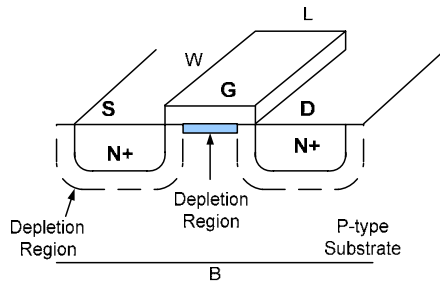


MOSFET Operation ($V_{GS} < V_T$)

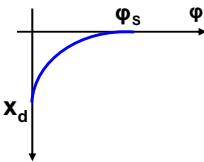
$V_{DS} = 0, V_{GS} > 0$ but $< V_T$
 $V_{GS} > 0 \Rightarrow +$ charge on gate
 - charge in substrate
 Electric field repels holes leaving ionized acceptor atoms \Rightarrow **depletion region**



$$\nabla^2 \phi = -\frac{\rho}{\epsilon} = \frac{qN_A}{\epsilon}$$

N_A = density of acceptors in p-type substrate
 q = electron charge (1.6×10^{-19} C)
 ϵ = permittivity of Si (1.04×10^{-12} F/cm)

Integrating in x direction $\phi_s = \frac{qN_A x_d^2}{\epsilon} \Rightarrow x_d = \sqrt{\frac{2\epsilon\phi_s}{qN_A}}$



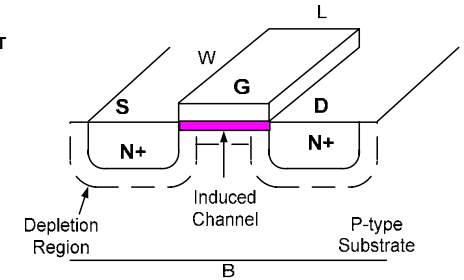
$Q_{dep} = qN_A x_d = \sqrt{2qN_A \epsilon \phi_s}$
 x_d = depletion layer depth
 Q_{dep} = depletion region charge

MOSFET Operation ($V_{GS} \approx V_T$)

$V_{DS} = 0, V_{GS}$ increasing and $\approx V_T$

Depletion region becomes deeper; almost as deep as that under the N+ regions

\Rightarrow **Surface is as attractive for electrons as N+ regions**
 \Rightarrow **Electrons can enter from the N+ regions**



The induced n-region under the gate is of n-type

\Rightarrow **Surface is inverted and the electron layer is called the inversion layer**

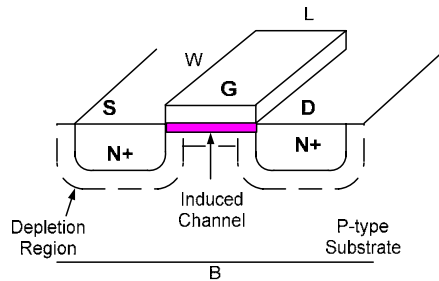
The inversion layer forms a channel for current flow when $V_{DS} > 0$

MOSFET Operation ($V_{GS} > V_T$)

$V_{DS} = 0, V_{GS} > V_T$

Once surface is inverted electrons can easily enter from the N+ regions

\Rightarrow **All additional gate charge is balanced by electrons in the inversion layer**
 \Rightarrow **Depletion layer depth and surface potential are fixed**



$$\phi_s (\text{strong inversion}) = 2V_{th} \ln \frac{N_A}{n_i} = 2\phi_F = \phi$$

V_{th} = thermal voltage ≈ 26 mV @ 300°K
 n_i = intrinsic carrier conc. (undoped Si) (1.5×10^{10} cm⁻³)
 ϕ_F = Fermi level

The gate voltage **required** to produce the inversion layer is called the **threshold voltage V_T**

The Threshold Voltage V_T

$$x_d = \sqrt{\frac{2\epsilon\phi_s}{qN_A}}$$

x_d = depletion layer depth
 Q_{dep} = depletion region charge

$$Q_{dep} = \sqrt{2qN_A \epsilon (2\phi_F + V_{SB})}$$

$$V_T = 2\phi_F + \frac{Q_{dep}}{C_{ox}} + \phi_{ms} - \frac{Q_{SS}}{C_{ox}}$$

ϕ_{ms} = work function difference between gate material and Si
 Q_{SS} = surface charge at Si - SiO₂ interface (**interface charge**)
 C_{ox} = capacitance per unit area of the gate oxide

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 3.45 \text{ fF}/\mu\text{m}^2 \text{ for } t_{ox} = 10 \text{ nm} = \text{oxide thickness}$$

$$V_T = 2\phi_F + \phi_{ms} - \frac{Q_{SS}}{C_{ox}} + \frac{Q_{dep0}}{C_{ox}} + \frac{Q_{dep} - Q_{dep0}}{C_{ox}} \quad Q_{dep0} = Q_{dep}(V_{SB} = 0)$$

$$\frac{Q_{dep} - Q_{dep0}}{C_{ox}} = \frac{\sqrt{2qN_A \epsilon}}{C_{ox}} \left(\sqrt{(2\phi_F + V_{SB})} - \sqrt{2\phi_F} \right)$$

V_T Calculation Example

$$\phi_{ms} = -0.6 \text{ V}, Q_{SS} = q \times 10^{11}/\text{cm}^2, N_A = 10^{16}/\text{cm}^3, t_{ox} = 10 \text{ nm}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = 3.45 \text{ fF}/\mu\text{m}^2 = 3.45 \times 10^{-7} \text{ F}/\text{cm}^2 \quad \gamma = \frac{\sqrt{2qN_A\epsilon}}{C_{ox}} = 0.17 \text{ V}^{1/2}$$

$$V_{TO} = 2\phi_F + \phi_{ms} - \frac{Q_{SS}}{C_{ox}} + \frac{Q_{dep0}}{C_{ox}}$$

$$\frac{Q_{SS}}{C_{ox}} = \frac{1.6 \times 10^{-19} \times 10^{11}}{3.45 \times 10^{-7}} = 0.046 \text{ V}$$

$$2\phi_F = 2V_{th} \ln \frac{N_A}{n_i} = 2 \times 26 \text{ mV} \times \ln \frac{10^{15}}{1.5 \times 10^{10}} = 0.58 \text{ V}$$

$$\frac{Q_{dep0}}{C_{ox}} = \frac{\sqrt{2qN_A\epsilon} 2\phi_F}{C_{ox}} = \gamma \sqrt{2\phi_F} = 0.13 \text{ V}$$

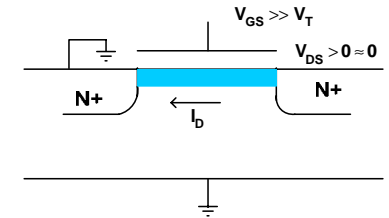
$$V_{TO} = -0.6 - 0.046 + 0.58 + 0.13 = 0.06 \text{ V} \Rightarrow V_T \text{ adjust implant (increase surface doping)}$$

Drain Current for V_{DS} > 0 but Small

$$V_{DS} > 0, V_{GS} \gg V_T$$

The charge induced in the channel is

$$Q_i = C_{ox} WL(V_{GS} - V_T)$$



$$R_{channel} = \rho \frac{L}{A} = \frac{1}{\sigma} \frac{L}{A} = \frac{1}{\sigma} \frac{L}{Wt}$$

$$\sigma = \mu_n \frac{Q_i}{WLt} = \mu_n \frac{C_{ox}(V_{GS} - V_T)}{t}$$

$$R_{channel} = \frac{1}{\mu_n C_{ox}(V_{GS} - V_T)} \frac{L}{Wt} = \frac{1}{\mu_n C_{ox}(V_{GS} - V_T)} \frac{L}{W}$$

$$I_D = \frac{V_{DS}}{R_{channel}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) V_{DS} = k' \frac{W}{L} (V_{GS} - V_T) V_{DS}$$

The inversion layer forms a channel for current flow when V_{DS} > 0

Drain Current in Nonsaturation

$$V_{DS} > 0, V_{GS} \gg V_T, V_{DS} < V_{GS} - V_T$$

Charge induced in the channel (per unit area)

$$Q_i = C_{ox}(V_{GS} - V_T - V(y))$$

$$dR = \frac{1}{W\mu_n Q_i(y)} dy$$

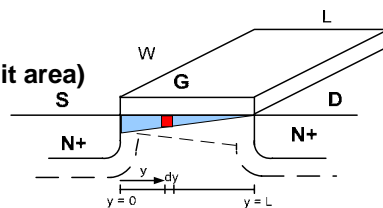
$$dV = I_D dR = \frac{I_D}{W\mu_n Q_i(y)} dy = \frac{I_D}{W\mu_n C_{ox}(V_{GS} - V_T - V(y))} dy$$

$$I_D dy = W\mu_n C_{ox}(V_{GS} - V_T - V(y)) dV$$

$$\int_0^L I_D dy = \int_0^{V_{DS}} W\mu_n C_{ox}(V_{GS} - V_T - V(y)) dV$$

Integrate from 0 to L

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$



Drain Current in Saturation

$$V_{GS} \gg V_T, V_{DS} \geq V_{GS} - V_T$$

For V_{DS} = V_{GS} - V_T

$$I_D = k' \frac{W}{L} \left[(V_{GS} - V_T)(V_{GS} - V_T) - \frac{(V_{GS} - V_T)^2}{2} \right]$$

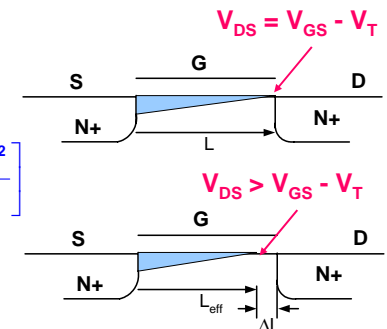
$$I_D = \frac{k'}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$$Q_i(L) = C_{ox} WL(V_{GS} - V_T - V_{DS}) = 0$$

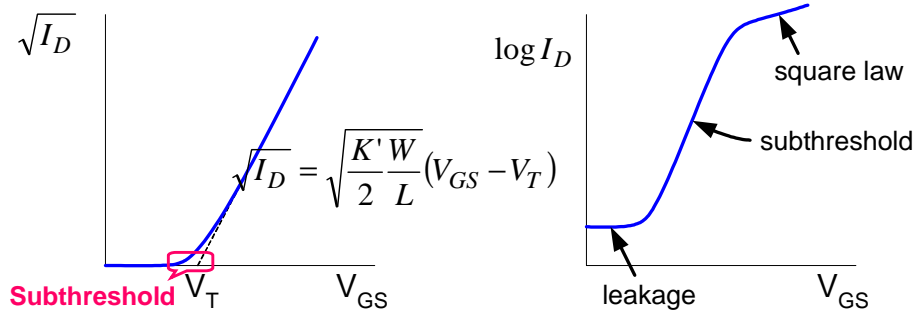
i.e., there is no mobile charge, which can not be true since current flows \Rightarrow only an approximate relation

$$I_D = \frac{K'}{2} \frac{W}{L_{eff}} (V_{GS} - V_T)^2 = \frac{K'}{2} \frac{W}{L - \Delta L} (V_{GS} - V_T)^2 \cong \frac{K'}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

where $\frac{1}{L - \Delta L} = \frac{1}{L(1 - \frac{\Delta L}{L})} \cong \frac{1}{L} \left(1 + \frac{\Delta L}{L}\right) \cong \frac{1}{L} (1 + \lambda V_{DS})$ is a first order model



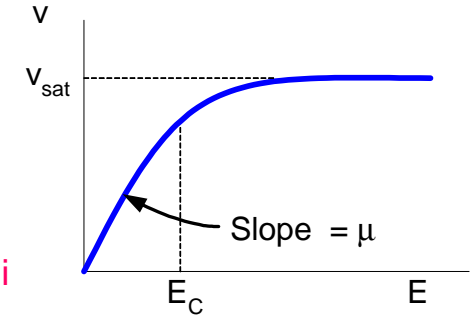
Subthreshold Conduction



$$I_D = K_x \frac{W}{L} e^{\frac{V_{GS}}{nV_{th}}} \left(1 - e^{-\frac{V_{DS}}{V_{th}}} \right)$$

Velocity Saturation

$$v = \frac{\mu E}{1 + E/E_C}$$

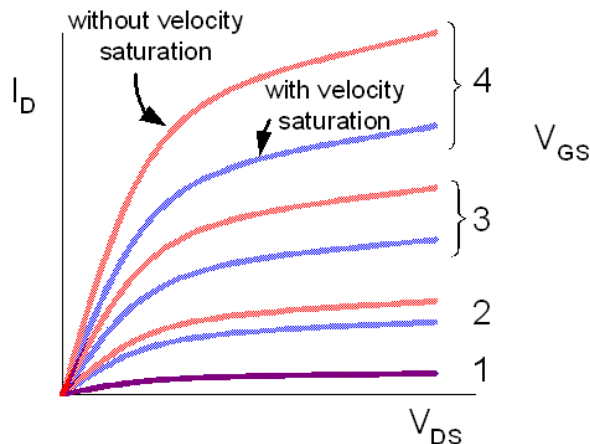


$E_C \cong 1 \times 10^5 \text{ V/cm for Si}$

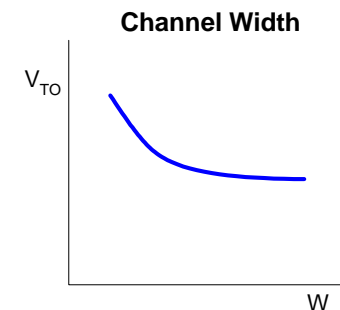
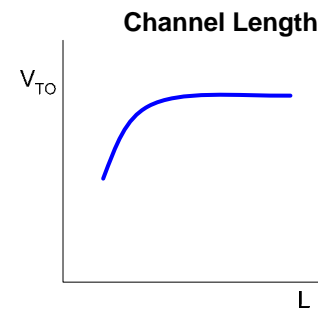
For a $1\mu\text{m}$ device with $V_{DS} > 2 \text{ V}$, the electron velocity is saturated

Velocity Saturation

$I_D \propto (V_{GS} - V_t)$ no longer a square law in saturation



Small Geometry Effects on V_{TO} and γ

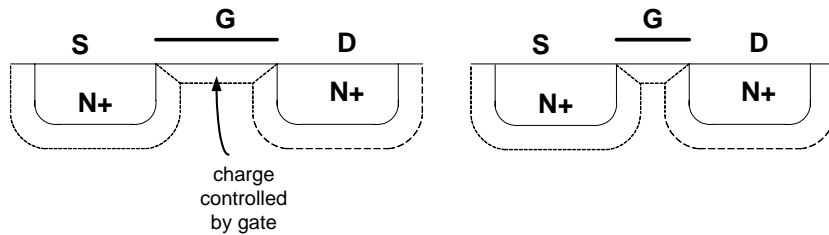


- V_{TO} decreases as L decreases because more of depletion region controlled by source and drain
- γ decreases because substrate has less effect

- V_{TO} increases as W decreases because field under gate weakened by fringing effects
- γ increases because more charge controlled by body

Drain Induced Barrier Lowering (DIBL)

The drain acts as a second gate and modulates the depletion charge \Rightarrow shift in V_{TO}



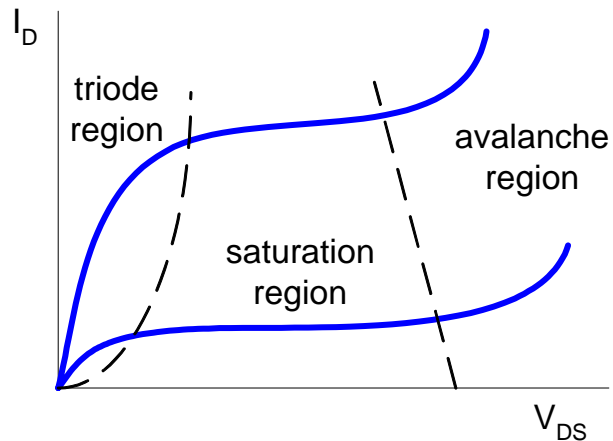
$$V_{TO}' = V_{TO} - \Delta V_T(L) - \Delta V_T(V_{DS}) + \Delta V_T(W)$$

Hot Electron Effects

- High field in drain region give enough energy for carriers to enter oxide
- Changes V_{TO} over time
- Leads to significant substrate currents
 $\Rightarrow r_{db}$ parasitic resistance

Avalanche Breakdown

Electron-hole pairs create additional e-h pairs at high V_{ds}



Punch Through

- The drain source depletion regions reach one another resulting in current below surface
– Halo or pocket implants used