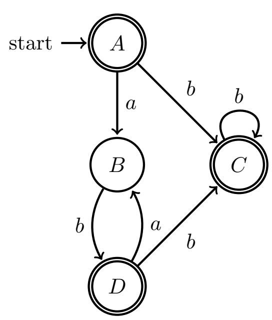
CS 321, Fall 2015 Midterm (45 min., 20 pts) Take Home due in class on Monday 11/2

Last = **First =** **ONID login =** **@oregonstate.edu**

1. (1 pt) (a) When is an NFA accepting no strings? (b) What is the language of that NFA? Use mathematical notations.

2. (3=2+1 pts) construct DFA and/or NFA for the following bitstring languages (Hint: it might be faster to skip drawing machines for L_1 or L_2, and focus on the semantics of the languages):
   (a) L_1=those with **odd** numbers of 0s; L_2=those with **even** numbers of 1s.
   $L_1 \cup L_2$ with DFA
   Now $L_1 - L_2$ with **NFA or DFA**:
   (b) those that **contain** 0100 using **either DFA or NFA**:

3. (2 pts) Minimize the following DFA using the partition algorithm.

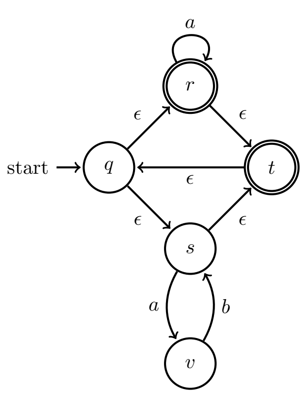


(a) Initial parition is:

(b) Final partition is:
because:

(c) Minimal DFA:

4. (3 pts) For this NFA, compute ϵ-closure for each state, and convert to DFA.



(a) ϵ-closures:

(b) DFA:

(c) $E(q)$ is defined to be **the smallest set** such that $q \in E(q)$ and if $p \in E(q)$, then:

(d) Convert this DFA to RE:

5. (2 pts) Given two ϵ-free NFAs $N_1 = (Q_1, \Sigma, \delta_1, q_{0,1}, F_1)$ and $N_2 = (Q_2, \Sigma, \delta_2, q_{0,2}, F_2)$, construct a new NFA $N = (Q_1 \times Q_2, \Sigma, \delta, q_0, F)$ so that $L(N) = L(N_1) \cap L(N_2)$ **without** first converting them to DFAs:

   $q_0 =$ $\delta((p, q), a) =$ $F =$ $\delta : (Q_1 \times Q_2) \times \Sigma \mapsto$

6. (3.5 pts) Complete the following partial proof of $(uv)^R = v^R u^R$ for any strings u and v.

   First define reverse: $\epsilon^R = \epsilon$, ______________________

   Now proof by induction on $|v|$. base case: $|v| = 0$ so: ______________________

   inductive case: assume the inductive hypothesis: ________________ holds for all: ____________

   Now for uv where $|v| = n + 1$, we can rewrite $(uv)^R =$__________________________

   By definition of reverse, ______________________________

   By IH, ____

   By definition of reverse, ______________________________

7. (5.5 pts) Write REs/NFAs for the following languages:

   (a) (2 pt) bitstrings that start and end with the same bit (at least one bit).
   RE:

   Now convert this RE to NFA:

   (b) (1 pt) Decimal integers, could be negative (-) but no need to write the plus (+) sign for positive numbers or zero. No leading zeros. Here are three examples: 2561 -89 0. But -0 is not allowed.
   To simpilify your notation, use $D = \{0..9\}$ for digits, and $\widetilde{D} = \{1..9\}$ for non-zero digits.
   RE:

   (c) (1.5 pts) An ONID password must have at least one uppercase letter, one lowercase letter, and a digit. (**hard**) Use $L = \{a..z\}$ for lowercase letters, $U = \{A..Z\}$ for uppercase letters, $D = \{0..9\}$ for digits, and Σ for all characters allowed in an ONID password (i.e., $\Sigma = L \cup U \cup D \cup \{$`!,@,#,$,%,^,`$\ldots\}$). **Draw NFA, but no RE**.

   (d) (1 pt) Actually besides the above requirements, a password *also* needs be at least 8 characters long. How would you construct the RE for this kind of password? Just describe your idea, but no need to implement it.

The End.