

Toward Intelligent Fault Detection in Turbine Blades: Variational Vibration Models of Damaged Pinned-Pinned Beams

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Abstract

Inaccuracies in the modeling assumptions about the distributional characteristics of the monitored signatures have been shown to cause frequent false positives in vehicle monitoring systems for high-risk aerospace applications. To enable the development of robust fault detection methods, this work explores the deterministic as well as variational characteristics of failure signatures. Specifically, we explore the combined impact of crack damage and manufacturing variation on the vibrational characteristics of turbine blades modeled as pinned-pinned beams. The changes in the transverse vibration and associated eigenfrequencies of the beams are considered. Specifically, a complete variational beam vibration model is developed and presented that allows variations in geometry and material properties to be considered, with and without crack damage. To simplify variational simulation, separation of variables is used for fast simulations. This formulation is presented in detail. To establish a baseline of the effect of geometric variations on the system vibrational response, a complete numerical example is presented that includes damaged beams of ideal geometry and damaged beams with geometric variation. It is shown that changes in fault detection monitoring signals caused by geometric variation are small with those caused by damage and impending failure. Also, when combined, the impact of geometric variation and damage appear to be independent.

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Nomenclature

a	crack depth
c	local compliance of damaged beam
$f(x,t)$	external force applied to a beam
$h(x)$	height of a beam's cross section
p	eigenfrequency of beam
t	time
$w(x,t)$	lateral deflection of beam as a function of space and time
x	horizontal coordinate
x_{crack}	location of crack in beam
A	cross sectional area of beam cross section
E	Young's modulus of elasticity
I	area moment of inertia
L	length of beam
β	non-dimensional horizontal crack location
ν	Poisson's ratio
ρ	density

1 Introduction

Vehicle health monitoring systems are becoming a critical safety boosting component in a significant portion of the aerospace industry. By monitoring and detecting impending failure, safety is increased through the replacement of damaged parts and the implementation of control and operation strategies that allow a damaged aerospace vehicle to return safely from an aborted mission. In addition, maintenance costs can be reduced by replacing only parts known to be faulty rather than those assumed to have reached the end of their service life based on theoretical models or statistical trends.

In spite of the clear advantages of, and need, for health monitoring in aerospace applications, robust (error-free) health monitoring and fault detection systems still face significant practical and fundamental challenges. Many of these challenges stem from differences between simulated fault detection signals that are based on theoretical models and monitored signals that are based on empirically-generated laboratory data and field measurements. One significant cause in the discrepancy is the variability in the operating environment that is not well modeled either with theoretical models or empirical models.

1.1 Background and Current Focus

Prior efforts have shown that aerospace systems exhibit significant variability in their operating environment [21, 28, 27]. For example, research in analyzing actual flight data has revealed several sources of variation that are not incorporated into conventional aircraft engine and helicopter transmission monitoring systems [14, 27]. In addition, analyses of test rig data for rotating machinery components have shown that, in addition to operational variations, variations introduced during design, manufacturing, and assembly significantly influence the final response characteristics of such systems [14, 28].

To develop robust fault detection systems, there is a need to improve the basic causal models and associated understanding of the behavior of realistic systems as measured for fault detection. In this context, a realistic system is one with structural and operational variation that deviates from the ideal case. Past

research has revealed causal model discrepancy as a major obstacle for identifying real-time component damage in complex systems [14, 28, 29, 19, 21]. Many sources of inaccuracies exist in the understanding of the data being monitored to detect anomalies, as well as the empirical and theoretical models that are used to make decisions on the state of the data. Empirical evidence from helicopter vibration data shows that for anomaly detection algorithms to work effectively, a more comprehensive framework must be established to represent known sources of design, manufacturing, operational, and random variation. For accurate anomaly detection, it is necessary to understand the probabilistic footprint of all these combined effects on the signals being measured. For model discrepancy evaluation, it is necessary to understand the likely change in functional relationships between forcing function parameters and component responses due to damage states.

To address these issues more thoroughly, this article reports on research that explores a simulation-based approach to help develop deterministic and probabilistic models of healthy and faulty data. The purpose is to provide enhanced signal models that will help with the roadblocks facing anomaly detection algorithms, and hence the vehicle health monitoring systems in which they are implemented. Previous work has been published which describes our attempt to capture the influence of design variations for dynamic systems (a cam-follower system) using probabilistic models [19]. Monte Carlo methods were used to vary characteristics such as the cam surface roughness and the return spring constant. In turn, complete distributions of system vibration were developed. The effect on the vibration response was explored to determine impact of manufacturing variation on fault detection signals [19]. The results showed that small changes in system characteristics cause significant variations in the acceleration signature of the cam-follower system.

Current efforts focus on modeling rare failure signatures to determine their distributional characteristics using simulation-based data. More specifically, the critical question that remained unanswered in the previous study was whether these variations, caused by variations in the manufacturing process, would have an impact on the monitored failure signature when combined with some impending failure in the system and the associated failure signature. Here, such a case is explored further by generating the failure signatures due to cracking, and exploring their variational characteristics when combined with inherent variations in the system and/or its components. The scope of this current article is not to suggest fault monitoring methods, but rather to understand the impact of having failure signatures combined with inherent variations, and establishing the role of simulation in generating the models and assumptions necessary for fault monitoring systems to function as designed. The purpose of the overall work is to provide enhanced signal models that will help with the roadblocks facing anomaly detection algorithms, and hence the vehicle health monitoring systems in which they are implemented. To develop robust fault detection systems, we need to improve the basic causal models and associated understanding of the behavior of realistic systems as measured for fault detection. We start the work from a hypothesis that fault detection is often done prematurely without establishing the true variational characteristics of the signals being monitored, from which decisions are made. Our goal here is to help avoid that pitfall.

1.2 Turbine Blade Crack Damage Detection

Turbine blades represent a critical area for developing variational failure models. Health monitoring of gas turbine engines is of great interest due to the use of the engines as the source of primary propulsion and power in many aerospace and military applications. The unexpected failure of a gas turbine can stop missions and in the worst case result in the death of crew members.

A crack in a blade rotating at high speed can devastate an engine beyond repair. Significant effort has been expended in understanding all aspects of turbine failure. Lifson et al. [18] analyze the practices, bene-

fits, and limitations of several types of vibration monitoring systems used with industrial gas turbines. Different global detection control methods for turbine engines have been investigated by Simani et al. [25, 24] and Diao [9]. To determine the approximate fatigue life of wind turbine blades, a case study has been performed by Sutherland et al. [26]. In an effort to understand the excitation caused by the operating environment, Panovsky has developed the turbine blade vibratory response due to upstream vane distress [22]. An extensive post fracture failure analysis of wind tunnel compressor blades was made by Hampton and Nelson [12], and reveals much about crack initiation and growth but this is far too late for many applications.

Significant effort has also been expended toward developing theoretically based damage models of vibrating turbine blades. Aretakis and Mathioudakis [2] use Wavelet Analysis to detect fouled rotor blades, a twisted rotor blade, and mistuned stator vanes. Ostachowicz develop a simplified vibration model of a turbine or compressor blade using finite element analysis [1]. Sekhar investigates the vibration characteristics of a rotor with two cracks using finite element methods [23]. Wu and Huang explore the vibration of a cracked rotating blade using the method of released energy and weighted residuals [31]. Ganesan et al. [10] use a finite element method to study the dynamic response of high speed rotors with variation in elastic modulus and mass density.

1.3 Simplified Beam Damage Models

In each of these efforts, the turbine blade geometry was simplified to enable model development and solution. Turbine blades have complex geometries and are subject to complex excitation signals. With a long term goal to completely understand the vibrational response of healthy and damaged turbine blades subject to different excitations, we too begin with the simpler problem of understanding the impact of manufacturing variations on healthy and damaged simple beams. As basic knowledge is developed, future work will explore the more complex turbine blade problem. The development of crack models for beams of simple geometry has a richer literature than that of turbine blades. This literature is briefly reviewed here.

A large effort has been made to explore the vibrational characteristics of damaged beams. Hu and Liang [13] use an integrated approach of a massless spring and a continuum damage concept to develop a crack detection technique. Gudmundson [11] develops models that enable the simulation of eigenfrequency changes of structures due to cracks and other geometrical changes. In Gudmundson [11] the models are benchmarked against finite element analysis and experimental results from Wetland [30]. Chondros et al. [7] develop a continuous vibration model for the lateral vibration of a cracked Euler-Bernoulli beam with open edge cracks. Chondros et al. have also explored a crack model for transverse, longitudinal, and other vibrations as well as beams with different end conditions [6, 8, 5]. Mengcheng [20] and Yokoyama [32] use a theoretical line-spring model and Euler-Bernoulli beam theory to approximate the response of cracked beam vibrations. Maiti [3] develops theoretical models for the vibration characteristics of cracked beams with a linearly variable cross section (wedge shaped beams). Using experimental methods, Ju et al. [16] diagnose the fracture damage of structures using modal frequency methods.

Though a large effort has been expended in developing models that can be used to simulate the vibrational characteristics of fractured beams, each of the models assume a beam with ideal geometry and material properties. Little work has been done to determine the combined effects of manufacturing variation and fractures on the vibrational characteristics of structures. In addition, much of the existing work in vibrational damage models relies on a finite element approximation (FEA) of the beam for a numerical solution. The FEA formulation handles cracks well. However, extending FEA methods to variational geometry and Monte Carlo simulation follows less naturally, the main difficulty being the requirement to construct a new finite element mesh for each change in geometry as different geometries are simulated.

In this light, the remainder of the article presents the development of a variational beam model, the inclusion of a crack model in this variational model, a formulation of the problem that allows variational simulation to be performed easily, a complete example, and results and discussion.

2 A Variational Damaged Beam Vibration Model

A critical and observable impact of fractures on beam vibrational response is the difference in the eigenfrequencies for the damaged and undamaged beam. Most theoretical models of cracked beams begin with a Euler-Bernoulli beam and develop special considerations to model crack damage. Here, the Euler-Bernoulli beam is developed with considerations for manufacturing variation. Crack models are added to the resultant model to explore the impacts of both manufacturing variation and structural damage.

A commonly used base model for beam vibration is the two dimensional Euler-Bernoulli Beam equation [15]:

$$\rho(x)A(x)\frac{\partial^2 w(x,t)}{\partial t^2} + \frac{\partial^2}{\partial x^2} \left(E(x)I(x)\frac{\partial^2 w(x,t)}{\partial x^2} \right) = f(x,t). \quad (1)$$

For the ideal case, material properties and geometry do not change with x . In such a case, the solution to Equation 1 can be found analytically using separation of variables. To simulate realistic beams with material and geometric variations, the change in material and geometry as a function of x must be considered. The solution approach that is employed here is to apply the chain rule to develop the complete variational Euler-Bernoulli Beam equation.

Applying the chain rule and collecting terms of Equation (1) gives

$$\begin{aligned} & E(x)I(x)\frac{\partial^4 w(x,t)}{\partial x^4} + \\ & \left(2I(x)\frac{\partial E(x)}{\partial x} + 2E(x)\frac{\partial I(x)}{\partial x} \right) \frac{\partial^3 w(x,t)}{\partial x^3} + \\ & \left(I(x)\frac{\partial^2 E(x)}{\partial x^2} + 2\frac{\partial E(x)}{\partial x}\frac{\partial I(x)}{\partial x} + E(x)\frac{\partial^2 I(x)}{\partial x^2} \right) \frac{\partial^2 w(x,t)}{\partial x^2} = -\rho(x)A(x)\frac{\partial^2 w(x,t)}{\partial t^2}. \end{aligned} \quad (2)$$

Dividing Equation 2 through by $\frac{1}{E(x)I(x)}$ and letting

$$\alpha_1(x) = \left(\frac{2}{E(x)}\frac{\partial E(x)}{\partial x} + \frac{2}{I(x)}\frac{\partial I(x)}{\partial x} \right), \quad (3)$$

$$\alpha_2(x) = \left(\frac{1}{E(x)}\frac{\partial^2 E(x)}{\partial x^2} + \frac{2}{E(x)I(x)}\frac{\partial E(x)}{\partial x}\frac{\partial I(x)}{\partial x} + \frac{1}{I(x)}\frac{\partial^2 I(x)}{\partial x^2} \right), \quad (4)$$

and

$$\alpha_3(x) = -\frac{\rho(x)A(x)}{E(x)I(x)} \quad (5)$$

gives Equation (2) as

$$\frac{\partial^4 w(x,t)}{\partial x^4} + \alpha_1(x)\frac{\partial^3 w(x,t)}{\partial x^3} + \alpha_2(x)\frac{\partial^2 w(x,t)}{\partial x^2} = -\alpha_3(x)\frac{\partial^2 w(x,t)}{\partial t^2}. \quad (6)$$

Equation 6 is a full variational expansion of the Euler-Bernoulli Beam equation. Using Equation 6, realistic variations in material properties and geometry can be taken into account.

2.1 Adding a Damage Model

The primary and critical failure mode for turbine blades and similar vibrating systems is the nucleation of a crack generally caused by fatigue though sometimes unexpected impact. As the crack grows, the structure suffers catastrophic failure. Thus, for this failure monitoring effort, the goal is to detect the existence of cracks in the turbine blade. A crack model is added to the beam to represent damage and indicate impending failure.

Different crack models have been proposed and explored computationally and experimentally. Here, a crack model from Chondros [7] is used. The ability of this crack model to predict changes in beam eigenfrequencies has shown good agreement with other models in the literature as well as experimental results [7].

The philosophy of the basic crack model is seen in Figure 1. Assuming that the crack is only apparent in its neighborhood, the crack is modeled as a local change in beam flexibility. The location of the beam which contains the crack is modeled as a torsion spring with a local compliance different than that of the undamaged beam. The local compliance of the beam is then

$$c = \frac{6\pi (1 - \nu^2) h \Phi \left(\frac{a}{h}\right)}{E(x)I(x)}. \quad (7)$$

Here, Φ is computed from

$$\begin{aligned} \Phi = & 0.6272 \left(\frac{a}{h}\right)^2 - 1.04533 \left(\frac{a}{h}\right)^3 + 4.5948 \left(\frac{a}{h}\right)^4 \\ & - 9.9736 \left(\frac{a}{h}\right)^5 + 20.2948 \left(\frac{a}{h}\right)^6 - 33.0351 \left(\frac{a}{h}\right)^7 \\ & + 47.1063 \left(\frac{a}{h}\right)^8 - 40.7556 \left(\frac{a}{h}\right)^9 + 19.6 \left(\frac{a}{h}\right)^{10}. \end{aligned} \quad (8)$$

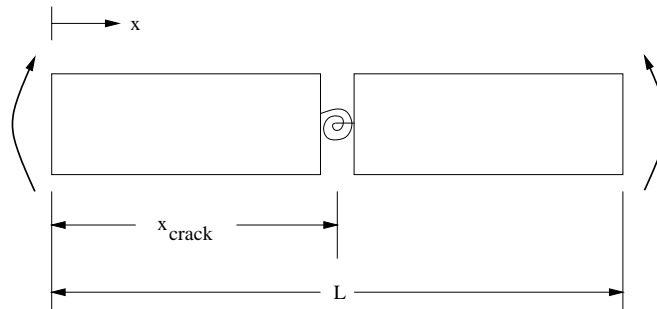


Figure 1: Crack model for the beam.

The compliance of this crack model gives boundary conditions at the location of the crack in the beam. The boundary conditions for the beam at some dimensionless crack location $\beta = x_{crack}/L$ are

$$\begin{aligned} w_l &= w_r \\ \frac{\partial w_r}{\partial x} - \frac{\partial w_l}{\partial x} &= E(x)I(x)c \frac{\partial^2 w_r}{\partial x^2} \\ \frac{\partial^2 w_l}{\partial x^2} &= \frac{\partial^2 w_r}{\partial x^2} \\ \frac{\partial^3 w_l}{\partial x^3} &= \frac{\partial^3 w_r}{\partial x^3}. \end{aligned} \quad (9)$$

Here, the subscripts l and r are used to denote the portions of the beam to the left and the right of the crack.

Combining Equation 6 with Equations 9, 7, and 8 at the location of the crack produce a complete variational damaged beam vibration model. Such a model can be used to simulate the impact of variation combined with damage on the vibrational response of a simple prismatic beam. In addition, by representing the turbine blade geometry as a parametric function of x , the directly models more complex geometries such as turbine blades.

3 Formulation for Numerical Solution

To solve Equation 6 and in turn the complete vibration problem for a damaged beam of realistic geometry, a separation of variables approach is used. Note here that $\alpha_1(x)$, $\alpha_2(x)$, and $\alpha_3(x)$ are functions of x and not t . Also, at each point x , the beam oscillates in time. Taking a solution of the form $w(x, t) = XT$ where X is only a function of x and T is only a function of t Equation (6) becomes

$$\frac{d^4X}{x^4}T + \alpha_1(x)\frac{d^3X}{x^3}T + \alpha_2(x)\frac{d^2X}{x^2}T = -\alpha_3(x)X\frac{d^2T}{t^2}. \quad (10)$$

Separating the terms dependent on only x and only t gives

$$\frac{\frac{d^4X}{x^4} + \alpha_1(x)\frac{d^3X}{x^3} + \alpha_2(x)\frac{d^2X}{x^2}}{\alpha_3(x)X} = -\frac{d^2T}{t^2} = p^2. \quad (11)$$

The solution to the time dependent term in Equation (11) is

$$T = D_1 \sin(pt) + D_2 \cos(pt) \quad (12)$$

With D_1 and D_2 determined by initial conditions. The spatially dependent term in Equation (11) can be written as

$$\frac{d^4X}{x^4} + \alpha_1(x)\frac{d^3X}{x^3} + \alpha_2(x)\frac{d^2X}{x^2} - p^2\alpha_3(x)X = 0. \quad (13)$$

The solution to Equation (13) produces different eigenvalues p with corresponding eigenfunctions X . The eigenvalues of Equation (13) give the natural frequencies or eigenfrequencies at which the beam vibrates with the corresponding mode shapes or eigenmodes of vibration, X . Equation (13) allows the impacts of spatially variable moments of inertia, area, density, and Young's modulus on natural frequency and mode shape to be explored. Equation (13) is a variational vibration beam model.

The derivation of Equation 13 has two important theoretical impacts. First, a critical and observable impact of fractures on beam vibrational response is the difference in the eigenfrequencies for the damaged and undamaged beam. Thus, the compounding factors of beam variation of natural frequency change need to be understood. Second, as the change in eigenfrequencies alone can be used to recognize the existence and severity of a crack, only equation 13 needs to be solved for eigenvalues and eigenmodes. The complete solution to Equation 6 need not be determined.

In addition to the basic theoretical advantages, Equation 13 offers some practical advantages. Material and geometric variations are contained within the α terms. Different parametric models, look up tables, or other models of geometric and material variation can be seeded with random numbers allowing Monte Carlo simulations and the resultant distributions to be determined for eigenmodes and eigenfrequencies. If the model derivation and solution approach was focused on the development of a finite element based solution

rather than the separation of variables approach, each time that the geometry and material were varied, a new material model and geometric decomposition or mesh would need to be developed. To accurately simulate the impacts of small variations in geometry and material properties, the mesh would have to be relatively small. Developing the meshing algorithm to automatically mesh a variational and damaged beam in conjunction with a Monte Carlo simulation is beyond the scope of this article. Making a quantitative comparison between the solution method presented in his article and a finite element analysis based approach remains future work. The solution method presented here is pursued initially as it is estimated that the required remeshing and the total computational time required for a Monte Carlo simulation using an finite element analysis based approach would be larger.

Though Equation 13 offers advantages as a variational beam vibration model, it also presents some challenges. Equation 13 is a fourth order boundary value problem with non-constant coefficients. These coefficients allow for variations in material properties and geometry but do not allow for closed form analytic solutions. Also, as an eigenvalue problem, there are an infinite number of solutions. When reduced to a numerical problem, the high order derivatives combined with the desire to find modes higher than the first produce terms of large differences in magnitude. Nevertheless, the problem is solvable and lends itself naturally to variational simulation. Before presenting the details of the solution method, a crack model is added to the variational beam model.

Returning to the crack model in Equation 9, the crack can be added to the variational beam model in Equation 6 and 13. Because we only need to solve Equation 13 to determine the behavior of a damaged beam, the inclusion of the crack model into Equation 6 is omitted.

The boundary conditions of Equation 9 are true for all time. Thus, the boundary conditions can be separated and written as

$$\begin{aligned} X_l &= X_r, \\ \frac{dX_r}{dx} - \frac{dX_l}{dx} &= E(x)I(x)c \frac{d^2X_r}{dx^2}, \\ \frac{d^2X_l}{dx^2} &= \frac{d^2X_r}{dx^2}, \text{ and} \\ \frac{d^3X_l}{dx^3} &= \frac{d^3X_r}{dx^3}. \end{aligned} \quad (14)$$

for the separated ordinary differential equation of Equation (13).

With the result of Equation 14, the development of an eigenvalue problem that represents the critical portion of the variational beam vibration model that includes damage is complete. Summarizing, the model is Equation 13 combined with Equations 14, 7, and 8. By solving these equations, the eigenfrequencies of a damaged beam with realistic geometric and material variations can be found.

4 Numerical Solution Methodology and Example

A review of the literature on vibrational models of damage structures showed that the primary numerical solution methodology was finite element analysis, though Chaudhari uses the method of Frobenius as well as finite element methods to model vibrations of a wedge shaped beam [4]. No references were found with the formulation as in Equation 13, thus the specific solution method used is presented here in enough detail so that it can be repeated and extended in the future.

In short, the eigenvalue problem of Equation (13) is solved using shooting point methods with a fitting point at the location of the crack. The specific methods are adapted from those described by Keller [17]. The first step is to reduce the fourth order ordinary differential equation to four first order differential equations.

Using the change of variables $v_1(x) = X$, $v_2(x) = v'_1(x)$, $v_3(x) = v'_2(x)$, and $v_4(x) = v'_3(x)$ yields

$$\begin{aligned} v'_1(x) &= v_2(x), \\ v'_2(x) &= v_3(x), \\ v'_3(x) &= v_4(x), \text{ and} \\ v'_4(x) &= -\alpha_1(x)v_4(x) - \alpha_2(x)v_3(x) + p^2\alpha_3v_1(x) \end{aligned} \quad (15)$$

where $'$ represents the derivative with respect to x .

To find unique (within a sign) eigenfunctions of Equation 13, Equation 15 is normalized as

$$\int_0^L v_1(x)^2 + v_2(x)^2 + v_3(x)^2 + v_4(x)^2 dx = 1. \quad (16)$$

This is equivalent to introducing the new variable

$$v'_5(x) = v_1(x)^2 + v_2(x)^2 + v_3(x)^2 + v_4(x)^2 \quad (17)$$

with the boundary conditions

$$\begin{aligned} v_5(0) &= 0 \text{ and} \\ v_5(L) &= 1. \end{aligned} \quad (18)$$

To handle the fact that the eigenvalue is an unknown constant the substitution

$$v'_0(x) = p^2 = 0 \quad (19)$$

is added to Equation 15.

Performing the substitution of variables on the boundary conditions of Equations 14 gives

$$\begin{aligned} v_{1,l}(x_{crack}) &= v_{1,r}(x_{crack}), \\ v_{2,r}(x_{crack}) - v_{2,l}(x_{crack}) &= E(x_{crack})I(x_{crack})c(x_{crack})v_{3,r}(x_{crack}), \\ v_{3,l}(x_{crack}) &= v_{3,r}(x_{crack}), \text{ and} \\ v_{4,l}(x_{crack}) &= v_{4,r}(x_{crack}). \end{aligned} \quad (20)$$

Here, the second subscript refers to approaching the crack location from the left (l) or the right (r) and x_{crack} is the crack location.

Equations 15, 17, and 19 along with boundary conditions from Equations 20, 18, and the physical boundary conditions are solved using a shooting point method.

4.1 Example

To explore the variation model of the cracked beam developed above, a computational example is used. So that a comparison can be made to published results, the specific details of the beam are the same as those used by Hu [13]. Different turbine designs may have blades that appear fixed at one end, pinned-pinned, or with other boundary conditions. For the example, the beam is pinned-pinned. The transverse

vibrations in the direction parallel to the beam's height (see Table 1) are those of interest for determining eigenfrequencies. The specific boundary conditions for the pinned-pinned beam are

$$\begin{aligned} w|_{x=0} &= 0, \\ \frac{\partial^2 w}{\partial x^2}|_{x=0} &= 0, \\ w|_{x=L} &= 0, \text{ and} \\ \frac{\partial^2 w}{\partial x^2}|_{x=L} &= 0. \end{aligned} \quad (21)$$

Because these boundary conditions are true for all time, these boundary conditions become

$$\begin{aligned} X|_{x=0} &= 0, \\ \frac{d^2 X}{dx^2}|_{x=0} &= 0, \\ X|_{x=L} &= 0, \text{ and} \\ \frac{d^2 X}{dx^2}|_{x=L} &= 0. \end{aligned} \quad (22)$$

and can be used with the separated ordinary differential equation of Equation (13). Performing the substitution of variables so that these boundary conditions can be used with the formulation of Equations 15, 17, and 19, these boundary conditions become

$$\begin{aligned} v_1(0) &= 0, \\ v_3(0) &= 0, \\ v_1(L) &= 0, \text{ and} \\ v_3(L) &= 0. \end{aligned} \quad (23)$$

The rest of the beam properties are shown in Table 1.

Young's modulus, E	28MPa
density, ρ	2350kg/m ³
Poisson's ratio, ν	0.33
length, L	10m
height	0.6 ± 0.00562m
width	0.2 ± 0.00562m

Table 1: Physical characteristics of beam used in example simulation.

To simulate the manufacturing variation, only the tolerance on height and width were considered. Young's modulus, density, and Poisson's ratio are assumed to be constant. Also, the parallel edges of the rectangular cross section are assumed to remain parallel. The area and area moment of inertia are varied by allowing a variation in the height and width of the beam. The height and width tolerances are modeled as slowly varying dimensions with nominal values as in Table 1 and variation generated as a random number drawn from a normal distribution with mean 0 and standard deviation 0.00562/3m. For each iteration in the Monte Carlo simulation, a "new" beam was generated with a different amplitude of variation. Two different cracks are explored in the example simulation. The properties of the crack are shown in Table 2. Ten thousand Monte Carlo samples were run for uncracked beams, beams with crack 1, and beams with crack 2.

	Crack 1	Crack 2
location	$\beta = 0.25$	$\beta = 0.45$
depth	$a = 0.04783m$	$a = 0.05917m$

Table 2: Crack properties used in example simulation.

4.2 Results

The results of the simulation of various simulations are below. The basic results of beams with ideal geometry agree with the published results of Hu [13].

For the first case, a beam of ideal geometry was considered. This beam had no variation in geometry. Simulations were run to determine the first two eigenfrequencies of the undamaged beam, a beam damaged with crack 1 and a beam damaged with crack 2. The results are shown in Table 3. Compared to the undamaged beam, natural frequencies for both the first and second modes of vibration are reduced. The reduction in the first eigenfrequency is 0.18% for crack 1 and 0.53% for crack 2. The reduction in the second eigenfrequency is 0.36% for crack 1 and 0.05% for crack 2.

	undamaged beam	crack 1	crack 2
p_1	59.007	58.900	58.693
p_2	236.029	235.176	235.906

Table 3: Summary for vibrational characteristics of a beam of ideal geometry. Frequencies are listed in radians/second.

For the second case, a beam with manufacturing variations was considered. Simulations were run to determine the first two eigenfrequencies of the undamaged beam, a beam damaged with crack 1 and a beam damaged with crack 2. The results are shown in Tables 4 and 5.

When comparing changes in the mean first eigenfrequency of the undamaged and damaged beam the results are the same as for a beam of ideal geometry. As with the ideal beam, the reduction in the first eigenfrequency is 0.18% for crack 1 and 0.54% for crack 2. Also note that the standard deviation is the same for the uncracked and cracked beam. The reduction in the second eigenfrequency is 0.37% for crack 1 and 0.05% for crack 2. There is some reduction in eigenfrequency standard deviation as the beams become cracked.

Histograms of the eigenfrequency distributions are shown in Figures 2 through 7. The shapes of the histograms show little change with the inclusion of manufacturing variation in the model.

	undamaged beam	crack 1	crack 2
μ_{p_1}	59.007	58.900	58.688
σ_{p_1}	0.007	0.007	0.007

Table 4: Summary of results for variational beam vibrational characteristics for the first eigenfrequency. Frequencies are listed in radians / second.

	undamaged beam	crack 1	crack 2
μ_{p_2}	236.041	235.165	235.915
σ_{p_2}	0.023	0.018	0.021

Table 5: Summary of results for variational beam vibrational characteristics for the second eigenfrequency. Frequencies are listed in radians/second.

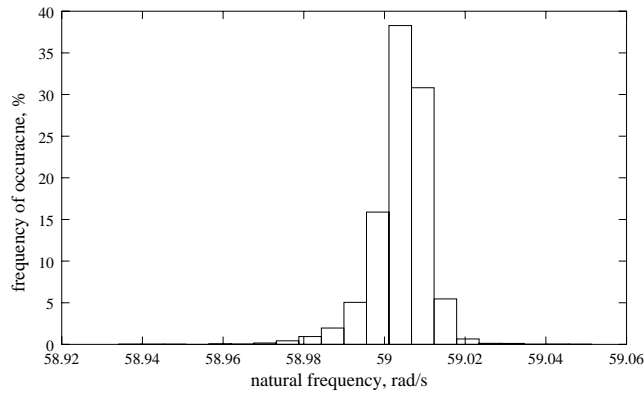


Figure 2: Histogram for the first eigenfrequency of the uncracked beam

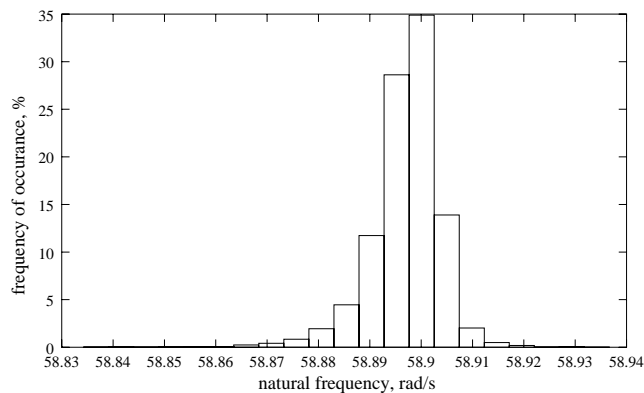


Figure 3: Histogram for the first eigenfrequency of the beam damaged with crack 1.

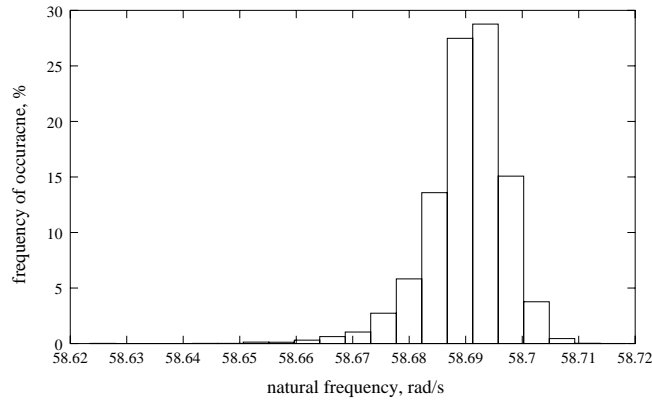


Figure 4: Histogram for the first eigenfrequency of the beam damaged with crack 2.

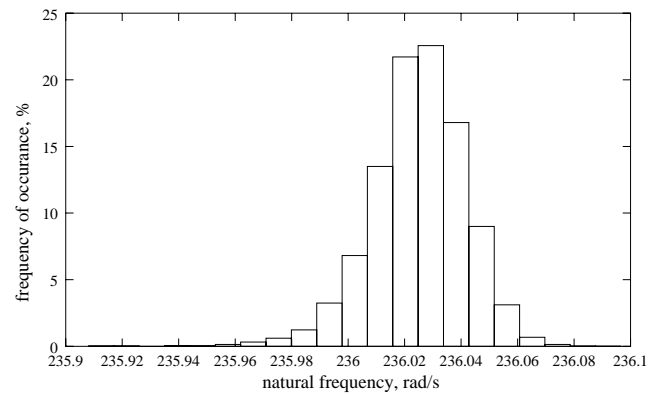


Figure 5: Histogram for the second eigenfrequency of the uncracked beam

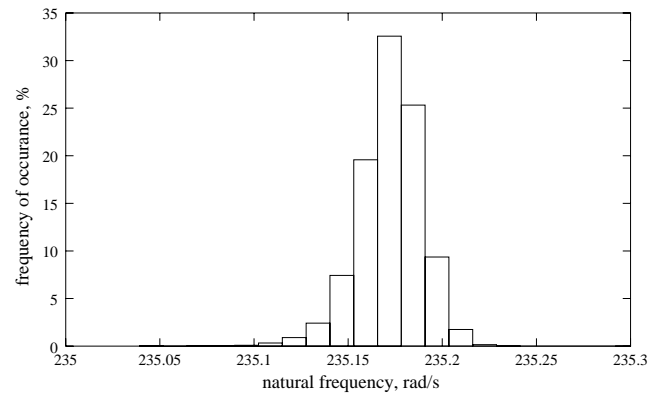


Figure 6: Histogram for the second eigenfrequency of the beam damaged with crack 1.

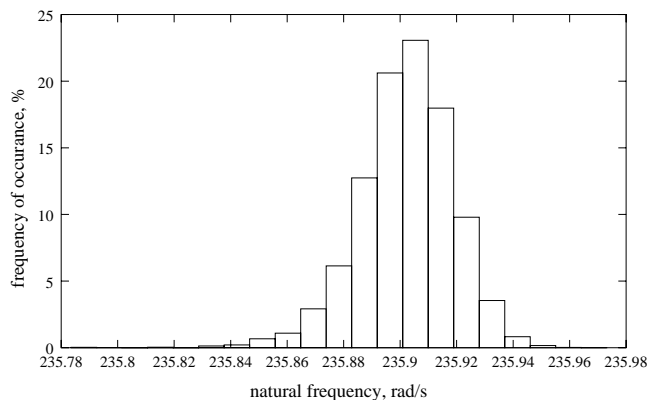


Figure 7: Histogram for the second eigenfrequency of the beam damaged with crack 2.

4.3 Discussion

The simulation results in this paper indicate that the impact of geometric variation on the vibrational characteristics of a pinned-pinned beam is small compared with the impact of a crack in the beam. This is an important result in our overall effort to “baseline” the variational characteristics of the vibrational response from simple beams, and eventually, from turbine blades. For the beams simulated here, the standard deviation on eigenfrequencies are significantly smaller than the change in eigenfrequency caused by the inclusion of damage in the beam. Also, the addition of manufacturing variation to the beam model appears to impact the vibration characteristics in a linear fashion. For the case of beams with both a crack and variational geometry, the mean eigenfrequencies are the same as those for a beam of ideal geometry with the same crack.

Based on the initial simulation results, it is clear that, in the presence of crack damage, the variation of real systems cannot be completely explained by geometric variations resulting from specific manufacturing processes. In the work presented here, only geometric variations in beam geometry are considered. Realistic variations in material stiffness and density were not considered. Note that a crack results in a local reduction in beam stiffness. Thus, variations in material stiffness may have a more complex interaction with the beam damage than the simple relationship between beam damage and geometric variation seen here. The basic variation model of Equation 6 allows variations in all the physically relevant beam properties. Future work will explore the individual and combined impact of geometric and material properties. Also, higher mode eigenfrequencies will be simulated to determine if vibrational trends are the same for low and high order eigenfrequencies.

5 Summary and Future Work

This article presents the modeling methods being developed toward a long term goal of generating a complete understanding of turbine blade behavior with respect to health monitoring. Specifically, the impact of operational variations on the vibrational behavior of healthy and damaged turbine blades modeled as pinned-pinned beams is explored. A complete variational vibration model for a damaged beam is developed and is presented as the means to study the variational characteristics of beam vibrational response. In addition, a crack failure model is developed and combined with the variational model. The proposed solution method

uses separation of variables and direct integration of the spatial portion to determine the vibrational characteristics of the damaged and undamaged beam. The specific formulation of the problem to allow numerical solution is presented.

To begin the process of baselining the impact of various operational variation sources, geometric variations are studied in this article, and combined with a crack failure. Based on the results of the work presented here, the impact of geometric variations on the vibrational characteristics of a pinned-pinned beam is small compared with the impact of a crack in the beam. This result provides an initial understanding of the impact of manufacturing variations on the vibrational response of turbine blades when monitoring the system response for impending failures. Critical future work includes the inclusion of additional variations beyond geometric, the extension of the beam model to more closely resemble a practical turbine blade, and validating the results of this article experimentally. With experimental validation, the models can be benchmarked and error assessed.

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