

Test 1 (10/28/08)

Total # Pages 6

Total # Problems 5

Attach your one page of notes to this test

Name SOLUTION

- 1. (20 points) _____
- 2. (25 points) _____
- 3. (30 points) _____
- 4. (10 points) _____
- 5. (15 points) _____

Total (100 points) _____

GOOD LUCK

Note: All bulk connections that are not shown are tied to the appropriate supply voltage.

$$I_d = \begin{cases} k' \frac{W}{L} \left[(V_{gs} - V_T)V_{ds} - \frac{V_{ds}^2}{2} \right] (1 + \lambda V_{ds}) & V_{gs} \geq V_T, V_{ds} \leq V_{gs} - V_T, \text{ linear} \\ \frac{k'}{2} \frac{W}{L} (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & V_{gs} \geq V_T, V_{ds} \geq V_{gs} - V_T, \text{ saturation} \\ 0 & V_{gs} < V_T, \text{ cutoff} \end{cases}$$

$$V_T = V_{T0} + \gamma (\sqrt{V_{sb} + \phi} - \sqrt{\phi})$$

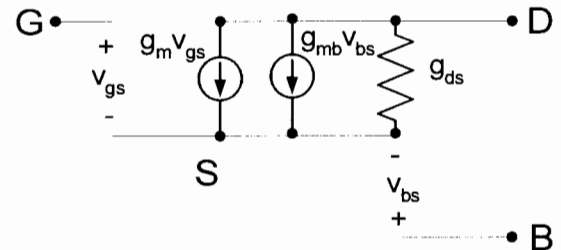
$$\text{In saturation, } V_{gs} = V_T + \sqrt{\frac{2I_d}{k' \frac{W}{L}}}$$

Small-signal parameters in saturation :

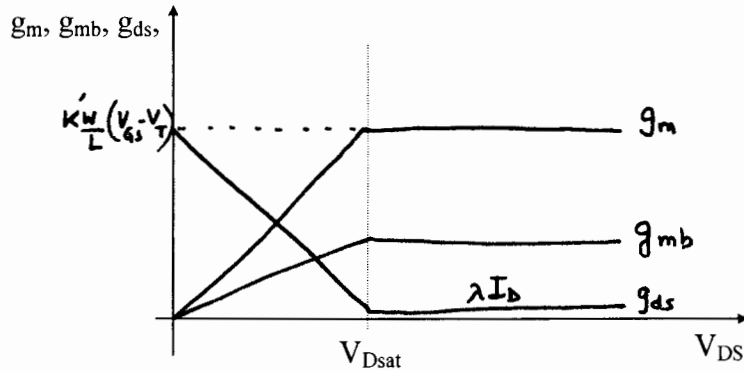
$$g_{ds} = g_o = \lambda I_d$$

$$g_m = \sqrt{2I_d k' \frac{W}{L}} = \frac{2I_d}{V_{gs} - V_T}$$

$$g_{mb} = \eta g_m \text{ where } \eta = \frac{\gamma}{2\sqrt{V_{sb} + \phi}}$$



1. a) Plot the g_m , g_{mb} , g_{ds} , for an n-channel MOSFET as a function of V_{DS} for a value of $V_{GS} > V_T$. (20 points)

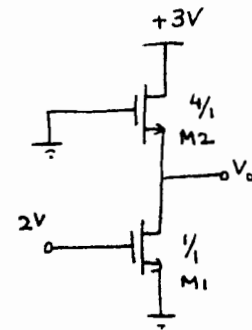
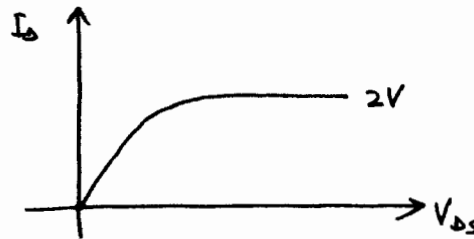


- b) Identify the regions of operation for M1 and M2 and calculate the voltage V_o as shown. $V_{TO} = 0.7\text{ V}$, $k' = 100\ \mu\text{A}/\text{V}^2$, $\gamma = 0$, $\lambda = 0$.

M2 cut off

M1 linear (nonsat, triode)

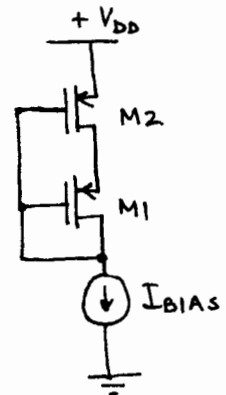
$$V_o = 0\text{ V}$$



- c) Identify the regions of operation for M1 and M2.

M2 linear (triode, nonsat)

M1 Saturation

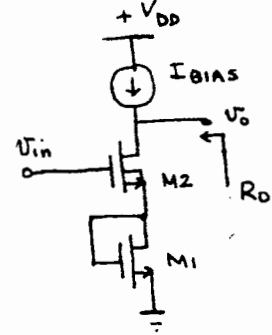


2. Write the expressions for the small-signal low-frequency quantities as shown. Assume all transistors are biased in saturation and have identical small-signal parameters. Ignore the body effect and assume that $g_m r_o \gg 1$ (25 points).

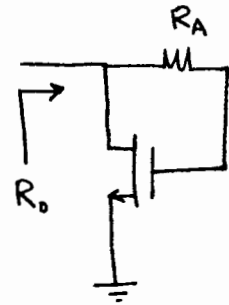
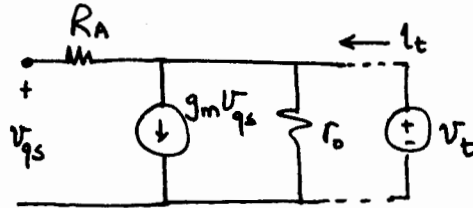
G_m (effective g_m) = $g_m/2$ R_o = $2r_o$

$G_m = \frac{g_m}{1+1} = \frac{g_m}{2}$

$R_o = \left(1 + g_m \cdot \frac{1}{g_m}\right) r_o = 2r_o$

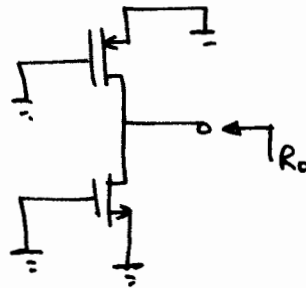
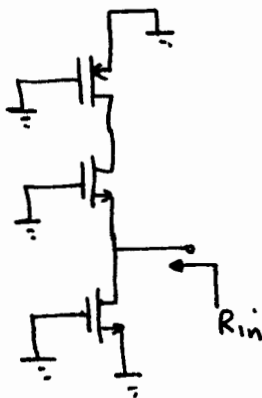


$R_o = \frac{1}{g_m}$

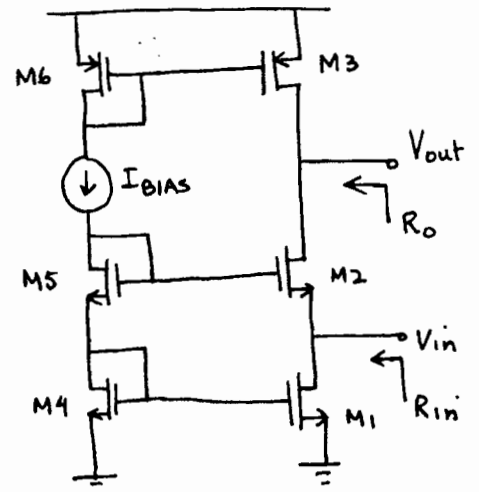


$R_{in} = \frac{2}{g_m}$

$R_o = \frac{r_o}{2}$



$R_o = r_o \parallel r_o = \frac{r_o}{2}$



$R_{in} = r_o \parallel \frac{1}{g_m} \left(1 + \frac{r_o}{r_o}\right)$

$= \frac{2}{g_m}$

3. a) For the amplifier shown calculate the dc gate bias V_{GS} for M1 to have a bias current of $100 \mu\text{A}$. Also calculate the values of the dc drain current of M3 and the drain voltage of M1 for this bias condition (15 points).

n-ch: $V_{TO} = 0.7 \text{ V}$, $k' = 100 \mu\text{A}/\text{V}^2$, $\gamma = 0.6 \text{ V}^{1/2}$, $\phi = 0.7 \text{ V}$, $\lambda = 0.02 \text{ V}^{-1}$

p-ch: $V_{TO} = -0.9 \text{ V}$, $k' = 50 \mu\text{A}/\text{V}^2$, $\gamma = 0.5 \text{ V}^{1/2}$, $\phi = 0.7 \text{ V}$, $\lambda = 0.02 \text{ V}^{-1}$

$V_{GS(M1)} = \underline{1.7 \text{ V}}$

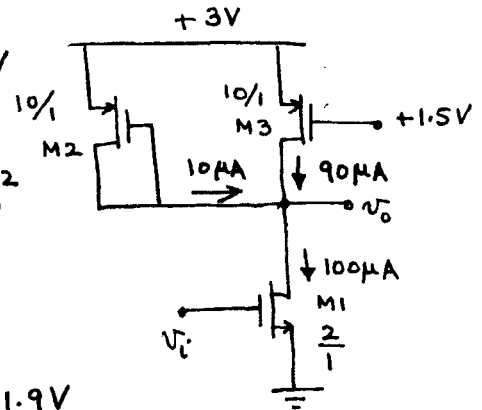
$I_{D(M3)} = \underline{90 \mu\text{A}}$

$V_{D(M1)} = \underline{1.9 \text{ V}}$

$$V_{GS} = V_T + \sqrt{\frac{2I_D}{k'W/L}}; \quad V_{GS1} = 0.7 + \sqrt{\frac{2 \times 100}{100 \times 2}} = 1.7 \text{ V}$$

$$I_{D(M3)} = \frac{k'}{2} \frac{W}{L} (V_{GS2} - |V_{T1}|)^2 = \frac{50}{2} \times 10 (1.5 - 0.9)^2 = 90 \mu\text{A}$$

$$|V_{GS2}| = 0.9 + \sqrt{\frac{2 \times 10}{50 \times 10}} = 1.1 \text{ V} \Rightarrow V_o = 1.9 \text{ V}$$



Check: M1 in saturation!

- b) Calculate the low frequency small-signal voltage gain $A_v = v_o/v_i$ (10 points).

$$A_v = -g_{m1} \cdot r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m2}}$$

$$\approx -\frac{g_{m1}}{g_{m2}}$$

$$g_{m1} = \sqrt{2I_D k'W/L} = \sqrt{2 \times 100 \times 100 \times 2} \mu\text{A}/\text{V} = 200 \mu\text{A}/\text{V}$$

$$g_{m2} = \sqrt{2 \times 10 \times 50 \times 10} = 100 \mu\text{A}/\text{V}$$

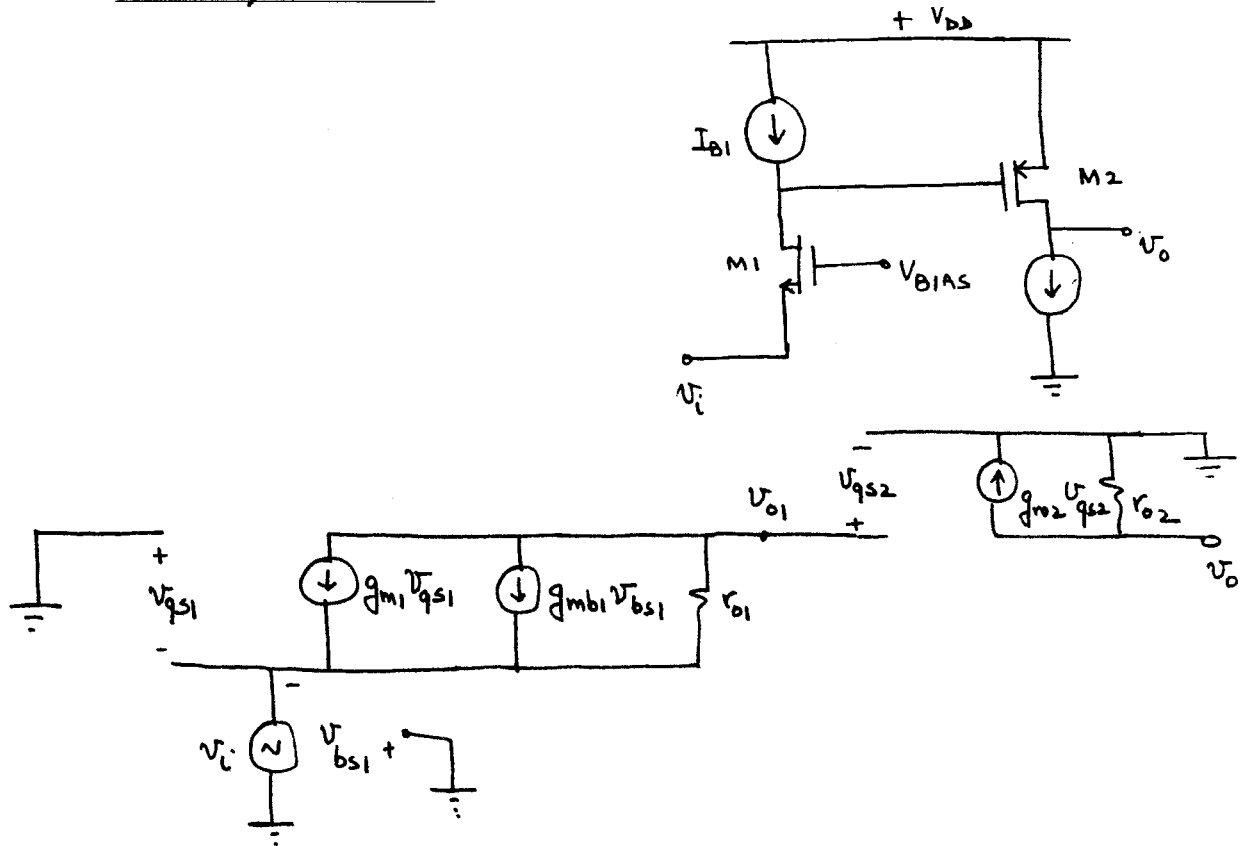
$$\therefore A_v = -2$$

- c) Calculate the output resistance of this amplifier (5 points).

$$R_o = r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m2}} \approx \frac{1}{g_{m2}} = 10 \text{ k}\Omega$$

4. Draw the low frequency small-signal circuit for the configuration shown and derive an expression for the voltage gain of this circuit. Assume $g_{mb} = \eta g_m$ and $g_m r_o \gg 1$ (10 points).

$$A_v = v_o/v_i = \underline{-(1+\eta) g_{m1} r_{o1} g_{m2} r_{o2}}$$



$$\frac{v_{o1}}{v_i} = (g_{m1} + g_{mb1}) r_{o1} \quad (\text{CG stage})$$

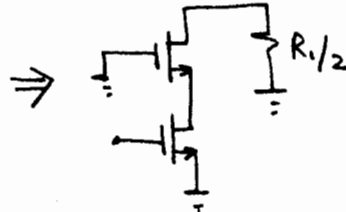
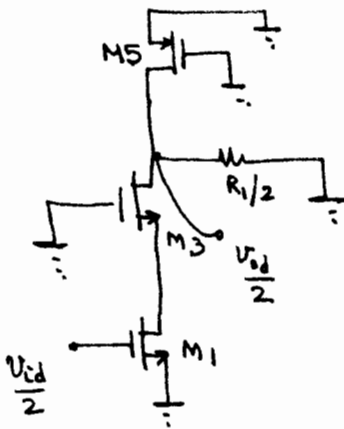
$$\frac{v_o}{v_{o1}} = -g_{m2} r_{o2} \quad (\text{CS stage})$$

$$\begin{aligned} A_v = \frac{v_o}{v_i} &= -(g_{m1} + g_{mb1}) r_{o1} g_{m2} r_{o2} \\ &= -(1+\eta) g_{m1} r_{o1} g_{m2} r_{o2} \end{aligned}$$

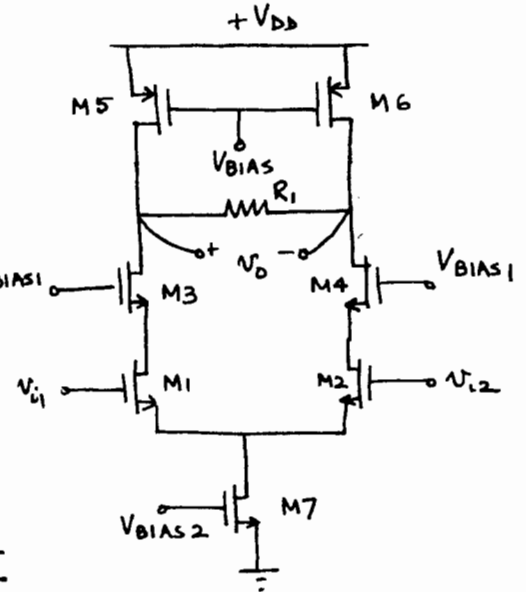
5. a) Derive the expression for the **differential** mode low frequency small-signal voltage gain for the amplifier as shown. Assume $g_{m1} = g_{m2} = g_{mx}$, $g_{m3} = g_{m4} = g_{my}$, $g_{m5} = g_{m6} = g_{mz}$. Take r_o to be infinity and ignore the body effect (10 points).

$$A_{dm} = \underline{-g_{m1} R_1 / 2}$$

differential half circuit:



$$A_{dm} = \frac{V_{od}}{V_{id}} = -g_{m1} \frac{R_1}{2}$$



- b) Draw the half circuit for calculating the common-mode gain. Assume $g_m r_o \gg 1$ and ignore the body effect (5 points).

