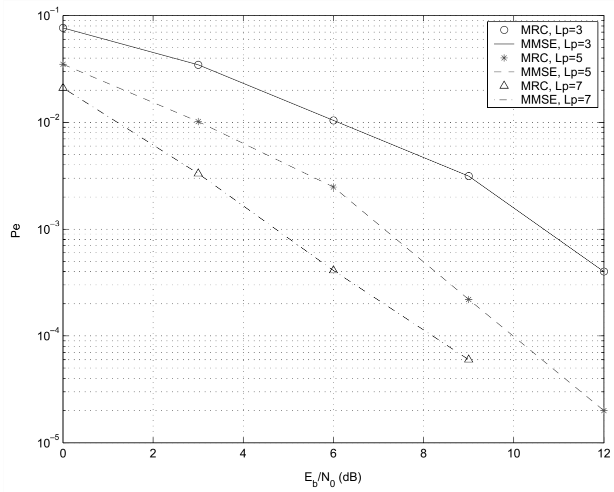

Fig. 1. Simulated BER versus E_b/N_0 curves of the generic MRC receiver and the optimum MMSE receiver when IPI caused by pulse overlapping is taken into consideration.

V. Conclusions

Inter-pulse interference caused by pulse overlapping could severely degrade the performance of impulse radio UWB systems in dense multipath environments. In this letter, the optimum linear MMSE receiver in the presence of pulse overlapping has been investigated. It has been found, by an analytical approach, that the generic RAKE receiver with MRC, which ignores IPI caused by pulse overlapping and noise correlation, performs the same as the optimum MMSE receiver. In simulations we have constructed the received signal waveforms and followed exactly the matched filtering process and the combining procedures. The waveform-based simulation results have validated this conclusion.

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