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Semidefinite Programming Methods for Alleviating Sensor Position Error in TDOA Localization

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ABSTRACT This paper develops a unified solution for time-difference-of-arrival (TDOA) localization in the presence of sensor position errors. This technique starts with maximum likelihood estimation (MLE), which is known to be nonconvex. A semidefinite programming technique to effectively transform the MLE problem into a convex optimization is proposed, together with a unified solution for four scenarios: 1) without a calibration emitter; 2) with a single calibration emitter, whose position is subject to measurement errors; 3) with a single calibration emitter, whose position is perfectly known; and 4) with a single calibration emitter, whose position is completely unknown. The results are finally extended to the case of multiple calibration emitters, whose positions are also subject to errors. Similar to the existing schemes that are known to have good performances, the proposed solution also reaches the Cramér–Rao lower bound when sensor position errors and TDOA measurement noise are sufficiently small. However, as TDOA measurement noise or sensor position errors increase, comparison with the existing state-of-the-art methods for each scenario shows that the proposed solution performs significantly better.

INDEX TERMS Source localization, sensor position error, time-difference-of-arrival (TDOA), semidefinite programming (SDP).

I. INTRODUCTION

Source localization finds a number of applications in wireless communications [1], [2], wireless sensor networks (WSNs) [3], [4], and navigation [5]. A popular positioning approach is to use the time-difference-of-arrival (TDOA) measurements between spatially distributed sensors. TDOA techniques require the sensors to be time-synchronized. There exist some time synchronization approaches in WSN [6]–[8]. Each TDOA measurement equation corresponds to one hyperbola/hyperboloid in 2-dimensional (2D) plane or 3D space. In the case of perfect TDOA measurements, the source position is the intersection of the hyperbolas/hyperboloids. There exist many source localization algorithms, which commonly assume that the sensor positions are perfectly known. The Taylor-series method [9] is an iterative scheme, which requires an initial value, and it cannot guarantee convergence. The closed-form solution in [10] is computationally efficient, and it does not have the divergence problem as the Taylor-series method. This solution can reach the Cramér-Rao lower bound (CRLB) in sufficiently small noise as the noise increases, however, its performance may degrade drastically.

Due to measurement errors, likely caused by the environment (e.g., multipath and noise), or sensor position errors, or both, the intersections of the multiple hyperbolas/hyperboloids will not be unique. In most practical scenarios, the sensor positions cannot be known or measured precisely. If not taken into account in the TDOA positioning algorithm, a slight sensor position error could drastically degrade the accuracy.

TDOA localization in the presence of sensor location errors has attracted considerable amount of research recently [11]–[13]. Ho *et al.* [14] analyze how much the source location accuracy degrades as a result of sensor position errors. They also propose an algorithm that takes the statistical distributions of the sensor positions into account to improve the localization accuracy, and the algorithm has a closed-form solution. This solution could approach the CRLB when the source is far away from the sensors.

Ho and Yang [15] also investigate the use of a single calibration emitter with accurately known position to reduce the loss in localization accuracy due to the random sensor position errors. This algorithm is based on weighted least-squares (WLS) in several stages and has a closed-form solution. It is shown that the algorithm can attain the CRLB when the TDOA measurement noise and the sensor position errors are sufficiently small.

Yang and Ho [16] extend the work in [15] to a more practical scenario where the exact position of a calibration emitter is unknown. By modeling the calibration emitter position error as an additive Gaussian noise, analysis results show that the performance penalty could be very high if the calibration position errors are ignored. A closed-form solution that accounts for the calibration emitter position error is proposed in [16], which is also extended to the case of multiple calibration emitters. Yang *et al.* conclude that their algorithm could reach the CRLB when TDOA measurement noise and the position errors of the sensors and the calibration emitter are sufficiently small.

Yang and Ho [17] consider another scenario that the calibration emitter position is completely unknown. In this case, the calibration emitter and the source are called as disjoint sources. A key assumption made in [17] is that the TDOA measurements from multiple disjoint sources are subject to the same sensor position displacement from the available sensor positions. Based on this, an algorithm that jointly estimates the unknown sources and sensor positions is proposed. Exploiting the coupling of the source and sensor positions in the measurement equations, Yang *et al.* introduce a hypothesized source location to form a set of pseudo-linear equations, thereby leading to a closed-form solution for source location estimates. This closed-form solution could reach the CRLB when the sensor position errors and the TDOA measurement noise are sufficiently small.

The novelty of this paper is that semidefinite programming (SDP) method is introduced for the problem of alleviating sensor position error in TDOA localization. Most of the state-of-art methods are based on WLS, which have bad performance when the noise are increase.

The goal of this paper is to develop a unified source positioning framework that employs SDP, which works efficiently for various scenarios: a single calibration emitter with accurately known position [15], a single calibration emitter whose position is subject to errors [16], a single calibration emitter with completely unknown position [17], no calibration emitters [11]–[14], and multiple calibration emitters whose positions are subject to errors [16]. The proposed unified solution starts from the asymptotically optimum maximum likelihood estimator (MLE) [18], which is known to be non-convex. However, its optimality motivates us to transform the MLE into a convex optimization for TDOA source localization in the presence of sensor position errors. In addition to the unified framework of the proposed solution, which can be conveniently adopted for any deployment scenarios, it significantly outperforms existing state-of-the-art schemes for each scenario, which will be verified in extensive amount of simulation.

The rest of this paper is organized as follows. Sec. II establishes the positioning network and measurement models. The proposed SDP solution for cases of no calibration emitters, a single calibration emitter whose position is subject to errors, a single calibration emitter with perfectly known position, and a single calibration emitter with a completely unknown position is derived in detail in Sec. III. Extension of the proposed solution to the case of multiple calibration emitters is discussed in IV. Extensive simulation results are obtained in Sec. V to compare the location estimation performances of the proposed unified solution and state-of-the-art methods for various deployment scenarios.

II. PROBLEM FORMULATION

The following notations are used throughout the paper. Bold lowercase and uppercase letters denote vectors and matrices, respectively; $\mathbf{A}(:, i)$ denotes the ith column of matrix $\mathbf{A}$, and $\mathbf{A}(i : j, k : l)$ denotes a submatrix of $\mathbf{A}$, formed from its ith row, kth column to its jth row, lth column; $\text{tr}(\mathbf{A})$ is the trace of $\mathbf{A}$; $\mathbf{I}_m$ is the $m \times m$ identity matrix; $\mathbf{1}_m$ denotes the column vector whose elements are all 1's; $\|\cdot\|$ is the l_2 norm and $\|\cdot\|_F$ is the Frobenius norm; $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

Consider a network with M sensors, where the unknown m-dimensional ($m = 2$ or 3) source location $\mathbf{u}$ is to be estimated. Let $\mathbf{s}_i^0$ and $\mathbf{s}_i$ represent, respectively, the true (unknown) location and the network acquired (available but inaccurate) position of the ith sensor. The known sensor positions can be denoted as

$$\mathbf{s}_i = \mathbf{s}_i^0 + \boldsymbol{\beta}_i, \quad i = 1, \cdots, M \tag{1}$$

where $\boldsymbol{\beta}_i$ is a zero-mean white Gaussian vector with covariance matrix $\delta_i^2 \mathbf{I}_m$ [15]. Thus, $d_i = \left\|\mathbf{u} - \mathbf{s}_i^0\right\|$ is the actual but unknown distance between the ith sensor and the source. Let sensor 1 be the reference sensor, without loss of generality. The range-difference-of-arrival (RDOA) measurements between sensor i and the reference sensor using the source's transmitted signal are expressed as [15]

$$r_{i1} = d_i - d_1 + n_{i1}, \quad i = 2, \cdots, M \tag{2}$$

where $n_{i1}, i = 1, \cdots, M$, are the measurement errors, called range difference noise here, and are modeled as zero-mean Gaussian random variables [15]. Let $\mathbf{n} = [n_{21}, \cdots, n_{M1}]^T$ with covariance matrix $\mathbf{Q}_d$.

It will improve the localization performance by adding an emitter(s) in the network, whose positions may or may not know accurately. The goal in this paper is to establish a unified solution for all these cases using an SDP approach.

III. UNIFIED LOCALIZATION USING SEMIDEFINITE PROGRAMMING

A. CALIBRATION EMITTER POSITION IS ACCURATELY KNOWN

In this case, the TDOAs of the signal from the calibration emitter at accurately known position $\mathbf{c}$ to the sensors are also measured [15], giving the calibration RDOA measurements

$$h_{i1} = h_i - h_1 + e_{i1}, \quad i = 2, \cdots, M \tag{3}$$

where h_{i1} is the measured calibration RDOA between sensor pair i and 1, $h_i = \left\|\mathbf{c} - \mathbf{s}_i^0\right\|$ is the unknown distance between the ith sensor and calibration emitter, and $\mathbf{e} = [e_{21}, \cdots, e_{M1}]^T$ is assumed to be a zero-mean Gaussian vector with covariance matrix $\mathbf{Q}_c$ [15].

The noise vectors $\mathbf{n}$, $\boldsymbol{\beta}_i$, $i = 1, \cdots, M$, and $\mathbf{e}$ are assumed to be independent of one another. Conditioned on independent Gaussian noises, the MLE problem is expressed as

$$\begin{aligned}\min_{\mathbf{u},\mathbf{s}_i^0} &\sum_{i=2}^{M}\sum_{j=2}^{M}\left(r_{i1}-\left\|\mathbf{u}-\mathbf{s}_i^0\right\|+\left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_d^{-1}\right]_{(i-1)(j-1)}\\ &\cdot\left(r_{j1}-\left\|\mathbf{u}-\mathbf{s}_j^0\right\|+\left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right)+\sum_{i=1}^{M}\frac{\left\|\mathbf{s}_i-\mathbf{s}_i^0\right\|^2}{\delta_i^2}\\ &+\sum_{i=2}^{M}\sum_{j=2}^{M}\left(h_{i1}-\left\|\mathbf{c}-\mathbf{s}_i^0\right\|+\left\|\mathbf{c}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_c^{-1}\right]_{(i-1)(j-1)}\\ &\cdot\left(h_{j1}-\left\|\mathbf{c}-\mathbf{s}_j^0\right\|+\left\|\mathbf{c}-\mathbf{s}_1^0\right\|\right).\end{aligned} \tag{4}$$

It will be shown next that this MLE problem is a nonconvex optimization problem, and a solution for it is difficult to obtain.

An SDP technique to solve the problem in (4) is described next. First, (4) can be rewritten in vector-matrix form as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{X},\mathbf{h}}\ &(\mathbf{r}_d-\mathbf{A}\mathbf{d})^T\mathbf{Q}_d^{-1}(\mathbf{r}_d-\mathbf{A}\mathbf{d})+(\mathbf{h}_d-\mathbf{A}\mathbf{h})^T\mathbf{Q}_c^{-1}\\ &\cdot(\mathbf{h}_d-\mathbf{A}\mathbf{h})+\left\|(\mathbf{X}(:,2:M+1)-\mathbf{S})\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \quad (5a)\\ \text{s.t. } & d_i = \|\mathbf{X}(:,1)-\mathbf{X}(:,i+1)\|, \quad i=1,\cdots,M, \quad (5b)\\ & h_i = \|\mathbf{c}-\mathbf{X}(:,i+1)\|, \quad i=1,\cdots,M \quad (5c)\end{aligned}$$

where $\mathbf{r}_d = [r_{21}, \cdots, r_{M1}]^T$, $\mathbf{d} = [d_1, \cdots, d_M]^T$, $\mathbf{A} = [-\mathbf{1}_{M-1}, \mathbf{I}_{M-1}]$, $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^0, \cdots, \mathbf{s}_M^0]$, $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_M]$, $\mathbf{W} = \text{diag}([\delta_1^{-2}, \cdots, \delta_M^{-2}])$, $\mathbf{h}_d = [h_{21}, \cdots, h_{M1}]^T$, and $\mathbf{h} = [h_1, \cdots, h_M]^T$.

Clearly, the constraints in (5b) and (5c) are nonconvex for the optimization variables; thus the above MLE problem is nonconvex. Let $\mathbf{D} = \mathbf{d}\mathbf{d}^T$, $\mathbf{H} = \mathbf{h}\mathbf{h}^T$, and $\mathbf{Y} = \mathbf{X}^T\mathbf{X}$. The objective function in (5) can then be written as

$$\begin{aligned}&\text{tr}(\mathbf{D}\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}) - 2\mathbf{d}^T\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{r}_d\\ &\quad+\text{tr}(\mathbf{W}\mathbf{Y}(2:M+1,2:M+1)) - 2\text{tr}(\mathbf{W}\mathbf{X}(:,2:M+1)^T\mathbf{S})\\ &\quad+\text{tr}(\mathbf{H}\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{A}) - 2\mathbf{h}^T\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{h}_d\end{aligned} \tag{6}$$

where three constant terms are discarded, since they do not affect the optimization results.

The constraints in (5b) and (5c) can be written as

$$D_{i,i} = Y(1,1) - 2Y(1,i+1) + Y(i+1,i+1), \tag{7a}$$
$$H_{i,i} = \mathbf{c}^T\mathbf{c} - 2\mathbf{c}^T\mathbf{X}(:,i+1) + Y(i+1,i+1). \tag{7b}$$

where $i = 1, \cdots, M$. Applying the Cauchy-Schwartz inequality [19] results in

$$D_{i,j} \geq |Y(1,1)-Y(1,i+1)-Y(1,j+1)+Y(i+1,j+1)|, \tag{8a}$$

$$H_{i,j} \geq |\mathbf{c}^T\mathbf{c}-\mathbf{c}^T(\mathbf{X}(:,i+1)+\mathbf{X}(:,j+1))+Y(i+1,j+1)|. \tag{8b}$$

where $1 \leq i < j \leq M$. However, the three nonconvex constraints in the form of equalities $\mathbf{D} = \mathbf{d}\mathbf{d}^T$, $\mathbf{H} = \mathbf{h}\mathbf{h}^T$, and $\mathbf{Y} = \mathbf{X}^T\mathbf{X}$ are still in the problem. We employ semidefinite relaxation (SDR) [20] to transform these constraints into convex inequalities $\mathbf{D} \succeq \mathbf{d}\mathbf{d}^T$, $\mathbf{H} \succeq \mathbf{h}\mathbf{h}^T$, and $\mathbf{Y} \succeq \mathbf{X}^T\mathbf{X}$. These convex inequalities can be written as linear matrix inequalities (LMI) [19]

$$\begin{bmatrix}1 & \mathbf{d}^T\\ \mathbf{d} & \mathbf{D}\end{bmatrix} \succeq \mathbf{0}, \quad \begin{bmatrix}1 & \mathbf{h}^T\\ \mathbf{h} & \mathbf{H}\end{bmatrix} \succeq \mathbf{0}, \quad \begin{bmatrix}\mathbf{I}_m & \mathbf{X}\\ \mathbf{X}^T & \mathbf{Y}\end{bmatrix} \succeq \mathbf{0}. \tag{9}$$

Note that $\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}$ and $\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{A}$ in the objective function are singular matrices. To deal with this issue, similar to the approach in [21], a penalty term $\eta\text{tr}(\mathbf{D}+\mathbf{H})$ is introduced into the objective function and the second-order-cone (SOC) constraints are added:

$$\|\mathbf{X}(:,1) - \mathbf{X}(:,i+1)\| \leq d_i, \quad i = 1, 2, \cdots, M, \tag{10a}$$
$$\|\mathbf{c} - \mathbf{X}(:,i+1)\| \leq h_i, \quad i = 1, 2, \cdots, M. \tag{10b}$$

to make the constraints tight.

These lead to the proposed SDP algorithm expressed as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{D},\mathbf{h},\mathbf{H},\mathbf{X},\mathbf{Y}}\ &\text{tr}(\mathbf{D}\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}) - 2\mathbf{d}^T\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{r}_d\\ &+\text{tr}(\mathbf{H}\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{A}) - 2\mathbf{h}^T\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{h}_d\\ &+\text{tr}(\mathbf{W}\mathbf{Y}(2:M+1,2:M+1))\\ &-2\text{tr}(\mathbf{W}\mathbf{X}(:,2:M+1)\mathbf{S})+\eta\text{tr}(\mathbf{D}+\mathbf{H})\end{aligned} \tag{11a}$$

$$\text{s.t. } D_{i,i} = Y(1,1) - 2Y(1,i+1) + Y(i+1,i+1), \tag{11b}$$

$$H_{i,i} = \mathbf{c}^T\mathbf{c} - 2\mathbf{c}^T\mathbf{X}(:,i+1) + Y(i+1,i+1), \tag{11c}$$

$$\|\mathbf{X}(:,1) - \mathbf{X}(:,i+1)\| \leq d_i, \tag{11d}$$
$$\|\mathbf{c} - \mathbf{X}(:,i+1)\| \leq h_i, \tag{11e}$$
$$D_{i,j} \geq |Y(1,1) - Y(1,i+1) - Y(1,j+1) + Y(i+1,j+1)|, \tag{11f}$$
$$H_{i,j} \geq |\mathbf{c}^T\mathbf{c} - \mathbf{c}^T(\mathbf{X}(:,i+1)+\mathbf{X}(:,j+1)) + Y(i+1,j+1)|, \tag{11g}$$
$$\begin{bmatrix}1 & \mathbf{d}^T\\ \mathbf{d} & \mathbf{D}\end{bmatrix} \succeq \mathbf{0}, \tag{11h}$$

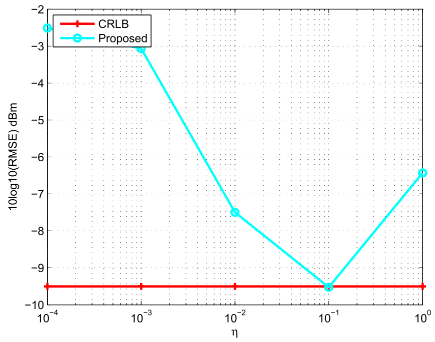

FIGURE 1. RMSE vs. η, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c} = [0, 0]^T m$, $\delta = 0.1m$, $\sigma = 0.1m$.

$$\begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H} \end{bmatrix} \succeq \mathbf{0}, \quad (11i)$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \quad (11j)$$

where η is the regularization parameter.

A suitable value of η is important to achieve a good estimation performance, as will be shown in Fig. 1. However, it is not easy to analytically obtain the optimal value of parameter η, which depends on the distance between the sensor and the source nodes as well as the RDOA noise level [21]. A proposed method is to create K values of $\eta, \eta_k,\ k = 1, 2, \cdots, K$, and for each η_k, (11) will be computed. Note that the K times of computation in (11) could take place in parallel. The estimated results $\hat{\mathbf{X}}_k,\ k = 1, 2, \cdots, K$, are then used to determine an optimal $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:, 1)$ that minimizes J_k expressed as

$$J_k = (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k)^T \mathbf{Q}_d^{-1}(\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k), \quad k = 1, 2, \cdots, K \quad (12)$$

where $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \cdots, \hat{d}_{kM}]^T$, and $\hat{d}_{ki} = \left\|\hat{\mathbf{X}}_k(:, 1) - \hat{\mathbf{X}}_k(:, i+1)\right\|, i = 1, 2, \cdots, M$.

B. CALIBRATION EMITTER LOCATION IS INACCURATE

Here the case of a single calibration emitter whose acquired location is not perfectly accurate [16] is considered. Let $\mathbf{c}^0$ and $\mathbf{c}$ be, respectively, the true (but unknown) and known (but erroneous) locations of the calibration emitter, and

$$\mathbf{c} = \mathbf{c}^0 + \boldsymbol{\epsilon} \quad (13)$$

where $\boldsymbol{\epsilon}$ is a zero-mean Gaussian vector with covariance matrix $\kappa^2 \mathbf{I}_m$ [16].

The noise vectors $\mathbf{n}$, $\boldsymbol{\beta}_i$, $i = 1, \cdots, M$, $\mathbf{e}$ and $\boldsymbol{\epsilon}$ are assumed to be independent of one another. The MLE problem for this case is formulated as

$$\begin{aligned} \min_{\mathbf{u},\mathbf{c}^0,\mathbf{s}_i^0} & \sum_{i=2}^{M}\sum_{j=2}^{M}\left(r_{i1} - \left\|\mathbf{u}-\mathbf{s}_i^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_d^{-1}\right]_{(i-1)(j-1)} \\ & \cdot \left(r_{j1} - \left\|\mathbf{u}-\mathbf{s}_j^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right) + \sum_{i=1}^{M}\frac{\left\|\mathbf{s}_i - \mathbf{s}_i^0\right\|^2}{\delta_i^2} \\ & + \sum_{i=2}^{M}\sum_{j=2}^{M}\left(h_{i1} - \left\|\mathbf{c}^0-\mathbf{s}_i^0\right\| + \left\|\mathbf{c}^0-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_c^{-1}\right]_{(i-1)(j-1)} \\ & \cdot \left(h_{j1} - \left\|\mathbf{c}^0-\mathbf{s}_j^0\right\| + \left\|\mathbf{c}^0-\mathbf{s}_1^0\right\|\right) + \frac{\left\|\mathbf{c}-\mathbf{c}^0\right\|^2}{\kappa^2}. \end{aligned} \quad (14)$$

Next, an SDP method will be developed to solve the above MLE problem. Similar to (4), (14) can also be written in vector-matrix form as

$$\begin{aligned} \min_{\mathbf{d},\mathbf{X},\mathbf{h}}\ & (\mathbf{r}_d - \mathbf{A}\mathbf{d})^T\mathbf{Q}_d^{-1}(\mathbf{r}_d - \mathbf{A}\mathbf{d}) + (\mathbf{h}_d - \mathbf{A}\mathbf{h})^T\mathbf{Q}_c^{-1} \\ & \cdot (\mathbf{h}_d - \mathbf{A}\mathbf{h}) + \left\|(\mathbf{X}(:, 2:M+2) - \mathbf{S})\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \quad (15a) \\ \text{s.t. } & d_i = \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+2)\|, \quad i = 1, \cdots, M, \quad (15b) \\ & h_i = \|\mathbf{X}(:, 2) - \mathbf{X}(:, i+2)\|, \quad i = 1, \cdots, M. \quad (15c) \end{aligned}$$

where $\mathbf{X} = [\mathbf{u}, \mathbf{c}^0, \mathbf{s}_1^0, \cdots, \mathbf{s}_M^0]$, $\mathbf{S} = [\mathbf{c}, \mathbf{s}_1, \cdots, \mathbf{s}_M]$, and $\mathbf{W} = \text{diag}([\kappa^{-2}, \delta_1^{-2}, \cdots, \delta_M^{-2}])$.

The SDP technique for the case of inaccurate calibration emitter position is expressed as

$$\begin{aligned} \min_{\mathbf{d},\mathbf{D},\mathbf{h},\mathbf{H},\mathbf{X},\mathbf{Y}}\ & \text{tr}(\mathbf{D}\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}) - 2\mathbf{d}^T\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{r}_d \\ & + \text{tr}(\mathbf{H}\cdot\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{A}) - 2\mathbf{h}^T\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{h}_d \\ & - 2\text{tr}(\mathbf{W}\mathbf{X}(:, 2:M+2)\mathbf{S}) \\ & + \text{tr}(\mathbf{W}\mathbf{Y}(2:M+2, 2:M+2)) + \eta\text{tr}(\mathbf{D}+\mathbf{H}) \quad (16a) \\ \text{s.t. } & D_{i,i} = Y(1,1) - 2Y(1, i+2) + Y(i+2, i+2), \quad (16b) \\ & H_{i,i} = Y(2,2) - 2Y(2, i+2) + Y(i+2, i+2), \quad (16c) \\ & \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+2)\| \le d_i, \quad (16d) \\ & \|\mathbf{X}(:, 2) - \mathbf{X}(:, i+2)\| \le h_i, \quad (16e) \\ & D_{i,j} \ge |Y(1,1) - Y(1, i+2) - Y(1, j+2) \\ & \qquad + Y(i+2, j+2)|, \quad (16f) \\ & H_{i,j} \ge |Y(2,2) - Y(2, i+2) - Y(2, j+2) \\ & \qquad + Y(i+2, j+2)|, \quad (16g) \\ & \begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0}, \quad (16h) \\ & \begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H} \end{bmatrix} \succeq \mathbf{0}, \quad (16i) \\ & \begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \quad (16j) \end{aligned}$$

where η is the regularization parameter.

As the previous case, a proper value of η is important for this algorithm, and again a proposed approach is to use different values of η_k, $k = 1, 2, \cdots, K$, to compute (16), and then use the estimated results $\hat{\mathbf{X}}_k$, $k = 1, 2, \cdots, K$ to determine $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:, 1)$ that minimizes J_k expressed as

$$J_k = (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k)^T\mathbf{Q}_d^{-1}(\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k), \quad k = 1, 2, \cdots, K. \quad (17)$$

where $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \cdots, \hat{d}_{kM}]^T$, and $\hat{d}_{ki} = \left\|\hat{\mathbf{X}}_k(:,1) - \hat{\mathbf{X}}_k(:,i+2)\right\|$, $i = 1, 2, \cdots, M$.

C. CALIBRATION EMITTER LOCATION IS COMPLETELY UNKNOWN

A more or less unrealistic case where the calibration emitter location is completely unknown [17] is considered in this section. Nevertheless, this scenario will be very useful to assess how much value an emitter adds in terms of performance improvement when no attempts are made to obtain its position. The corresponding MLE problem is expressed as

$$\begin{aligned}\min_{\mathbf{u},\mathbf{c},\mathbf{s}_i^0} & \sum_{i=2}^{M}\sum_{j=2}^{M}\left(r_{i1} - \left\|\mathbf{u}-\mathbf{s}_i^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_d^{-1}\right]_{(i-1)(j-1)} \\ &\cdot \left(r_{j1} - \left\|\mathbf{u}-\mathbf{s}_j^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right) + \sum_{i=1}^{M}\frac{\left\|\mathbf{s}_i-\mathbf{s}_i^0\right\|^2}{\delta_i^2} \\ &+ \sum_{i=2}^{M}\sum_{j=2}^{M}\left(h_{i1} - \left\|\mathbf{c}-\mathbf{s}_i^0\right\| + \left\|\mathbf{c}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_c^{-1}\right]_{(i-1)(j-1)} \\ &\cdot \left(h_{j1} - \left\|\mathbf{c}-\mathbf{s}_j^0\right\| + \left\|\mathbf{c}-\mathbf{s}_1^0\right\|\right). \qquad (18)\end{aligned}$$

This MLE problem can be written in vector-matrix form as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{X},\mathbf{h}} \ & (\mathbf{r}_d - \mathbf{A}\mathbf{d})^T\mathbf{Q}_d^{-1}(\mathbf{r}_d - \mathbf{A}\mathbf{d}) + (\mathbf{h}_d - \mathbf{A}\mathbf{h})^T\mathbf{Q}_c^{-1} \\ & \cdot (\mathbf{h}_d - \mathbf{A}\mathbf{h}) + \left\|(\mathbf{X}(:,3:M+2) - \mathbf{S})\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \qquad (19a)\\ \text{s.t. } & d_i = \left\|\mathbf{X}(:,1) - \mathbf{X}(:,i+2)\right\|, \quad i = 1,\cdots,M, \qquad (19b)\\ & h_i = \left\|\mathbf{X}(:,2) - \mathbf{X}(:,i+2)\right\|, \quad i = 1,\cdots,M. \qquad (19c)\end{aligned}$$

where $\mathbf{X} = [\mathbf{u}, \mathbf{c}, \mathbf{s}_1^0, \cdots, \mathbf{s}_M^0]$, $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_M]$, and $\mathbf{W} = \text{diag}([\delta_1^{-2}, \cdots, \delta_M^{-2}])$.

The proposed SDP algorithm for this case is written as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{D},\mathbf{h},\mathbf{H},\mathbf{X},\mathbf{Y}} \ & \text{tr}(\mathbf{D}\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}) - 2\mathbf{d}^T\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{r}_d \\ & + \text{tr}(\mathbf{H}\cdot\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{A}) - 2\mathbf{h}^T\mathbf{A}^T\mathbf{Q}_c^{-1}\mathbf{h}_d \\ & - 2\text{tr}(\mathbf{W}\mathbf{X}(:,3:M+2)\mathbf{S}) \\ & + \text{tr}(\mathbf{W}\mathbf{Y}(3:M+2,3:M+2)) + \eta\,\text{tr}(\mathbf{D}+\mathbf{H}) \qquad (20a)\\ \text{s.t. } & D_{i,i} = Y(1,1) - 2Y(1,i+2) + Y(i+2,i+2), \qquad (20b)\\ & H_{i,i} = Y(2,2) - 2Y(2,i+2) + Y(i+2,i+2), \qquad (20c)\\ & \left\|\mathbf{X}(:,1) - \mathbf{X}(:,i+2)\right\| \le d_i, \qquad (20d)\\ & \left\|\mathbf{X}(:,2) - \mathbf{X}(:,i+2)\right\| \le h_i, \qquad (20e)\\ & D_{i,j} \ge |Y(1,1) - Y(1,i+2) - Y(1,j+2) \\ & \qquad + Y(i+2,j+2)|, \qquad (20f)\\ & H_{i,j} \ge |Y(2,2) - Y(2,i+2) - Y(2,j+2) \\ & \qquad + Y(i+2,j+2)|, \qquad (20g)\\ & \begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D}\end{bmatrix} \succeq \mathbf{0}, \qquad (20h)\\ & \begin{bmatrix} 1 & \mathbf{h}^T \\ \mathbf{h} & \mathbf{H}\end{bmatrix} \succeq \mathbf{0}, \qquad (20i)\\ & \begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y}\end{bmatrix} \succeq \mathbf{0}. \qquad (20j)\end{aligned}$$

Again, K different values of η, η_k, $k = 1, 2, \cdots, K$, can be used to compute (20), and the estimated results $\hat{\mathbf{X}}_k$, $k = 1, 2, \cdots, K$, can then be used to determine a proper $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:,1)$ that minimizes J_k expressed as

$$J_k = (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k)^T\mathbf{Q}_d^{-1}(\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k), \quad k = 1, 2, \cdots, K \qquad (21)$$

where $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \cdots, \hat{d}_{kM}]^T$, and $\hat{d}_{ki} = \left\|\hat{\mathbf{X}}_k(:,1) - \hat{\mathbf{X}}_k(:,i+2)\right\|$, $i = 1, 2, \cdots, M$.

D. NO CALIBRATION EMITTERS

In the absence of a calibration emitter, the MLE problem is formed as

$$\begin{aligned}\min_{\mathbf{u},\mathbf{s}_i^0} & \sum_{i=2}^{M}\sum_{j=2}^{M}\left(r_{i1} - \left\|\mathbf{u}-\mathbf{s}_i^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right)\left[\mathbf{Q}_d^{-1}\right]_{(i-1)(j-1)} \\ &\cdot \left(r_{j1} - \left\|\mathbf{u}-\mathbf{s}_j^0\right\| + \left\|\mathbf{u}-\mathbf{s}_1^0\right\|\right) + \sum_{i=1}^{M}\frac{\left\|\mathbf{s}_i-\mathbf{s}_i^0\right\|^2}{\delta_i^2} \qquad (22)\end{aligned}$$

which can be formulated in vector-matrix form as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{X},\mathbf{h}} \ & (\mathbf{r}_d - \mathbf{A}\mathbf{d})^T\,\mathbf{Q}_d^{-1}\,(\mathbf{r}_d - \mathbf{A}\mathbf{d}) \\ & + \left\|(\mathbf{X}(:,2:M+1) - \mathbf{S})\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \qquad (23a)\\ \text{s.t. } & d_i = \left\|\mathbf{X}(:,1) - \mathbf{X}(:,i+1)\right\|, \quad i = 1,\cdots,M, \qquad (23b)\end{aligned}$$

where $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^0, \cdots, \mathbf{s}_M^0]$, $\mathbf{S} = [\mathbf{s}_1, \cdots, \mathbf{s}_M]$, and $\mathbf{W} = \text{diag}([\delta_1^{-2}, \cdots, \delta_M^{-2}])$.

The proposed SDP algorithm for this case is formulated as

$$\begin{aligned}\min_{\mathbf{d},\mathbf{D},\mathbf{X},\mathbf{Y}} \ & \text{tr}(\mathbf{D}\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{A}) - 2\mathbf{d}^T\mathbf{A}^T\mathbf{Q}_d^{-1}\mathbf{r}_d \\ & + \text{tr}(\mathbf{W}\mathbf{Y}(2:M+1,2:M+1)) \\ & - 2\text{tr}(\mathbf{W}\mathbf{X}(:,2:M+1)\mathbf{S}) + \eta\,\text{tr}(\mathbf{D}) \qquad (24a)\\ \text{s.t. } & D_{i,i} = Y(1,1) - 2Y(1,i+1) + Y(i+1,i+1), \qquad (24b)\\ & \left\|\mathbf{X}(:,1) - \mathbf{X}(:,i+1)\right\| \le d_i, \qquad (24c)\\ & D_{i,j} \ge |Y(1,1) - Y(1,i+1) - Y(1,j+1) \\ & \qquad + Y(i+1,j+1)|, \qquad (24d)\\ & \begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D}\end{bmatrix} \succeq \mathbf{0}, \qquad (24e)\\ & \begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y}\end{bmatrix} \succeq \mathbf{0}. \qquad (24f)\end{aligned}$$

A proposed method to compute (24) is to consider its K different values, η_k, $k = 1, 2, \cdots, K$, and then use the estimated results $\hat{\mathbf{X}}_k$, $k = 1, 2, \cdots, K$ to select the optimal $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:, 1)$ that minimizes J_k expressed as

$$J_k = (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k)^T \mathbf{Q}_d^{-1} (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k), \quad k = 1, 2, \cdots, K \tag{25}$$

where $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \cdots, \hat{d}_{kM}]^T$, and $\hat{d}_{ki} = \left\| \hat{\mathbf{X}}_k(:, 1) - \hat{\mathbf{X}}_k(:, i+1) \right\|$, $i = 1, 2, \cdots, M$.

IV. WITH MULTIPLE CALIBRATION EMITTERS

The focus so far has been on cases without or with a single calibration emitter. The goal here is to extend the proposed SDP solutions to the case when multiple calibration emitters are available [16]. The analysis and algorithm development so far have shown that the case when the position of the calibration emitter is subject to error is a general case; the cases of emitter whose position is perfectly known or completely unknown can be treated as special cases. Thus, here only the case when all calibration emitters' locations are available but are subject to errors is considered.

Let $\mathbf{c}_l^0$, $l = 1, \cdots, N$, be the true but unknown locations of the lth calibration emitter. The acquired but inaccurate locations of the emitters can be written as

$$\mathbf{c}_l = \mathbf{c}_l^0 + \boldsymbol{\epsilon}_l, \quad l = 1, \cdots, N \tag{26}$$

where $\boldsymbol{\epsilon}_l$ is a zero-mean Gaussian vector with covariance matrix $\kappa_l^2 \mathbf{I}_m$ [16].

The TDOAs of the signal from the N calibration emitters to the sensors are also measured [16], giving the following calibration RDOA measurements:

$$h_{i1}^l = h_i^l - h_1^l + e_{i1}^l, \quad i = 2, \cdots, M, \ l = 1, \cdots, N. \tag{27}$$

where h_{i1}^l is the estimated RDOA between the lth calibration emitter and sensor pair i and 1, $h_i^l = \left\| \mathbf{c}_l^0 - \mathbf{s}_i^0 \right\|$, and $\mathbf{e}^l = [e_{21}^l, \cdots, e_{M1}^l]^T$ is assumed to be a zero-mean Gaussian vector with covariance matrix $\mathbf{Q}_c$.

Under the assumption that the noise vectors $\mathbf{n}$, $\boldsymbol{\beta}_i$, $i = 1, \cdots, M$, $\mathbf{e}^l$, $l = 1, \cdots, N$ and $\boldsymbol{\epsilon}_l$, $l = 1, \cdots, N$ are mutually independent, the MLE problem is formulated as

$$\begin{aligned}\min_{\mathbf{u}, \mathbf{c}_l^0, \mathbf{s}_i^0} & \sum_{i=2}^{M} \sum_{j=2}^{M} \left(r_{i1} - \left\| \mathbf{u} - \mathbf{s}_i^0 \right\| + \left\| \mathbf{u} - \mathbf{s}_1^0 \right\| \right) \left[\mathbf{Q}_d^{-1} \right]_{(i-1)(j-1)} \\ & \cdot \left(r_{j1} - \left\| \mathbf{u} - \mathbf{s}_j^0 \right\| + \left\| \mathbf{u} - \mathbf{s}_1^0 \right\| \right) + \sum_{i=1}^{M} \frac{\left\| \mathbf{s}_i - \mathbf{s}_i^0 \right\|^2}{\delta_i^2} \\ & + \sum_{l=1}^{N} \sum_{i=2}^{M} \sum_{j=2}^{M} \left(h_{i1}^l - \left\| \mathbf{c}_l^0 - \mathbf{s}_i^0 \right\| + \left\| \mathbf{c}_l^0 - \mathbf{s}_1^0 \right\| \right) \\ & \cdot \left[\mathbf{Q}_c^{-1} \right]_{(i-1)(j-1)} \left(h_{j1}^l - \left\| \mathbf{c}_l^0 - \mathbf{s}_j^0 \right\| + \left\| \mathbf{c}_l^0 - \mathbf{s}_1^0 \right\| \right) \\ & + \sum_{l=1}^{N} \frac{\left\| \mathbf{c}_l - \mathbf{c}_l^0 \right\|^2}{\kappa_l^2}. \end{aligned} \tag{28}$$

Similarly, (28) can be written in vector-matrix form as

$$\begin{aligned}\min_{\mathbf{d}, \mathbf{X}, \mathbf{h}^l} \ & (\mathbf{r}_d - \mathbf{A}\mathbf{d})^T \mathbf{Q}_d^{-1} (\mathbf{r}_d - \mathbf{A}\mathbf{d}) + \sum_{l=1}^{N} (\mathbf{h}_d^l - \mathbf{A}\mathbf{h}^l)^T \mathbf{Q}_c^{-1} \\ & \cdot (\mathbf{h}_d^l - \mathbf{A}\mathbf{h}^l) + \left\| (\mathbf{X}(:, 2:M+N+1) - \mathbf{S}) \mathbf{W}^{\frac{1}{2}} \right\|_F^2 \end{aligned} \tag{29a}$$

$$\text{s.t. } d_i = \left\| \mathbf{X}(:, 1) - \mathbf{X}(:, i+N+1) \right\|, \quad i = 1, \cdots, M, \tag{29b}$$

$$\begin{aligned} & h_i^l = \left\| \mathbf{X}(:, 1+l) - \mathbf{X}(:, i+N+1) \right\|, \\ & i = 1, \cdots, M, \quad l = 1, \cdots, N. \end{aligned} \tag{29c}$$

where $\mathbf{h}_d^l = [h_{21}^l, \cdots, h_{M1}^l]^T$, $\mathbf{h}^l = [h_1^l, \cdots, h_M^l]^T$, $\mathbf{X} = [\mathbf{u}, \mathbf{c}_1^0, \cdots, \mathbf{c}_N^0, \mathbf{s}_1^0, \cdots, \mathbf{s}_M^0]$, $\mathbf{S} = [\mathbf{c}_1, \cdots, \mathbf{c}_N, \mathbf{s}_1, \cdots, \mathbf{s}_M]$, and $\mathbf{W} = \text{diag}([\kappa_1^{-2}, \cdots, \kappa_N^{-2}, \delta_1^{-2}, \cdots, \delta_M^{-2}])$.

The proposed SDP algorithm for this case is formulated as

$$\begin{aligned}\min_{\mathbf{d}, \mathbf{D}, \mathbf{h}^l, \mathbf{H}^l, \mathbf{X}, \mathbf{Y}} \ & \text{tr}(\mathbf{D}\mathbf{A}^T \mathbf{Q}_d^{-1} \mathbf{A}) - 2\mathbf{d}^T \mathbf{A}^T \mathbf{Q}_d^{-1} \mathbf{r}_d \\ & + \sum_{l=1}^{N} [\text{tr}(\mathbf{H}^l \mathbf{A}^T \mathbf{Q}_c^{-1} \mathbf{A}) - 2\mathbf{h}^{l^T} \mathbf{A}^T \mathbf{Q}_c^{-1} \mathbf{h}_d^l] \\ & - 2\text{tr}(\mathbf{W}\mathbf{X}(:, 2:M+N+1)\mathbf{S}) \\ & + \text{tr}(\mathbf{W}\mathbf{Y}(2:M+N+1, 2:M+N+1)) \\ & + \eta \text{tr}(\sum_{l=1}^{N} \mathbf{H}^l + D) \end{aligned} \tag{30a}$$

$$\begin{aligned}\text{s.t. } D_{i,i} = \ & Y(1,1) - 2Y(1, i+N+1) \\ & + Y(i+N+1, i+N+1), \end{aligned} \tag{30b}$$

$$\begin{aligned} H_{i,i}^l = \ & Y(l+1, l+1) - 2Y(l+1, i+N+1) \\ & + Y(i+N+1, i+N+1), \end{aligned} \tag{30c}$$

$$\left\| \mathbf{X}(:, 1) - \mathbf{X}(:, i+N+1) \right\| \le d_i, \tag{30d}$$

$$\left\| \mathbf{X}(:, l+1) - \mathbf{X}(:, i+N+1) \right\| \le h_i^l, \tag{30e}$$

$$\begin{aligned} D_{i,j} \ge \ & |Y(1,1) - Y(1, i+N+1) - Y(1, j+N+1) \\ & + Y(i+N+1, j+N+1)|, \end{aligned} \tag{30f}$$

$$\begin{aligned} H_{i,j}^l \ge \ & |Y(l+1, l+1) - Y(l+1, i+N+1) \\ & - Y(l+1, j+N+1) + Y(i+N+1, j+N+1)|, \end{aligned} \tag{30g}$$

$$\begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0}, \tag{30h}$$

$$\begin{bmatrix} 1 & \mathbf{h}^{l^T} \\ \mathbf{h}^l & \mathbf{H}^l \end{bmatrix} \succeq \mathbf{0}, \tag{30i}$$

$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \tag{30j}$$

Again, K different values of η_k, $k = 1, 2, \cdots, K$, can be used to compute (30), and then the estimated results $\hat{\mathbf{X}}_k$, $k = 1, 2, \cdots, K$, can be used to determine a proper $\hat{\mathbf{u}} = \hat{\mathbf{X}}(:, 1)$ that minimizes J_k expressed as

$$J_k = (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k)^T \mathbf{Q}_d^{-1} (\mathbf{r}_d - \mathbf{A}\hat{\mathbf{d}}_k), \quad k = 1, 2, \cdots, K \tag{31}$$

TABLE 1. Comparison of the proposed SDP algorithms and existing algorithms.

Configuration	Proposed	Existing Algorithm
A single calibration emitter whose location is known accurately	(11)	[15]
A single calibration emitter whose location is not perfectly known	(16)	[16]
A single calibration emitter whose location is completely unknown	(20)	[17]
No calibration emitters	(24)	[13], [14], [17]
Two calibration emitters whose locations are not perfectly known	(30)	[16]

where $\hat{\mathbf{d}}_k = [\hat{d}_{k1}, \hat{d}_{k2}, \cdots, \hat{d}_{kM}]^T$, and $\hat{d}_{ki} = \left\| \hat{\mathbf{X}}_k(:,1) - \hat{\mathbf{X}}_k(:, i+N+1) \right\|$, $i = 1, 2, \cdots, M$.

V. SIMULATION RESULTS

The performances of the proposed SDP algorithms (labeled as Proposed) and state-of-the-art existing algorithms for various cases listed in Table 1 are compared in this section.

The proposed model is suitable for networks and also depend on the networks size. A network with four sensors and one source is considered. The unknown true positions of the sensor are $[10, 10]^T m$, $[10, -10]^T m$, $[-10, 10]^T m$, $[-10, -10]^T m$. The calibration emitter can take any of the two positions at $[0, 0]^T m$ or $[5, 5]^T m$. The RDOA measurements are created by adding to the true values a zero-mean Gaussian noise vector with covariance matrices $\mathbf{Q}_d = \mathbf{Q}_c = \sigma^2 \mathbf{R}$, where the diagonal elements of matrix $\mathbf{R}$ are all equal to 1 and the off-diagonal elements of $\mathbf{R}$ are all equal to 0.5 [15].

The ith ($i = 1, \cdots, M$) noisy sensor position is created by adding to its true position a zero-mean Gaussian noise vector with covariance matrix $\delta^2 \mathbf{I}_2$. The jth ($j = 1, \cdots, N$) noisy calibration emitter's position is created by adding to its true position a zero-mean Gaussian noise vector with covariance matrix $\kappa^2 \mathbf{I}_2$.

The performance metric is the root mean-square error (RMSE), which is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N_c} \sum_{j=1}^{N_c} \left\| \hat{\mathbf{u}}_j - \mathbf{u} \right\|^2} \tag{32}$$

where $\hat{\mathbf{u}}_j$ is the the source position estimate in the jth run and N_c is the number of Monte Carlo runs. A total of 500 realizations are generated in the following simulations. The proposed SDP algorithm is implemented in CVX toolbox [22] by using SeDuMi as a solver [23], and the precision is set to best. The simulation is executed by Matlab 2012b in personal computer (CPU I3, 2.4GHz).

Fig. 1 shows the RMSE of the proposed algorithm (11) versus the penalty parameter η, which varies from 10^{-4} to 10^0. These results show that a suitable value of η is important to achieve a good estimation performance. However, it is not easy to analytically obtain the optimal value of parameter η, which depends on the distance between the sensor and the source nodes as well as the RDOA noise level [21]. In the following simulations, $\eta_1 = 10^{-4}$, $\eta_2 = 10^{-3}$, $\eta_3 = 10^{-2}$, $\eta_4 = 10^{-1}$, $\eta_5 = 10^0$ for the five proposed algorithms, i.e., (11), (16), (20), (24), and (30), respectively.

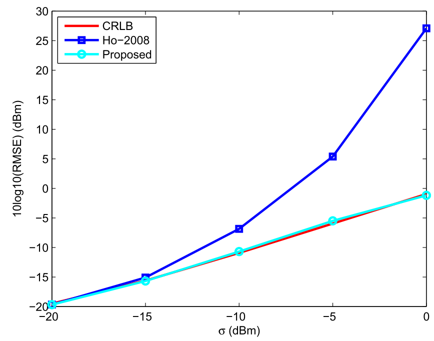

FIGURE 2. RMSE vs. σ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c} = [0, 0]^T m$, $\delta = 0.01m$.

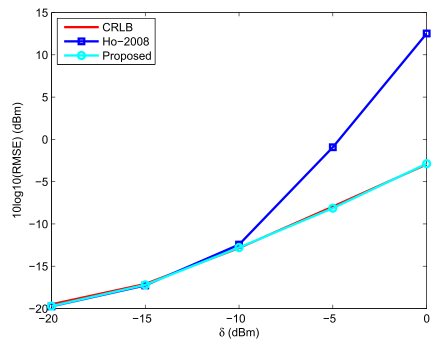

FIGURE 3. RMSE vs. δ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c} = [0, 0]^T m$, $\sigma = 0.01m$.

Fig. 2 and Fig. 3 evaluate, respectively, the performance of the proposed algorithm versus σ and δ, assuming a single calibration emitter located at $\mathbf{c} = [0, 0]^T m$ (accurately known). These results lead to the following observations. First, both the algorithm in [15] and the proposed algorithm can reach the CRLB when the RDOA measurement noise and sensor position errors are sufficiently small. Second, the proposed algorithm is superior to the one in [15] when the RDOA measurement noise or sensor position errors are large.

The results in Figs. 4 and 5 are also for a single calibration emitter located at $\mathbf{c} = [0, 0]^T m$, but this position is not accurately known to the algorithms. It is observed that the algorithm in [16] reaches the CRLB only when the RDOA measurement noise, sensor position errors, and calibration emitter position errors are sufficiently small, and its performance degrades significantly as σ and δ increase. On the other hand, the performance of the proposed algorithm approaches the CRLB for all values of σ and δ evaluated.

The case of a single calibration emitter placed at $\mathbf{c} = [0, 0]^T m$ (but this position is completely unknown to the

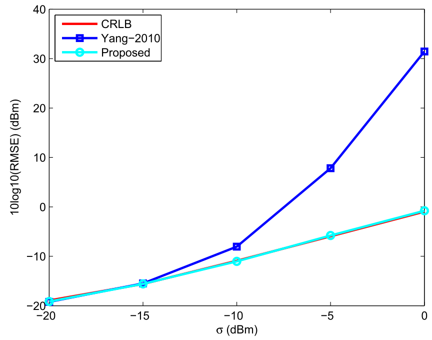

FIGURE 4. RMSE vs. σ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c}^0 = [0, 0]^T m$, $\delta = 0.01m$, $\kappa = 0.01m$.

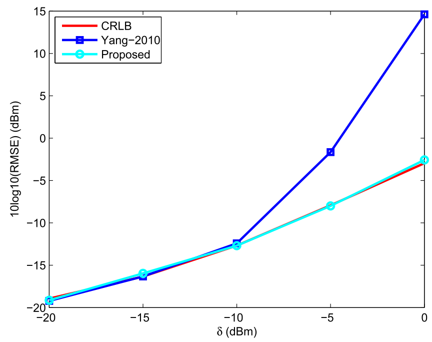

FIGURE 5. RMSE vs. δ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c}^0 = [0, 0]^T m$, $\sigma = 0.01m$, $\kappa = 0.01m$.

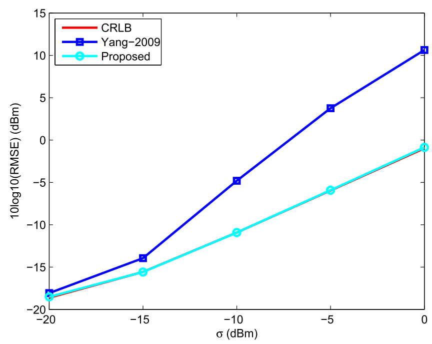

FIGURE 6. RMSE vs. σ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c} = [0, 0]^T m$, $\delta = 0.01m$.

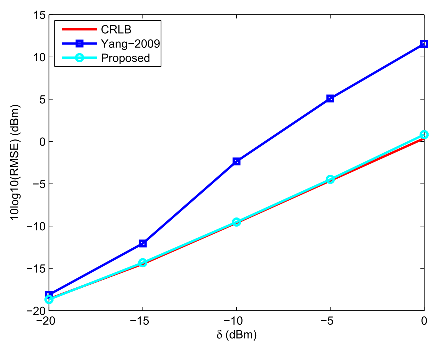

FIGURE 7. RMSE vs. δ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c} = [0, 0]^T m$, $\sigma = 0.01m$.

network) is assessed in Fig. 6 and Fig. 7. Again, for all values of σ and δ evaluated, the proposed algorithm approaches the CRLB and significantly outperforms the algorithm in [17].

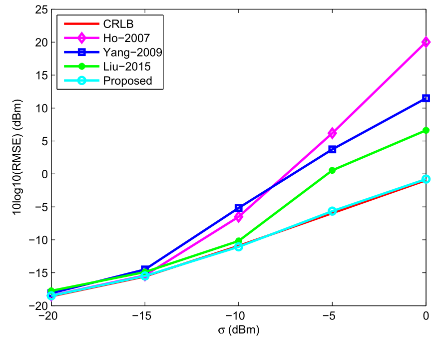

FIGURE 8. RMSE vs. σ, $\mathbf{u} = [2, 8]^T m$, $\delta = 0.01m$.

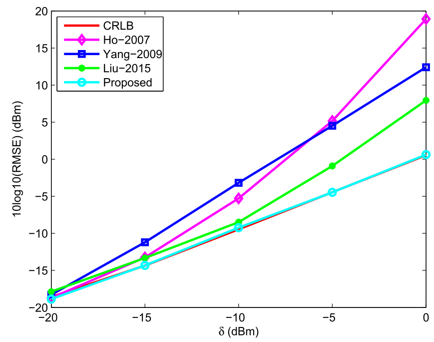

FIGURE 9. RMSE vs. δ, $\mathbf{u} = [2, 8]^T m$, $\sigma = 0.01m$.

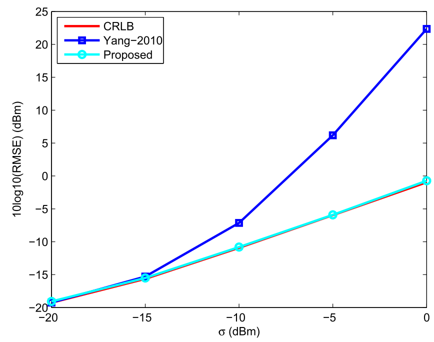

FIGURE 10. RMSE vs. σ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c}_1^0 = [0, 0]^T m$, $\mathbf{c}_2^0 = [5, 5]^T m$, $\delta = 0.01m$, $\kappa = 0.01m$.

Figs. 8 and 9 compare the performances of the proposed algorithm, and the algorithms proposed in [13], [14], and [17], all addressing the same configuration. It is found that the proposed algorithm can reach the CRLB in the whole range of noise levels evaluated, while the other three algorithms approach the CRLB only when σ and δ are sufficiently small.

The last case listed in Table 1—two calibration emitters whose positions are not accurately known—is studied in Figs. 10 and 11. Similar to the previous cases studied, it is observed that the location estimation performances of the algorithm in [16] can reach the CRLB only when the RDOA measurement noise, sensor position errors, and calibration emitter position errors are sufficiently small, whereas the proposed algorithm performs close to the CRLB for the whole range of noise levels evaluated.

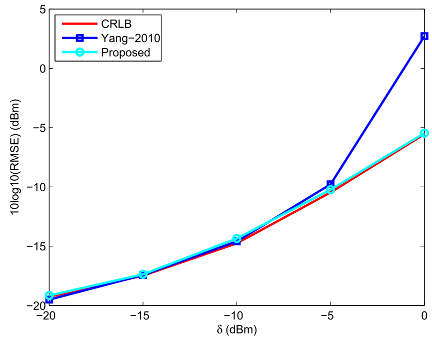

FIGURE 11. RMSE vs. δ, $\mathbf{u} = [2, 8]^T m$, $\mathbf{c}_1^0 = [0, 0]^T m$, $\mathbf{c}_2^0 = [5, 5]^T m$, $\sigma = 0.01m$, $\kappa = 0.01m$.

Note that the computational complexity of the proposed SDP algorithms are higher than that of the existing least-squares based algorithms. However, for many indoor pisitioning applications, the estimation performance is more important than the computational complexity, for which the proposed algorithms will be attractive.

VI. CONCLUSIONS

A unified solution of TDOA-based source localization in the presence of random sensor position errors is established in this paper. After investigating three possible cases of a network with a single calibration emitter and the case without a calibration emitter, the algorithms developed are then generalized to the case of multiple calibration emitters. The proposed unified solution is based on SDP for all these cases. It starts from the MLE problems (nonconvex), which are transformed into convex optimization problems. In addition to a unified solution that is applicable for various scenarios, another key feature of the proposed algorithms is that their performances reach the CRLB, whereas existing state-of-the-art algorithms can reach the CRLB only when the TDOA measurements noise, the sensor position errors, and calibration emitter position errors are sufficiently small. This is verified via extensive amount of simulation results.

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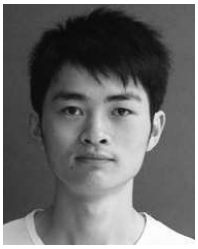

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