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A Single-Well Tracing Method for Estimating Regional Advective Velocity in a Confined Aquifer: Theory and Preliminary Laboratory Verification

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An equation is derived by which advective groundwater velocity in a confined aquifer may be estimated by a single-well tracer test in which a single tracer pulse is allowed to drift from the well and then pumped back to the well and sampled to obtain a breakthrough curve. Although similar in methodology to preexisting methods, this method differs in that it takes into account ambient groundwater movement during the pumpback phase. Using sodium chloride as a tracer solution, a series of small-scale tests were run in a laboratory sand tank model to test the theory. Results of linear flow tracer tests through the model, simulating unperturbed regional advective flow at known velocities, were compared to results of single-well drift-and-pumpback tests conducted during linear flow through the model. Advective velocities computed by both types of tests were identical, thus proving the validity of the equation.

INTRODUCTION

The estimation of regional advective groundwater velocity is best accomplished through groundwater tracer experiments in which the investigator can actually measure the travel time of a tracer label (center of mass) between an input well and an output well. Tracer tests involving several wells can be expensive and may be logistically prohibitive under certain conditions. In some cases, the information required may be a good estimate of advective velocity at one well rather than a more expensive and time consuming determination involving several wells.

Such rapidly acquired estimates obtained from several single-well tests over a wide area, rather than from one multiwell test in one locality, might be useful for estimating potential water soluble contaminant migration rates from an existing or potential waste repository in a scoping study of a wide area. Such a test could also be performed in existing wells of convenience without the necessity of drilling and developing new wells.

The purpose of this paper is to present the theory and laboratory verification of a single-well tracer test method in which a single pulse of conservative tracer is emplaced in a well and allowed to drift with subsequent pumpback. The advective velocity is then computed from the total elapsed time from emplacement to retrieval of the center of mass (label) of the pulse. The method is probably the simplest and least expensive tracing method in existence and should be useful in a wide variety of hydrogeologic settings. One must know regional hydraulic gradient direction but not its magnitude in order to determine the direction of tracer movement. Effective porosity and uniform aquifer thickness must also be known a priori.

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OTHER PERTINENT TRACING METHODS

Other methods developed for the express purpose of determining regional advective groundwater velocity include the following.

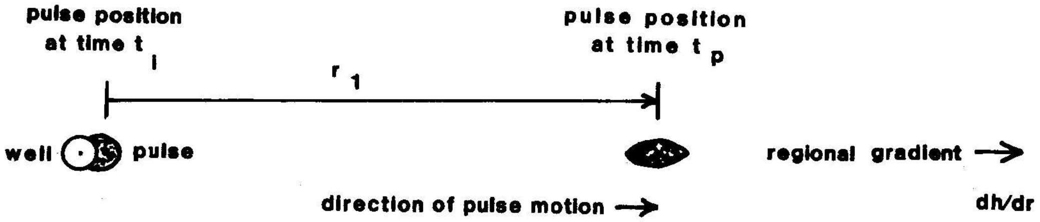
1. Multiple-well drift method [*Gaspar and Oncescu*, 1972, p. 133]: This method employs one well for pulse emplacement and one or more wells downstream for detection. Porosity and aquifer thickness are not required a priori.
2. Single-well, point dilution method [*Drost and Neumaier*, 1974; *Klotz et al.*, 1979; *Grisak et al.*, 1977; *Freeze and Cherry*, 1979]: This technique does not require a priori knowledge of porosity or aquifer thickness, but it does require an estimate of hydraulic conductivity around the well-bore gravel pack.
3. Two-well, two-pulse method: This method was very recently developed by the senior author and was reported by *Leap* [1985]. It employs an upgradient emplacement well and a downgradient pumping well arranged in tandem along the estimated direction of the regional hydraulic gradient.

This method also does not require a prior knowledge of hydraulic conductivity or porosity or thickness and yields a regional velocity characteristic of the aquifer volume between the two wells.

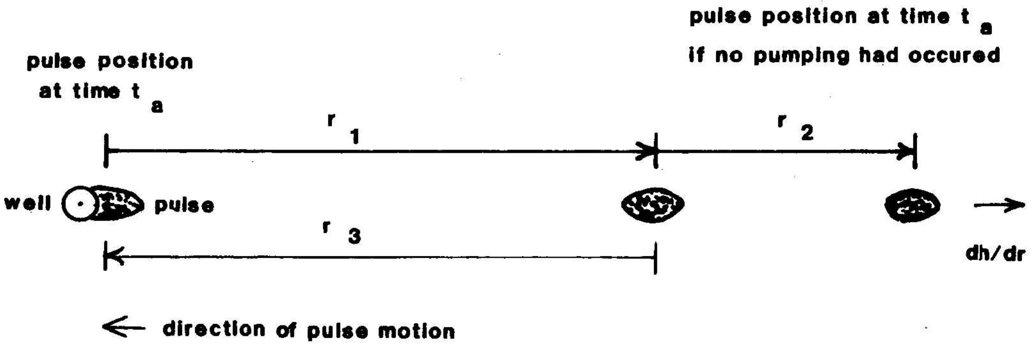
4. One-well, emplacement and pumpback method: This method is described by *Borowczyk et al.* [1966] and also by *Gaspar and Oncescu* [1972]. *Fried* [1975] also describes the method and extends its use to determination of dispersion coefficients.

The method is used for regional advective velocity determination by emplacing a pulse of tracer in a well, forcing it into the aquifer from the well bore, and then letting it drift for a time t_*. At the end of this time it is pumped back to the well at a constant rate Q over a time t_p which is the elapsed time from the beginning of pumpback until the center of mass of the breakthrough curve is retrieved. The regional advective velocity is then computed as

$$v = (Qt_p/\pi bn)^{1/2}/t_* \tag{1}$$



a. During pulse drift (no pumping).



b. During pulse pumpback.

Fig. 1. Schematic plan view diagram of tracer pulse history, including drift and pumpback.

where

b	aquifer thickness;
n	effective porosity.

It is assumed that the regional velocity is low enough that regional groundwater movement during pumpback time t_p is negligible. One advantage of this method, as in the case of the point dilution method, is that the direction or magnitude of the regional hydraulic gradient does not have to be known. Although suitable for low regional velocity, the method can give erroneous estimates when regional velocity is rather high.

Our new method, described in the following discussion, is very similar to the latter method and employs a very similar equation, but it also accounts for regional velocities that are too high to be neglected during pumpback.

THEORY

The volume of water pumped from or injected into a cylindrical volume of a confined aquifer within a certain elapsed time by a well whose screened or open hole part completely penetrates the aquifer is given by:

$$V = \left| \int Q(t)\, dt \right| \tag{2}$$

where $Q(t)$ = volumetric flow rate as a function of time. If the flow rate is constant, then integration of (2) yields

$$V = |Qt| \tag{3}$$

where

Q	constant volumetric flow rate;
t	elapsed pumping or injection time.

If the aquifer is homogeneous and isotropic and possesses no regional hydraulic gradient, then the volume in (3) will be described as

$$V = \pi r^2 nb \tag{4}$$

assuming a constant thickness. Equating (3) and (4) and rearranging gives

$$r = (Q/\pi nb)^{1/2} t^{1/2} \tag{5}$$

which expresses the radius of an enlarging cylinder of water as a function of the injection or pumping time.

Steady state unidirectional flow is expressed by Darcy's law as

$$\begin{aligned} q/n &= (K/n)(dh/dx) \\ q/n &= v_a \\ q/n &= x/\tau \end{aligned} \tag{6}$$

where

q	specific discharge;
K	hydraulic conductivity;
h	hydraulic head;
x	unidirectional coordinate;
v_a	advective regional velocity;
τ	travel time.

Figure 1 shows a schematic diagram of a well in which a single pulse of conservative tracer is emplaced and allowed to drift with the regional gradient for some time and then pumped back to the well and sampled. The pulse is emplaced and begins drifting at time t_i and continues drifting along the regional gradient direction r at the regional advective velocity until time t_p when pumping begins at the well in order to retrieve the pulse. After pumping at a constant rate Q, the pulse label, i.e., the center of mass on the breakthrough curve, returns to the well at time t_a (Figure 2). The apparent radial

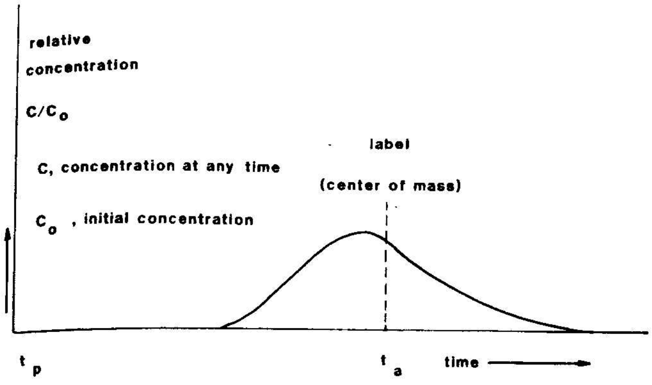


Fig. 2. Typical tracer breakthrough curve during pulse pumpback.

position of the label from the well can be found from the relationship

$$r = r_1 + r_2 + r_3 \tag{7}$$

where

r_1 radial displacement of the label between times t_i and t_p due to regional flow away from the well;
r_2 radial displacement of the label between times t_p and t_a due to regional flow way from the well if there were no pumping;
r_3 radial displacement between times t_p and t_a due to pumping toward the well.

The instantaneous advective velocity of the label can be written from (7) as

$$dr/dt = dr_1/dt + dr_2/dt + dr_3/dt \tag{8}$$

from which

$$\int_0^{r_1} dr_1 = v_a \int_{t_i}^{t_p} dt \tag{9}$$

$$\int_{r_1}^{r_2} dr_2 = v_a \int_{t_p}^{t_a} dt \tag{10}$$

Evaluating (9) and (10) give, respectively,

$$r_1 = v_a(t_p - t_i) \tag{11}$$

$$r_2 = v_a(t_a - t_p) \tag{12}$$

From (5)

$$r_3{}^2 = (Q/\pi nb)t \tag{13}$$

$$2r_3(dr/dt) = (Q\pi nb) \tag{14}$$

$$2\int_r^0 r_3\, dr = (Q/\pi nb)\int_{t_p}^{t_a} dt \tag{15}$$

and thus

$$r_3 = -(Q/\pi nb)^{1/2}(t_a - t_p)^{1/2} \tag{16}$$

Therefore from (7), (11), (12), and (16) we get

$$r = v_a(t_p - t_i) + v_a(t_a - t_p) - (Q/\pi nb)^{1/2}(t_a - t_p)^{1/2} \tag{17}$$

Letting

$$T = t_a - t_i \tag{18}$$

$$t = t_a - t_p \tag{19}$$

Then, from (17) we get

$$r = v_a T - (Q/\pi nb)^{1/2} t^{1/2} \tag{20}$$

At the well, where all observations are made,

$$r = 0 \tag{21}$$

thus

$$v_a = [(Q/\pi nb)^{1/2} t^{1/2}]/T \tag{22}$$

which allows one to determine advective regional velocity while considering the effects of regional velocity during pumpback [*Kaplan*, 1985; *Kaplan and Leap*, 1985].

Experimental Verification

Experimental testing and verification of (22) was accomplished with small-scale experiments in a sand tank (a model aquifer) in a laboratory where variables and parameters could be closely controlled and observed.

Model Aquifer Construction

The actual sand tank (Figure 3) was constructed from 3/4-inch plywood as an open top box 4 x 8 feet in lateral dimension and 8 inches deep. All interior wood surfaces were made waterproof with three coats of polyurethane varnish and a finishing coat of polyester fiberglass resin. A perforated polyvinyl chloride pipe, serving as the well, was inserted through the bottom in the middle of the box.

Attached to each end of the box was a plywood tank, also water sealed. The tanks served to maintain constant head at each end of the box. Through the bottom of each tank a sliding drain tube was inserted which when adjusted to the proper height, regulated the hydraulic head of the upstream and downstream ends and thus maintained a constant gradient and velocity of flow across the length of the tank.

The porous medium consisted of a 5-cm thickness of well-sorted Ottawa sand with a medium grain diameter of approximately 0.5 mm. Porosity of the sand was found to be 0.38 by the volumetric displacement method. Confining conditions were simulated by placing over the sand, flush with its surface, a 4 x 8 foot sheet of clear acrylic plastic, 1/2 inch in thickness. All seams and joints were sealed with clear silicone sealant.

A network of 2 x 4 inch lumber was placed on top of the acrylic sheet and two 100-pound sacks of sand were placed on the lumber to tightly compress the acrylic against the sand. The top of the acrylic was tapped numerous times with a rubber mallet to insure maximum settling and compaction of the sand after saturation. The entire system was leveled with hydraulic jacks below the bottom of the model.

The upstream tank was supplied with deionized water at a constant flow rate from a laboratory supply line at a temperature of approximately 25°C; the temperature did not vary enough to cause significant changes in viscosity.

Drainage from the downstream tank was by gravity through the drain tube in the tank bottom. Likewise, pumpage from the well was simulated by controlled gravity drainage from the well tube through the bottom of the tank.

Instrumentation

Both the downstream drainage tube and the well drainage tube were equipped with glass, floating-ball Gilmont flow meters with a range of 10–850 mL per min. Calibration checks showed both to be accurate to within plus or minus 5%.

Detection of the sodium chloride tracer solution was performed at the well by a commercial in-line conductance cell with platinum electrodes. Detection within the porous medium downstream from the well was accomplished by a laboratory-constructed conductance cell consisting of an elec-

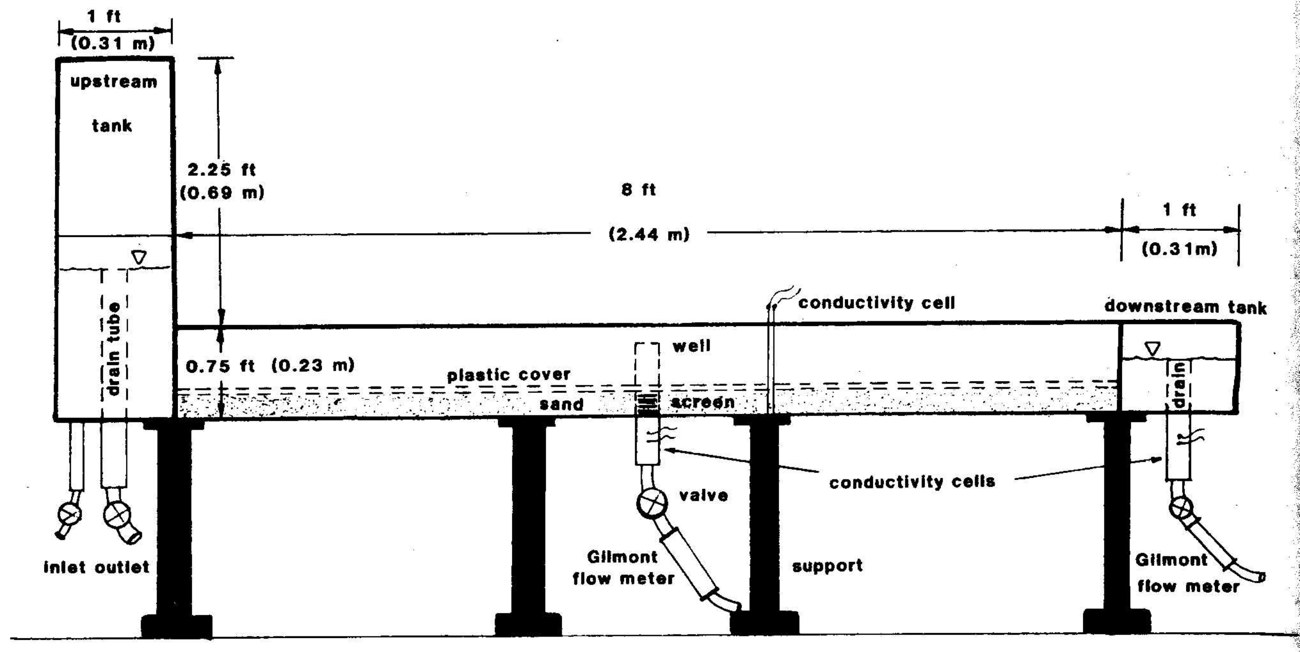


Fig. 3. Cross-sectional schematic view of sand tank used for testing single-well tracer test theory.

trode pair constructed from two stainless steel hypodermic needles and imbedded within the porous medium 23 cm downstream from the side of the well.

Both conductivity cells could be attached to a Yellow Springs Instrument Company model 32 conductance meter with a range of 0.1 micromhos to 200 millimhos to monitor tracer concentrations over time, both at the well and downstream from the well. The conductance meter was attached to a Linear model 355 chart recorder with variable input voltages and chart speeds which made a continuous record of specific conductance versus time.

Tracer Experiments

Sodium chloride was selected as the tracer because of its conservative behavior in quartzitic sand [*Kaufmann and Orlob*, 1956; *Cahill*, 1966, 1973], ease of detection by conductivity methods, and low cost and availability.

Two types of tracer tests were conducted with the model aquifer in order to test the validity of (22). Both types of tests were run after constant heads had been established in the end tanks, and thus a condition of steady state flow had been established and maintained from one end of the model to the other which was monitored by the in-line Gilmont flow meter in the discharge line from the downstream end tank.

Type I tests consisted of emplacing pulses of tracer in the well, allowing them to drift with the advective background velocity for a predetermined time and then pumping them back to the well where they were detected and monitored by the in-line flow meter in the discharge line from the well. This type of test is the laboratory simulation of the actual type of test that would be performed in the field.

Breakthrough curves were drawn by the strip chart recorder of conductivity values of the tracer returning to the well. The center of mass of each curve was determined, and the advective velocity through the model was calculated for each test from (22), taking values of porosity and thickness to be 0.38 and 5 cm, respectively, for all cases. These tests were run for various combinations of hydraulic gradients and thus background velocity and pumping rates at the well. Results are shown in Table 1.

Tests of type II were simple tests to determine advective background velocity for constant gradients and discharge

TABLE 1. Results of Eight Type I Tests for Determining Regional Advective Velocity by the Sequence of Emplacement, Drift, and Pumpback

Trial Number	Drift Time, min	t, s	T, s	Q, cm^3/s	n	b cm	v_a, cm/s
23	20	360	1560	2.08	0.38	5	0.007
24	20	192	1392	3.42	0.38	5	0.008
25	25	202	1702	6.08	0.38	5	0.008
26	30	339	2139	5.83	0.38	5	0.009
27	35	606	2706	6.08	0.38	5	0.009
28	40	849	3249	6.25	0.38	5	0.009
29	40	1116	3516	6.00	0.38	5	0.010
30	45	1476	4176	6.42	0.38	5	0.010

Regional advective velocities calculated at a head difference of 7.5 cm (gradient = 0.031). T, $ta-ti$ (equation (18)); t, $ta-tp$ (equation (19)); Q, pumpback discharge rate; n, porosity; b, aquifer thickness; v_a, computed regional advective velocity.

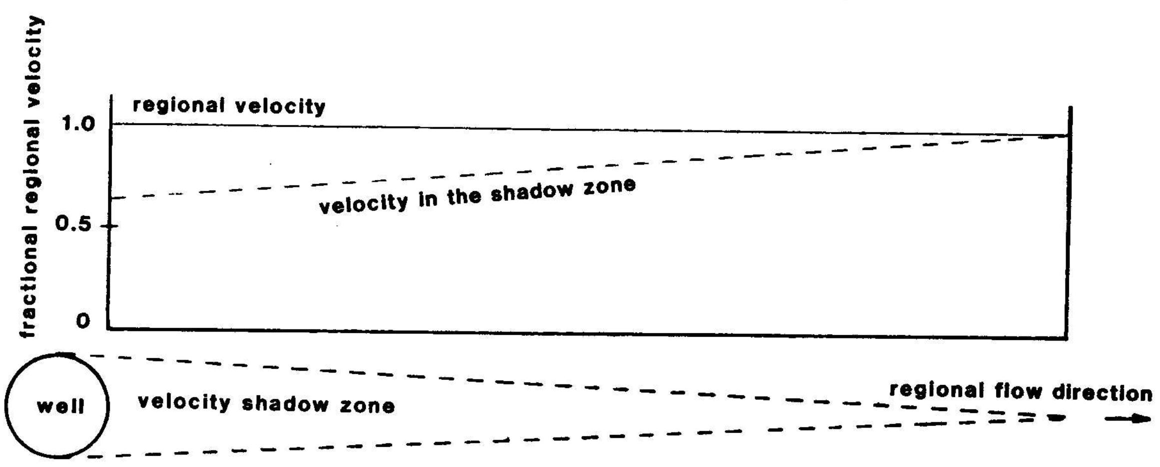


Fig. 4. Plan view diagram showing effect of the velocity shadow zone downgradient from a well.

rates from the downstream end tank. The background advective velocity obtained from these tests was then compared to the background velocity obtained by type I tests which showed the accuracy of type I test in determining advective background or regional velocity.

The type II testing procedure involved emplacing a pulse of tracer in the well and allowing it to drift with the advective velocity through the model under the gradient imposed by the difference in heads at the end tanks. The pulse was detected at the electrode pair embedded within the sand 23 cm downstream from the well. The breakthrough curve of the pulse passing the electrode pair was analyzed for its center of mass whose arrival time t' and travel distance x' was used in the formula for linear background advective velocity

$$v_L = x'/t' \tag{23}$$

$$v_L = 23\ \text{cm}/t' \tag{24}$$

Thirty-six type I tests were run, and it was originally planned to perform several type I tests for specific constant values of upstream and downstream heads and end-tank discharge but with varying drift times to be followed by a type II test at the same head and discharge settings. With this sequence of tests, the drift and pumpback values of advective velocities could be compared with velocity from the one-way drift test under the same sets of conditions.

Unfortunately, due to repeated failures of the main deionized water system, which was beyond the control of the investigators, it was possible to compare results from the two test types for only one set of hydraulic parameters, which included type I tests numbers 23–30.

For these tests the upstream head was set at 13 cm and the downstream head at 5.5 cm, making a head difference of 7.5 cm across the 250-cm length of the model to produce a gradient of 0.03. Discharge from the downstream end tank was maintained at 9.3 cm^3/s. These hydraulic parameters were the same for the subsequent type II tests used for comparison.

Data Analysis and Interpretation

Examination of the advective velocity values obtained from the type I tests listed in Table 1 show that as drift time increased after emplacement at the well, the value of the computed advective velocity v_a also increased so that both trials 29 and 30 with drift times of 40 and 45 min, respectively, gave an advective velocity 0.01 cm/s while in test 23 a drift time of 20 min yielded a velocity of only 0.007 cm/s. This phenomenon is believed to be due to a "velocity shadow" downstream from the well, i.e., regional flow past the well is faster than flow directly downgradient from the well for some distance (Figure 4).

Assuming that 0.01 cm/s is the terminal velocity, that is, the maximum velocity or the velocity at the downstream end of the velocity shadow, then a drift time of 40 min after emplacement would indicate a total downstream travel distance from the well of 24 cm. Likewise, a drift time of 45 min would indicate a total distance of 27 cm.

The validity of the advective velocity of 0.01 cm/s as determined by type I tests was assured when the velocity was computed from the type II test. A drift time of 38.4 min from emplacement at the well until the pulse label was detected at the embedded electrode pair 23 cm downstream from the well also yielded an advective velocity of 0.01 cm/s. Thus (22) was shown to be valid.

SUMMARY AND CONCLUSIONS

An equation was derived for determining regional advective groundwater velocity from the results of a single-well drift-and-pumpback tracer test which accounts for actual regional advection during pumpback by employing the following steps.

1. A pulse of conservative tracer is emplaced into a well and allowed to drift with the regional gradient for an arbitrary length of time. The actual direction of flow will be along the regional gradient, assuming isotropicity and homogeneity.
2. The pulse is then pumped back to the well at a constant rate and sampled for concentration versus time, and a breakthrough curve is constructed from these values.
3. The center of mass of the breakthrough curve is found, and the total length of time of pumpback is taken to be the time between commencement of pumpback and arrival of the center of mass at the well.
4. The rate of pumpback, drift time, pumpback time, effective porosity, and aquifer thickness are then used in the equation to compute the regional advective groundwater velocity.

It was also found from the laboratory experiments that a velocity shadow exists for some distance downgradient from a well in which advective velocity may be slightly less than at a greater distance downstream or less than flow past the well.

Doubling the travel distance changed the velocity by 30%. This fact should be taken into consideration in any kind of groundwater tracer test involving flow from or past wells, and more study needs to be done on this problem to determine the range of velocity perturbations.

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