Signal Analysis Using Karhunen-Loève Transformation: Application to Hydrodynamic Forces [1]

Irem Y. Tumer
Research Scientist
Computational Sciences
NASA Ames Research Center
Moffett Field, California, 94035
(650) 604 2976
itumer@ptolemy.arc.nasa.gov

Raul G. Longoria
Associate Professor
Mechanical Engineering
The University of Texas
Austin, Texas, 78712-1063
(512) 471 0530
r.longoria@mail.utexas.edu

Kristin L. Wood
Associate Professor
Mechanical Engineering
The University of Texas
Austin, Texas, 78712-1063
(512) 471 0095
wood@mail.utexas.edu

Abstract

This paper describes a signal analysis method using the Karhunen-Loève (KL) transform, and applies the method to the analysis of hydrodynamic force data collected from controlled laboratory experiments. To demonstrate feasibility, the study focuses on inline forces induced on fixed cylinders exposed to oscillatory flows, with increasing Keulegan-Carpenter numbers. This preliminary example illustrates how this force makes a transition from the inertia-dominated region at low Keulegan-Carpenter numbers up into higher Keulegan-Carpenter numbers where the nonlinear effects of the flow-structure interaction can be observed through the increased influence of clearly defined modes.

1 Introduction

This paper introduces a signal analysis technique and applies it to the analysis of the nature of inline forces exerted on rigidly-mounted cylinders. The signal analysis approach based on the Karhunen-Loève transform was extended and used by the first author for identifying statistically and physically-significant trends in measured signals [8, 9, 10]. In this paper, we demonstrate how the inline force signals can be analyzed by this technique, and how critical modes can be extracted without a reference to a particular model.

Because it is known that flow-induced inline forces can be decomposed in a way consistent with a KL-based analysis, we choose here to focus on these forces only. The signals analyzed are related to inline forces induced on slender cylinders and exposed to planar oscillatory flows in a laboratory tunnel [3]. These flow conditions have been traditionally used for the purpose of studying inline and transverse forces in an ideal setting. This setup allows fundamental models to be tested and evaluated before application in real ocean environments. In this paper, the intent is to use laboratory-generated data to demonstrate the utility of the KL-based analysis technique.

Further experiments are needed to demonstrate that the KL-based method has the potential of being used as a pre-conditioning tool, or for identifying the critical modes in inline force signals. By understanding how the modes depend

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on the incident flow field, it is suggested that a method might exist for performing long-term monitoring of forces in mode realistic situations. However, that is not the main point of this paper. Here we seek only to demonstrate how a KL-based analysis method offers some unique tools for analyzing force signals.

1.1 Inline Hydrodynamic Forces on Cylinders

The study of hydrodynamic forces exerted on circular cylinders have helped form the basis for understanding complex fluid-structure interactions that are present in real-world applications [4, 11]. Building an understanding of the dependence of the inline force, for example, on the Keulegan-Carpenter number (*KC*), Reynolds number (*Re*), and in some cases, surface roughness (k/D), has relied on idealized experiments. For example, using results from these experiments, it is possible to show that the *KC* number, which provides a measure of the displacement of the fluid across the cylinder, can be used to classify the forces into distinct regions (e.g., inertia/drag regime, etc.). The work of Williamson [11] has illustrated how certain regions of *KC* are associated with specific flow phenomena governed by shed and interacting vortices about the cylinder body. The shedding and convection of vorticity has been shown to govern the strongly nonlinear character of forces induced on slender cylinders exposed to oscillatory flows [5].

1.2 Experimental Data

In the present study, data collected from the experimental setup shown in Figure 1 is used to demonstrate an alternative approach for analyzing these hydrodynamic force signals, using the Karhunen-Loève (KL) transformation. The data

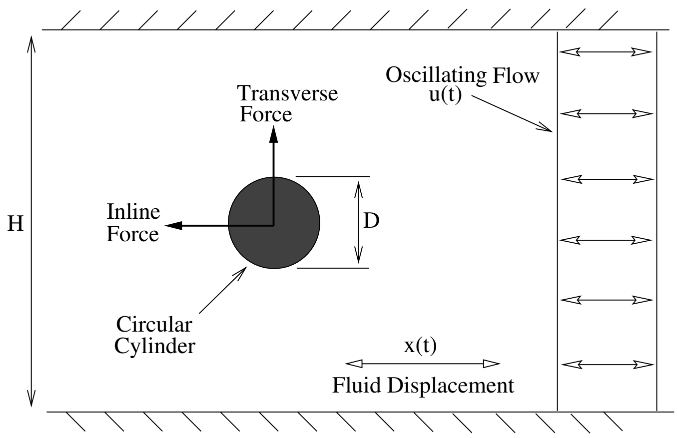


Figure 1: Laboratory Model for Forces on Circular Cylinders in Planar Flow.

collected corresponds to inline forces measured on fixed cylinders exposed to a planar oscillatory flow. The laboratory experiments are used to collect force and fluid motion data over a time interval when the amplitude (X_m) and period of oscillation are held fixed. Increasing the amplitude of the fluid motion corresponds to increasing the *KC* and *Re* numbers. Experiments in which the period of oscillation is fixed are also parameterized by the frequency parameter, $\beta = D^2/\nu T$, where ν is the kinematic viscosity of the fluid [4].

2 Karhunen-Loève Transform

The Karhunen-Loève (KL) transform is used to decompose measured signals into uncorrelated empirical basis functions. This optimal set of basis functions represents the main modes in the data, based on the variability information. The data is projected onto this optimal set of basis functions, as opposed to predetermined basis functions such as sines and cosines, as in the case of the standard Fourier transform. This property allows for the detection of nonstationary changes in the signals. Nonstationarities are time-varying changes which are averaged out using the standard Fourier spectrum methods. Furthermore, nonstationary trends often appear as a low-frequency component using the Fourier spectrum, as well as altering the true values of the magnitudes. Using the KL transform, the significant components are identified correctly, regardless of their nature. A comparison example of the KL-based information and the FT-based information is provided in [9].

2.1 KL in Signal Processing

The KL transform is used in a variety of signal processing applications [1, 2, 6, 7, 12], mainly as a means of detecting "dominant" characteristics in a set of signals. A thorough literature review is provided in previous publications by the first author [9, 10]. For example, Sirovich et al. apply the KL transform to pattern recognition, attacking the general problem of characterizing, identifying, and distinguishing individual patterns drawn from a well-defined class of patterns [6]. The patterns are pictures of faces of people, taken from a random sample of males. They show that a compressed version of the original faces can be reconstructed with reduced dimensionality to generate recognizable faces [6]. In addition, Ball et al. use the KL transform in the analysis of low-Reynolds number turbulent channel flow. Snapshots of the velocity field $u(x,t)$ are taken and the entire ensemble of velocity measurements are decomposed into spatial modes (basis eigenvectors) and time-dependent amplitude coefficients [1].

2.2 Technique for Signal Decomposition

The KL decomposition of the input data is performed as follows: (1) the covariance matrix is computed first using the M zero-mean input vectors: $\hat{\mathbf{S}} = \frac{1}{M}\sum_{j=1}^{M}\mathbf{X}_j\mathbf{X}_j^T = \frac{1}{M}\mathbf{X}^T\mathbf{X}$; (2) the eigenvectors $\blacksquare_i$ and eigenvalues λ_i of the covariance matrix are computed using the following matrix relationship: $\hat{\mathbf{S}}\Phi = \Phi\Lambda$; (3) the coefficient vectors $\mathbf{Y}_i$ are computed using the matrix transformation: $\mathbf{Y} = \Phi^T\mathbf{X}$. The eigenvalues and eigenvectors are listed in descending order and their energies compared to determine each component's significance. Note that the coefficients are computed as a projection of the signal onto the new domain spanned by the KL eigenvectors. For each significant eigenvector, there are as many coefficients as there are input vectors. Low-dimensional estimates of the original signals are then computed using the m significant modes and corresponding coefficients, as follows: $\mathbf{X_j} = \mathbf{X}_{ave} + \sum_{i=1}^{m} y_{ij}\blacksquare_i$.

2.3 Identifying Critical Modes and Changes: An Example

This section presents a set of numerically-generated data that simulates multicomponent signals with nonstationary changes. Analysis of signals for critical trends sometimes requires qualitative judgment, made using a set of data that may or may not contain statistically-significant information. One benefit of using the KL approach is that the information is ordered by energy content (eigenvalues) which indicate the modes of importance versus noise components. The KL decomposition acts as a filter in extracting the modes (eigenvectors) of significance, and provides the changes in these modes with respect to time, in the form of weights for each mode (coefficient vectors).

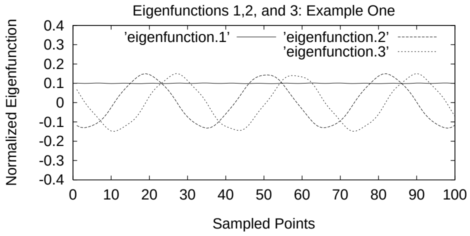
In this following example, snapshots of a multicomponent signal composed of two sinusoidal modes and a linear mode are collected and used as inputs to the KL transform. The multicomponent signal is assumed to be a time-varying signal of the following form: $x(t) = Asin(\omega_1 t) + Bsin(\omega_2 t) + Ct$, where A and B are the amplitudes of the sinusoidal functions with frequencies ω_1 and ω_2, respectively, and where C is the amplitude of the linear function represented by the third component [9, 10].

To perform the KL decomposition, snapshots of these multicomponent signals are collected to compute the covariance matrix, whose resulting eigenvectors are shown in Figures 2(a) and 2(b). These eigenvectors successfully extract the main components of the input snapshots, namely, a linear component, a low-frequency sinusoidal component, and a high-frequency sinusoidal component. Notice that the sinusoidal modes are represented by a "pair" of eigenvectors instead of a single eigenvector. This is due to the fact that additional phase information is captured by a second sinusoidal eigenvector. Changes in the amplitude structure can be monitored by means of either one of the eigenvectors in the pair.

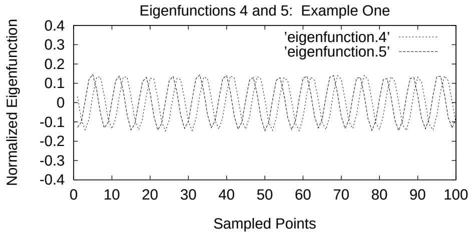
Changes in these fundamental eigenvectors are reflected by means of the corresponding coefficient vectors. For example, possible changes in the linear mode, extracted as the first eigenvector in Figure 2(a), are shown in Figures 2(c) and 2(d). The first change corresponds to a case where a linear trend is introduced in the data; the coefficient vector indicates the relative severity of this nonstationary change in the signal characteristics. Another example is a nonstationary change in the offset level of the data, which is indicated by an offset change in Figure 2(d), where the exact location of the change and its severity are determined. Changes in the sinusoidal components are in turn reflected by the coefficient vectors corresponding to the sinusoidal eigenvectors extracted from the KL decomposition. Specifically, a change in the magnitude of the low-frequency sinusoidal component is introduced at the $20th$ snapshot, and corrected at the $40th$ snapshot. This change is indicated exactly by the corresponding coefficient vector, as shown in Figure 2(e). Also notice that no changes are introduced to the high-frequency component in this case, which is reflected by the corresponding coefficient vector shown in Figure 2(f).

3 Analysis and Monitoring of Inline Force Signals

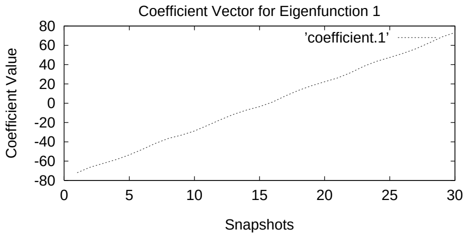
Following the example analysis given in the previous section, we examine experimental inline force signals to isolate significant modes and their changes over time.



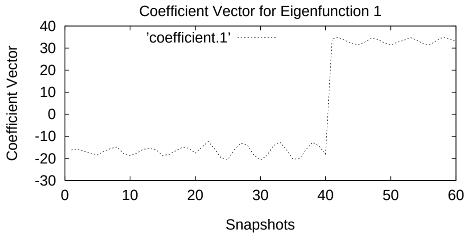
(a) Linear and Low-Frequency Sinusoidal Modes.



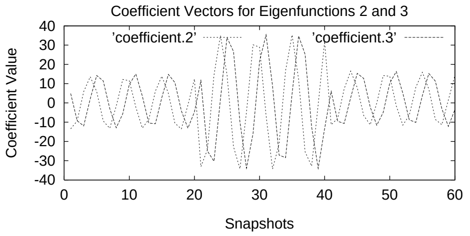
(b) High-Frequency Sinusoidal Mode.



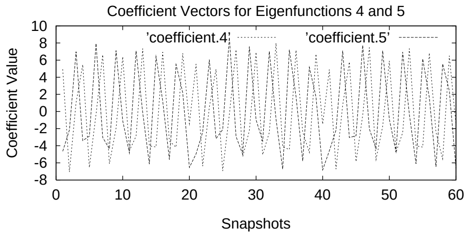
(c) Example Coefficient for Linear Mode: Trend Change.



(d) Example Coefficient for Linear Mode: Offset Change.



(e) Coefficient for Low-Frequency Sinusoidal Mode: Sudden Magnitude Change.

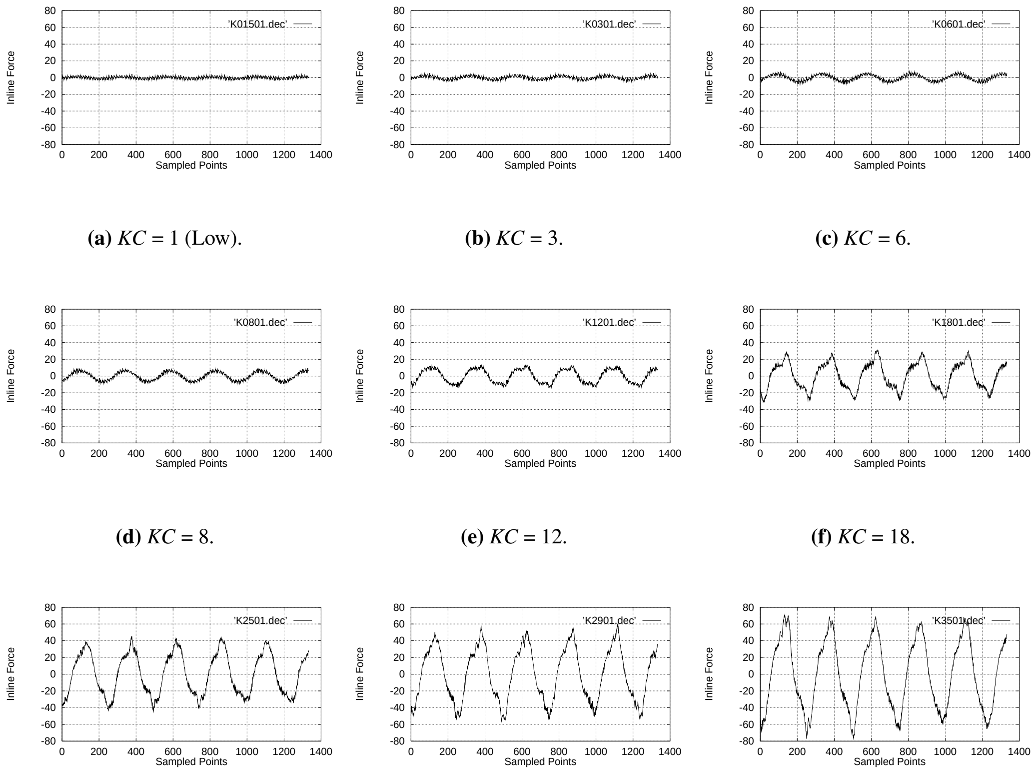


(f) Example Coefficient for High-Frequency Sinusoidal Mode: No Magnitude Change.

Figure 2: Examples of KL Decomposition using Numerical Signals.

3.1 Input Data

In order to capture the change in the force signals for increasing KC numbers, inline force data are arranged by increasing Keulegan-Carpenter (KC) number. Each data sequence has $N = 4000$ sampled points. For each KC number, five measurements (signal realizations) are collected. The data are split into three sequences, resulting in $N = 1200$ sampled points for each data sequence. This results in $M = 15$ data sequences for each KC number. Furthermore, the data is adjusted so that the starting points are different (random phase). In this manner, variability in the input data is assured so that the rank of the covariance matrix is significant [10]. The sampling interval for this data is $t_s = 400\ \mu s$, implying a sampling frequency of $f_s = 2500\ points/sec$. Examples of input data for increasing KC numbers are shown in Figure 3.



(a) KC = 1 (Low). **(b)** KC = 3. **(c)** KC = 6.

(d) KC = 8. **(e)** KC = 12. **(f)** KC = 18.

(g) KC = 25. **(h)** KC = 29. **(i)** KC = 35 (High).

Figure 3: Inline Force Data for Increasing Keulegan-Carpenter Number (rounded to the nearest integer).

3.2 Monitoring Inline Forces for Increasing Keulegan-Carpenter Numbers

In order to observe the changes in the inline force data with respect to a change in the KC number, $M = 240$ data sequences are used, corresponding to 16 different KC numbers ranging from $KC = 1.5$ to $KC = 27$. The results obtained here use the information from all KC values to extract dominant modes in the inline force.

3.2.1 Eigenvalues

The eigenvalues are presented in Table 1. As is observed from the normalized values, the first four eigenvalues are distinct and dominant, whereas the remaining eigenvalues are similar in magnitude. Note that $M = 240$ input vectors result in $M = 240$ eigenvalues. Among these, the first $m = 20$ eigenvalues total 99.4 % of the total energy, as shown in Table 1.

Table 1: KL Eigenvalues for Increasing Keulegan-Carpenter Number Data.

index	Eigenvalue	Normalized Eigenvalue	Cumulative Energy
1	101789.914062	0.4801	0.4801
2	97352.257812	0.4592	0.9393
3	4459.701172	0.0210	0.9603
4	4036.065186	0.0190	0.9793
5	353.807495	0.0017	0.9810
6	344.137146	0.0016	0.9826
7	321.727264	0.0015	0.9841
8	274.834961	0.0013	0.9854
9	266.313843	0.0013	0.9867
10	245.370132	0.0012	0.9879
11	226.065231	0.0011	0.9889
12	197.003265	0.0009	0.9898
13	167.830078	0.0008	0.9906
14	140.000854	0.0007	0.9913
15	131.660675	0.0006	0.9919
16	121.481461	0.0006	0.9925
17	109.443764	0.0005	0.9930
18	99.683731	0.0005	0.9935
19	94.961273	0.0004	0.9939
20	87.422913	0.0004	0.9943

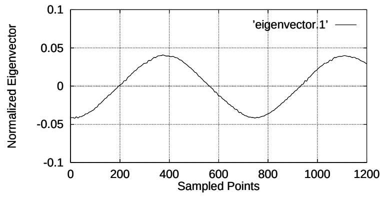
3.2.2 Eigenvectors and Coefficient Vectors

The eigenvectors and the corresponding coefficient vectors for this case are presented in Figure 4. The eigenvectors indicate the average dominant pattern in all of the input data, whereas the coefficient vectors represent the change in the magnitude of each eigenvector over the different inputs, hence for increasing KC number. Recall that for each KC number, there are $M = 15$ input data; hence, the first $M = 15$ inputs correspond to $KC = 1.5$, etc.

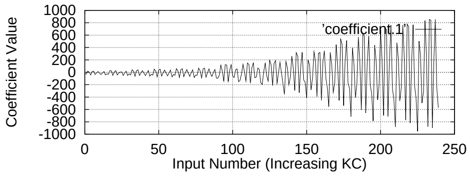
The first four eigenvectors correspond to the dominant sinusoidal patterns in the data. Note that each sinusoidal mode results in a pair of eigenvectors, with the second one representing the random phase introduced due to sampling [10]. Hence the first mode, shown in Figure 4(a) corresponds to a dominant low-frequency mode in the data at a frequency of $f = 3.4\ cycles/sec$. The corresponding coefficient vector, shown in Figure 4(b) indicates a gradual increase in the magnitude of this mode as the KC number increases.

The next mode, shown in Figure 4(c) corresponds to a second sinusoidal mode with a frequency of $f = 10.2\ cycles/sec$. The corresponding coefficient vector, shown in Figure 4(d) indicates that this mode is nonexistent for the low KC number data.

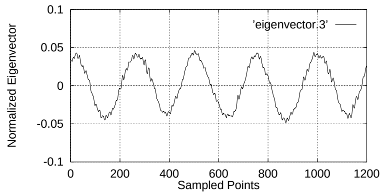
The second mode appears at around $KC = 8$, which is when the inline force becomes highly complex [3]. Also notice from the scale that the magnitude of the second sinusoidal mode is less significant than the first sinusoidal mode for the higher KC number data.



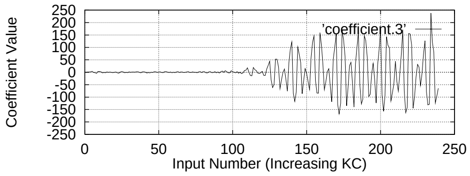
(a) Eigenvector 1.



(b) Coefficient Vector 1.



(c) Eigenvector 3.



(d) Coefficient Vector 3.

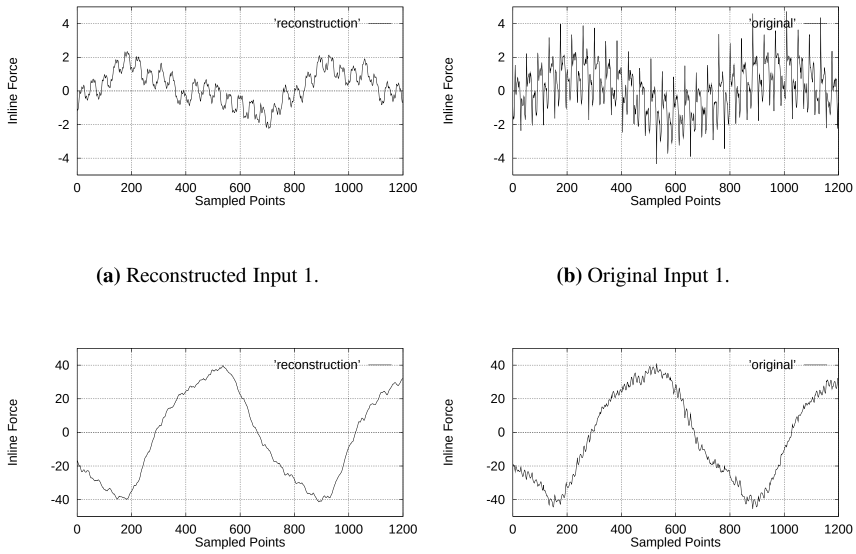
Figure 4: KL Eigenvectors and Coefficient Vectors: Increasing Keulegan-Carpenter Number.

3.2.3 Reconstruction and Noise Extraction

Estimates of the original input vectors are reconstructed using the dominant eigenvectors, as shown in Figure 5. Reconstruction provides a means to eliminate the less-significant (noise) components from the data, which is an important benefit of the KL method. The reconstruction of a low KC number data sequence indicates that the reconstructed estimate for data with $KC = 1.5$ (Figure 5(a)) lacks some high-frequency modes when compared to the original input (Figure 5(b)). However, most of the high-frequency modes are actually electrical noise. The noise levels are, of course, less severe for high KC conditions. The reconstruction estimate for data with $KC = 27$ (Figure 5(c)) compares fairly well with the original input (Figure 5(d)).

3.2.4 Discussion

The KL decomposition of inline force data provides insight on the dominant fundamental modes induced by the flow-structure interaction. Specifically, an increase in the magnitude of the dominant mode is indicated by the first eigenvector and coefficient vector pair. This fundamental mode and its time-varying change can be monitored as the KC value increases. The results show that this component increases in significance as the KC number increases, as might



(a) Reconstructed Input 1.

(b) Original Input 1.

(c) Reconstructed Input 240.

(d) Original Input 240.

Figure 5: KL Reconstruction Using Six Eigenvectors vs. Original: Increasing Keulegan-Carpenter Number.

be expected. Despite the fact that *KC* varies widely, the KL method is able to extract a dominant mode that is largely harmonic.

An additional frequency component is detected through a second pair of eigenvectors. This mode has a fundamental frequency that is clearly three times that of the fluid motion frequency of oscillation. The corresponding coefficient vector indicates that this mode is insignificant for low *KC* values, and that it becomes significant as *KC* approaches a value of about 8, increasing significantly beyond that point. This result illustrates how the eigenvector and coefficient vector can be used together to monitor the appearance of such nonlinear effects, or to develop a warning scheme based on force data. The eigenvalues shown in Table 1 indicate the presence of additional modes of lower significance, and a detailed study of either fixed-cylinder or flexibly-mounted cylinder data might reveal how certain modes change for different flow conditions.

The observed mode must be interpreted in light of the fact that it is composed of data over a wide range of *KC* values, and for one data set with $\beta = Re/KC = 1024$. A more extensive study would examine how these modes change as β is increased (for example, since this would indicate increased Reynolds number). For a given β, additional insight can be gained by focusing on a certain region of *KC*.

4 Conclusions and Future Work

This paper presents preliminary results from the analysis of experimental data collected from laboratory studies of inline forces induced on fixed circular cylinders by sinusoidal oscillatory (planar) flow. The analysis of the data using a Karhunen-Loève-based method has uncovered the dominant modes in the force data. Using this method, findings and expectations based on empirical models are confirmed for one specific case. The method is applied to inline force data analysis in this paper to show its potential utility in the field of offshore force analysis. Much remains to be done in examining the data over a wider range of flow conditions.

The analysis example used in this paper shows that the inline forces are primarily composed of two dominant components, which are isolated as fundamental eigenvectors. In addition, noise components in the experimental measurement of inline forces are also isolated as lower-eigenvalue eigenvectors. These components can be discarded when analyzing inline force data; however, their structure (i.e., frequency) and time dependence can provide invaluable insight when, for example, designing the experimental configuration or monitoring an instrumented offshore system.

The two dominant components of the inline force data are analyzed further to observe the changes as the dimensionless Keulegan-Carpenter number is varied. The results validate the empirical observations that a second component appears as KC approaches a value of 8. In addition, a constant change in significance of the modes is observed in the dominant low-frequency mode as the KC number increases. Such accurate information about the changes in the inline force data can potentially provide important models to complement the empirical observations in the field.

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