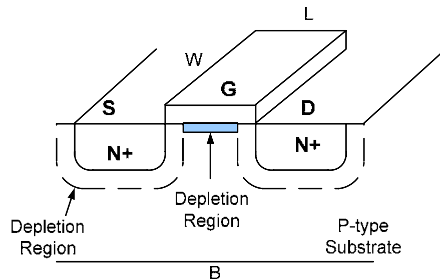
MOSFET Operation ($V_{GS} < V_T$)

$V_{DS} = 0$, $V_{GS} > 0$ but $< V_T$

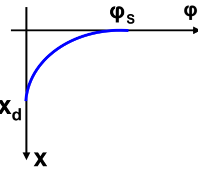
$V_{GS} > 0 \Rightarrow +$ charge on gate
- charge in substrate

Electric field repels holes leaving ionized acceptor atoms $\Rightarrow$ depletion region



$$\nabla^2\varphi = -\frac{\rho}{\varepsilon} = \frac{qN_A}{\varepsilon}$$

N_A = density of acceptors in p-type substrate
q = electron charge (1.6×10^{-19} C)
ε = permittivity of Si (1.04×10^{-12} F/cm)



Integrating in x direction $\varphi_S = \frac{qN_A}{\varepsilon}\frac{x_d^2}{2} \Rightarrow x_d = \sqrt{\frac{2\varepsilon\varphi_S}{qN_A}}$

$$Q_{dep} = qN_A x_d = \sqrt{2qN_A\varepsilon\varphi_S}$$

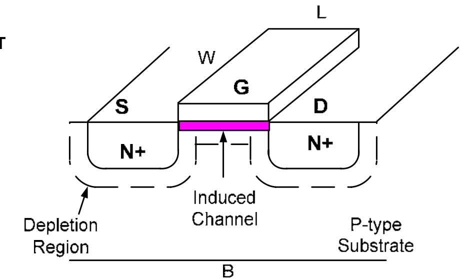
x_d = depletion layer depth
Q_{dep} = depletion region charge

MOSFET Operation ($V_{GS} \approx V_T$)

$V_{DS} = 0$, V_{GS} increasing and $\approx V_T$

Depletion region becomes deeper; almost as deep as that under the N+ regions

$\Rightarrow$ Surface is as attractive for electrons as N+ regions
$\Rightarrow$ Electrons can enter from the N+ regions



The induced n-region under the gate is of n-type
$\Rightarrow$ Surface is inverted and the electron layer is called the inversion layer

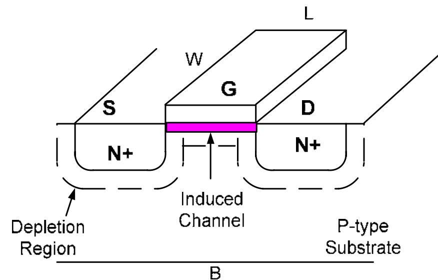
The inversion layer forms a channel for current flow when $V_{DS} > 0$

MOSFET Operation ($V_{GS} > V_T$)

$V_{DS} = 0$, $V_{GS} > V_T$

Once surface is inverted electrons can easily enter from the N+ regions

$\Rightarrow$ All additional gate charge is balanced by electrons in the inversion layer
$\Rightarrow$ Depletion layer depth and surface potential are fixed



$$\varphi_S(\text{strong inversion}) = 2V_{th}\ln\frac{N_A}{n_i} = 2\varphi_F = \varphi$$

V_{th} = thermal voltage ≈ 26 mV @ 300°K
n_i = intrinsic carrier conc. (undoped Si) (1.5×10^{10} cm^{-3})
ϕ_F = Fermi level

The gate voltage required to produce the inversion layer is called the threshold voltage V_T

The Threshold Voltage V_T

$$x_d = \sqrt{\frac{2\varepsilon\varphi_S}{qN_A}}$$

x_d = depletion layer depth
Q_{dep} = depletion region charge

$$Q_{dep} = \sqrt{2qN_A\varepsilon(2\varphi_F + V_{SB})}$$

$$V_T = 2\varphi_F + \frac{Q_{dep}}{C_{ox}} + \varphi_{ms} - \frac{Q_{SS}}{C_{ox}}$$

ϕ_{ms} = work function difference between gate material and Si
Q_{SS} = surface charge at Si – SiO_2 interface (interface charge)
C_{ox} = capacitance per unit area of the gate oxide

$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.45\text{fF}/\mu\text{m}^2$ for t_{ox} = 10 nm = oxide thickness

$$V_T = \underbrace{2\varphi_F + \varphi_{ms} - \frac{Q_{SS}}{C_{ox}} + \frac{Q_{dep0}}{C_{ox}}}_{V_{T0}} + \frac{Q_{dep} - Q_{dep0}}{C_{ox}}$$

$Q_{dep0} = Q_{dep}(V_{SB} = 0)$

$$\frac{Q_{dep} - Q_{dep0}}{C_{ox}} = \underbrace{\frac{\sqrt{2qN_A\varepsilon}}{C_{ox}}}_{\gamma}\left(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F}\right)$$

$2\varphi_F$: φ

V_T Calculation Example

$\phi_{ms} = -0.6$ V, $Q_{SS} = q \times 10^{11}/cm^2$, $N_A = 10^{16}/cm^3$, $t_{ox} = 10$ nm

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 3.45 fF/\mu m^2 = 3.45\times10^{-7} F/cm^2 \qquad \gamma = \frac{\sqrt{2qN_A\varepsilon}}{C_{ox}} = 0.17V^{1/2}$$

$$V_{TO} = 2\varphi_F + \varphi_{ms} - \frac{Q_{SS}}{C_{ox}} + \frac{Q_{dep0}}{C_{ox}}$$

$$\frac{Q_{SS}}{C_{ox}} = \frac{1.6\times10^{-19}\times10^{11}}{3.45\times10^{-7}} = 0.046\,V$$

$$2\varphi_F = 2V_{th}\ln\frac{N_A}{n_i} = 2\times26\text{ mV}\times\ln\frac{10^{15}}{1.5\times10^{10}} = 0.58\,V$$

$$\frac{Q_{dep0}}{C_{ox}} = \frac{\sqrt{2qN_A\varepsilon 2\varphi_F}}{C_{ox}} = \gamma\sqrt{2\varphi_F} = 0.13\,V$$

$$V_{TO} = -0.6 - 0.046 + 0.58 + 0.13 = 0.06\,V$$

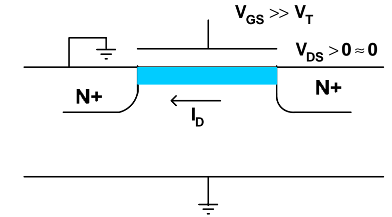
$\Rightarrow$ V_T adjust implant (increase surface doping)

Drain Current for V_{DS} > 0 but Small

$V_{DS} > 0$, $V_{GS} >> V_T$

The charge induced in the channel is

$$Q_i = C_{ox}WL(V_{GS} - V_T)$$



$$R_{channel} = \rho\frac{L}{A} = \frac{1}{\sigma}\frac{L}{A} = \frac{1}{\sigma}\frac{L}{Wt}$$

$$\sigma = \mu_n\frac{Q_i}{WLt} = \mu_n\frac{C_{ox}(V_{GS}-V_T)}{t}$$

$$R_{channel} = \frac{1}{\mu_n\frac{C_{ox}(V_{GS}-V_T)}{t}}\frac{L}{Wt} = \frac{1}{\mu_n C_{ox}(V_{GS}-V_T)}\frac{L}{W}$$

$$I_D = \frac{V_{DS}}{R_{channel}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T)V_{DS} = k'\frac{W}{L}(V_{GS}-V_T)V_{DS}$$

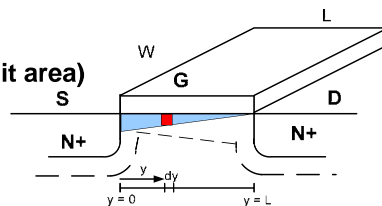
The inversion layer forms a channel for current flow when V_{DS} > 0

Drain Current in Nonsaturation

$V_{DS} > 0$, $V_{GS} >> V_T$, $V_{DS} < V_{GS} - V_T$

Charge induced in the channel (per unit area)

$$Q_i = C_{ox}(V_{GS} - V_T - V(y))$$



$$dR = \frac{1}{W\mu_n Q_i(y)}dy$$

$$dV = I_D dR = \frac{I_D}{W\mu_n Q_i(y)}dy = \frac{I_D}{W\mu_n C_{ox}(V_{GS} - V_T - V(y))}dy$$

$$I_D dy = W\mu_n C_{ox}(V_{GS} - V_T - V(y))dV$$

$$\int_0^L I_D dy = \int_0^{V_{DS}} W\mu_n C_{ox}(V_{GS} - V_T - V(y))dV$$

Integrate from 0 to L

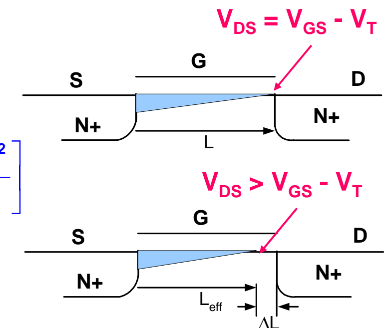
$$I_D = \mu_n C_{ox}\frac{W}{L}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$$

k' ($= \mu_n C_{ox}$)

Drain Current in Saturation

$V_{GS} >> V_T$, $V_{DS} \geq V_{GS} - V_T$

For $V_{DS} = V_{GS} - V_T$



$$I_D = k'\frac{W}{L}\left[(V_{GS}-V_T)(V_{GS}-V_T) - \frac{(V_{GS}-V_T)^2}{2}\right]$$

$$I_D = \frac{k'}{2}\frac{W}{L}(V_{GS}-V_T)^2$$

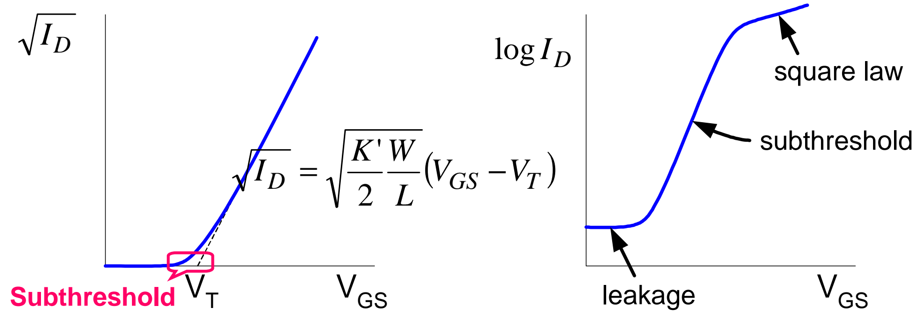
$$Q_i(L) = C_{ox}WL(V_{GS} - V_T - V_{DS}) = 0$$

i.e., there is no mobile charge, which can not be true since current flows $\Rightarrow$ only an approximate relation

$$I_D = \frac{K'}{2}\frac{W}{L_{eff}}(V_{GS}-V_T)^2 = \frac{K'}{2}\frac{W}{L-\Delta L}(V_{GS}-V_T)^2 \cong \frac{K'}{2}\frac{W}{L}(V_{GS}-V_T)^2(1+\lambda V_{DS})$$

where $\frac{1}{L-\Delta L} = \frac{1}{L\left(1-\frac{\Delta L}{L}\right)} \cong \frac{1}{L}\left(1+\frac{\Delta L}{L}\right) \cong \frac{1}{L}(1+\lambda V_{DS})$ is a first order model

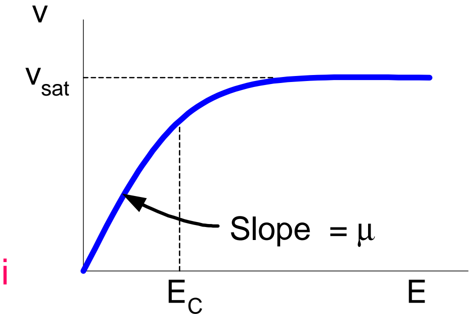
Subthreshold Conduction



$$I_D = K_x \frac{W}{L} e^{V_{GS}/nV_{th}} \left(1 - e^{-V_{DS}/V_{th}}\right)$$

Velocity Saturation

$$\mathbf{v} = \frac{\mu \mathbf{E}}{1 + \mathbf{E}/\mathbf{E_C}}$$

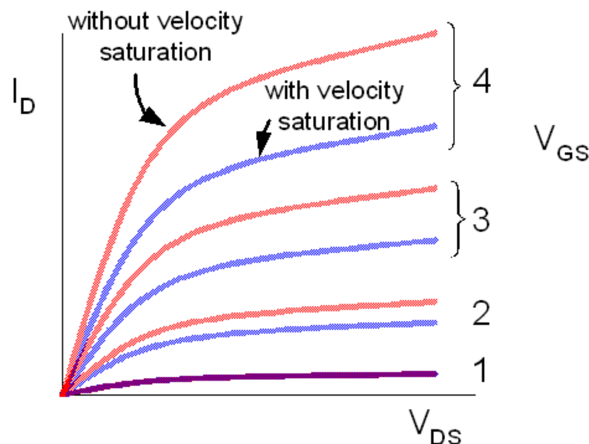


$E_C \cong 1 \times 10^5$ V/cm for Si

For a 1μm device with $V_{DS} > 2$ V, the electron velocity is saturated

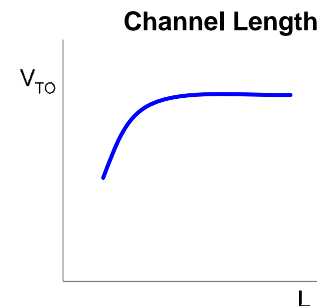
Velocity Saturation

$I_D \propto (V_{GS} - V_t)$ no longer a square law in saturation

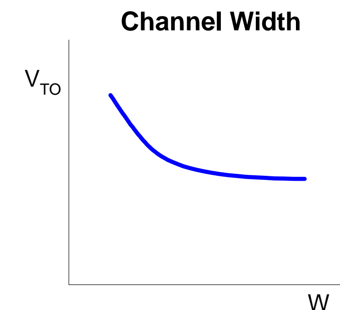


Small Geometry Effects on V_{TO} and γ

Channel Length



Channel Width

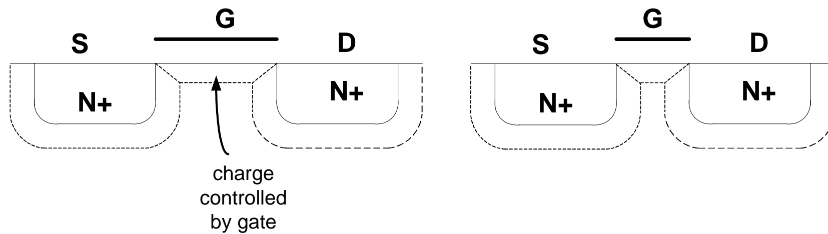


- V_{TO} decreases as L decreases because more of depletion region controlled by source and drain
- γ decreases because substrate has less effect

- V_{TO} increases as W decreases because field under gate weakened by fringing effects
- γ increases because more charge controlled by body

Drain Induced Barier Lowering (DIBL)

The drain acts as a second gate and modulates the depletion charge ⇒ shift in V_{TO}



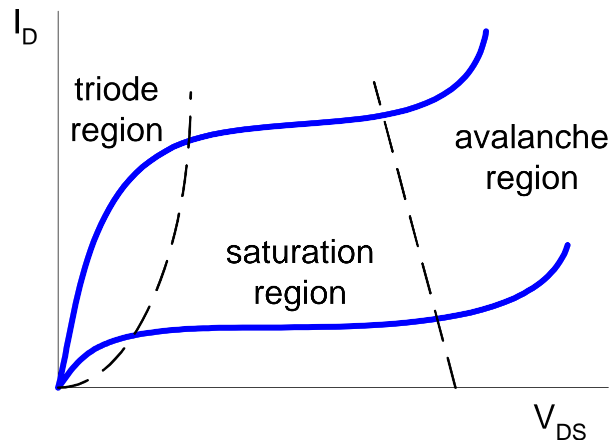
$$V_{TO}' = V_{TO} - \Delta V_T(L) - \Delta V_T(V_{DS}) + \Delta V_T(W)$$

Hot Electron Effects

- **High field in drain region give enough energy for carriers to enter oxide**
- **Changes V_{TO} over time**
- **Leads to significant substrate currents**
  ⇒ r_{db} parasitic resistance

Avalanche Breakdown

Electron-hole pairs create additional e-h pairs at high V_{ds}



Punch Through

- **The drain source depletion regions reach one another resulting in current below surface**
  - Halo or pocket implants used