ECE 521 Lecture #8 Oct. 17, 2016

Note Title 10/17/2016

HW # 1 Graded handed back

Grading Part 1 pulse grading

✓ ✓- ✓--

Part 2 scored out of 75

Stamping errors : - sign errors

- completely incorrect stamps

Gyrator — full MNA formulation

$i_1 = -g v_2$

$i_2 = g v_1$

} voltage controlled formulation

⇒ plain nodal stamp

Transformer only 1 Branch current

2 branch currents not needed

branchNum = mappedBranchName( )..

branchNum1 = branchNum + 1

NumBranches ++

branchNum1 = NumBranches

r1 node1 vsrc 1

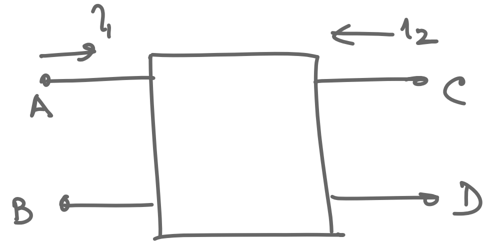
Vsrc node1 0 1

Stamping of '0' entries not required

Test circuits — 1 line circuit

Two ports:

n1 A B C D 'n'



n1 C D A B

Testing:

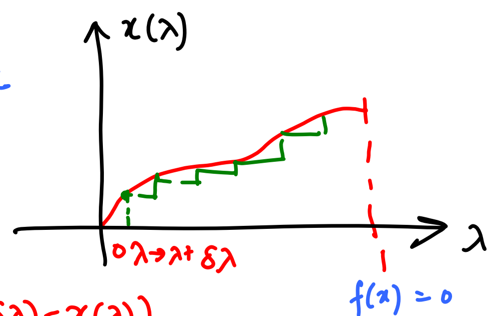
Continuation methods

- Source stepping – natural parameter continuation
- GMIN stepping (SPICE3, HSPICE)

Artificial parameter continuation methods

$$\tilde{f}(x(\lambda), \lambda) = 0$$ sweep λ from 0 to 1

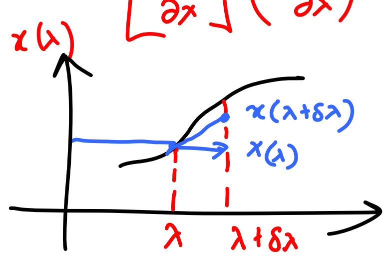
$$\tilde{f}(x(\lambda), \lambda) = \lambda f(x) + (1-\lambda)x$$



$$\tilde{f}\left(x(\lambda+\delta\lambda), \lambda+\delta\lambda\right) = \tilde{f}(x(\lambda),\lambda) + \frac{\partial \tilde{f}}{\partial x}\left(x(\lambda+\delta\lambda) - x(\lambda)\right) + \frac{\partial \tilde{f}}{\partial \lambda}\,\delta\lambda$$

$$\frac{\partial \tilde{f}}{\partial x}\left(x(\lambda+\delta\lambda) - x(\lambda)\right) = -\frac{\partial \tilde{f}}{\partial \lambda}\,\delta\lambda$$

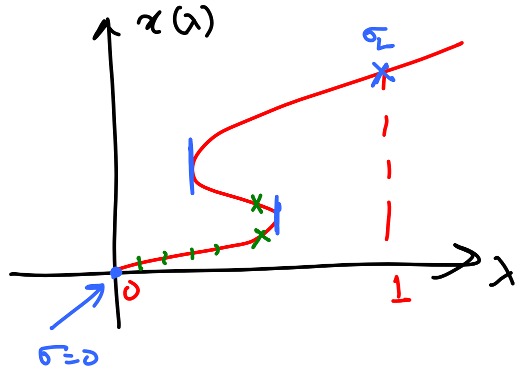
$$x(\lambda+\delta\lambda) = x(\lambda) + \left[\frac{\partial \tilde{f}}{\partial x}\right]^{-1}\left(-\frac{\partial \tilde{f}}{\partial \lambda}\right)\delta\lambda$$



Parameterized in terms of the arc length (σ)

$\sigma = 0$ to σ_L

$x(\sigma), \quad \lambda(\sigma)$



$$\delta\sigma = \sqrt{(\delta\lambda)^2 + \delta x^2}$$

$$\|\delta x\|^2 + |\delta\lambda|^2 = \delta\sigma^2$$

$$\left\|\frac{dx}{ds}\right\|^2 + \left|\frac{d\lambda}{ds}\right|^2 = 1 \qquad \text{actual arc length } (s)$$

$$\|\dot{x}(s)\|^2 + |\dot{\lambda}(s)|^2 = 1$$

HOMPACK is a public domain solver
(homotopy methods)

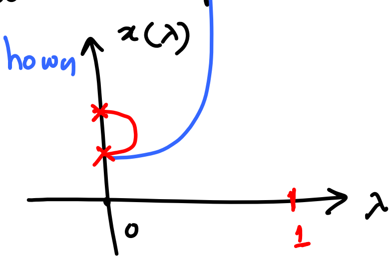
- Choice of embedding function

$$\tilde{f}(x(\lambda), \lambda) = \lambda f(x) + (1-\lambda)x$$

Choice of $\tilde{f}$ affects convergence

- $\tilde{f}(x(0), 0)$ must have a unique solution

avoid the situation shown



- The solution $x(\lambda)$ must be bounded

This ensures that the track doesn't go to infinity

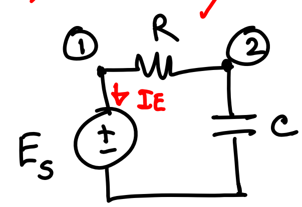
DC operating point analysis

DC transfer curve "

Small-signal AC analysis

- Frequency domain analysis of linear circuits
- provides information on magnitude/phase of response or transfer functions
  $\Rightarrow$ phase margin, stability

MNA equations



$$G(V_1 - V_2) + I_E = 0$$
$$G(V_2 - V_1) + \frac{C\,dV_2}{dt} = 0$$
$$V_1 = E_s$$

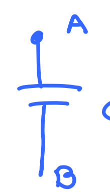
$$x = x_0 + x_{ac} e^{j\omega t}$$

$$\frac{C\,dV_2}{dt} \to j\omega C V_2$$

$$G(V_1 - V_2) + I_E = 0$$
$$G(V_2 - V_1) + j\underline{\omega C} V_2 = 0$$
$$V_1 = E_s$$

AC stamp of a linear capacitor

$$\begin{array}{ccc} & V_A & V_B \\ A & \left[\begin{array}{c} +j\omega C \\ -j\omega C \end{array}\right. & \left.\begin{array}{c} -j\omega C \\ +j\omega C \end{array}\right] \\ B & & \end{array}$$



We will now have a system of complex linear equations

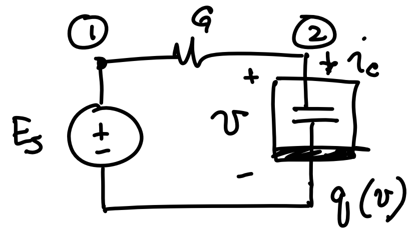
Sparse 1.4 can solve such systems

Procedure

for $\omega = \omega_{initial}$ to ω_{final} {
  assemble circuit equations for each ω
  Solve $A(j\omega)\,x = b$
    LU Factorization of $A(j\omega)$
    Forward/Back solves

x is a function of frequency

Nonlinear capacitor

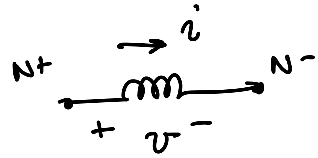


$$i_c = \frac{dq}{dt} = \frac{dq(v)}{dt}$$

$$= \underbrace{\frac{\partial q(v)}{\partial v}}\ \frac{dv}{dt}$$

incremental capacitance at a DC operating point $\quad C(v_{op}) = \left.\frac{\partial q(v)}{\partial v}\right|_{op.\ pt.}$

Linear Inductor



$$v = L\frac{di}{dt}$$

$$= j\omega L\, I$$

$$V_{N+} - V_{N-} - j\omega L\, I = 0 \Rightarrow I = \frac{V_{N+} - V_{N-}}{j\omega L}$$

Current controlled element $\Rightarrow$ MNA stamp

$$\begin{array}{c} \\ N+ \\ N- \\ I \end{array}
\begin{array}{c} \begin{array}{ccc} N+ & N- & I \end{array} \\
\begin{bmatrix} & & +1 \\ & & -1 \\ 1 & -1 & -\underline{\underline{j\omega L}} \end{bmatrix} \end{array}$$

For DC

$$\begin{bmatrix} & & +1 \\ & & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Nodal stamp

$$\begin{bmatrix} +\frac{1}{j\omega L} & -\frac{1}{j\omega L} \\ -\frac{1}{j\omega L} & +\frac{1}{j\omega L} \end{bmatrix}$$

$\omega = 1, \quad L = 1\,nH, \quad C = 1\,pF$

$\omega C = 10^{-12}; \quad \frac{1}{\omega L} = 10^{9} \quad [10^{-12}, 10^{9}]$

$\omega L = 10^{-9} \quad$ dynamic range $[10^{-12}, 1]$

Summary

- For small-signal ac analysis (linearize around a DC operating point)
  - nonlinear capacitors replaced by incremental capacitors at dc op. pt.
  - nonlinear conductances ($i = i(v)$) replaced by linear conductances ($\frac{\partial i}{\partial v}$) calculated at the dc. op. pt.
- Inductors — use MNA formulation
- Assemble a linear complex system of equations
- Solve equations at each frequency.

$$A(j\omega)\, x = b$$

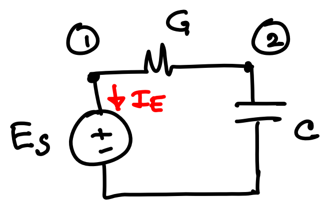
Frequency independent

$$[G + j\omega C]\, x = b$$

Transient analysis

- large-signal time-domain analysis of nonlinear circuits
- input source(s) time varying (sine, pulse, PWL - - -)
  determine currents/voltage waveforms as a function of time
- Used as the basis for a Fourier analysis (.FOUR in SPICE)

Consider a linear circuit



$$G(V_1 - V_2) + I_E = 0$$

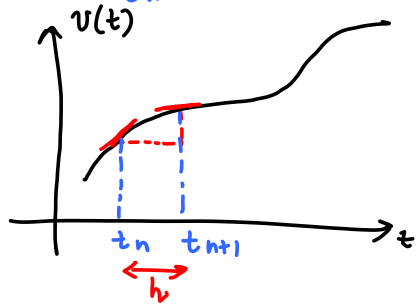
$$G(V_2 - V_1) + C\frac{dV_2}{dt} = 0$$

$$V_1 = E_s(t)$$

Given $E_s(t)$ find $V_1(t)$, $V_2(t)$, $I_E(t)$

Suppose we can express $\frac{dV_2}{dt} = \alpha V_2 + \beta$

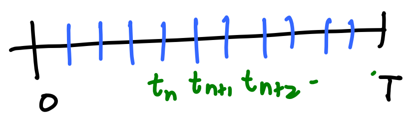
The derivative is the slope



B.E. (Backward Euler) $\left.\frac{dv}{dt}\right|_{t_{n+1}} = \frac{v(t_{n+1}) - v(t_n)}{h}$

F.E. (Forward Euler) $\left.\frac{dv}{dt}\right|_{t_n} = \frac{v(t_{n+1}) - v(t_n)}{h}$

Observe that we are discretizing time



Questions ?

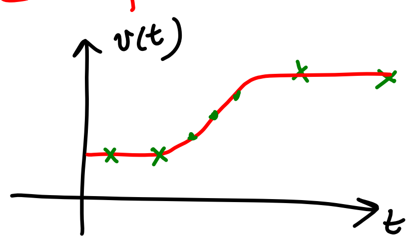
What should 'h' be. Error control

which integration method should be used (FE, BE, or others ?)

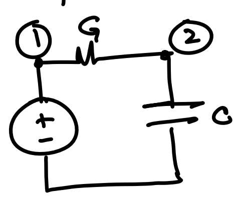
⇒ properties of integration methods

Should 'h' be uniform or nonuniform

⟹ variable timesteps



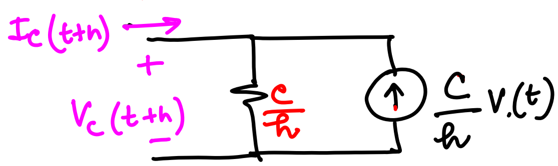
$G(V_2 - V_1) + C\frac{dV_2}{dt} = 0$

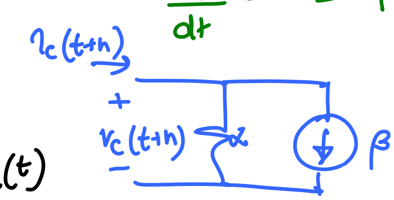


$C \left.\frac{dv}{dt}\right|_{t+h} = C\,\frac{V(t+h) - V(t)}{h}$

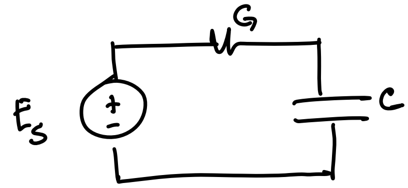
$I_c(t+h) = \frac{C}{h} V(t+h) - \frac{C}{h} V(t)$

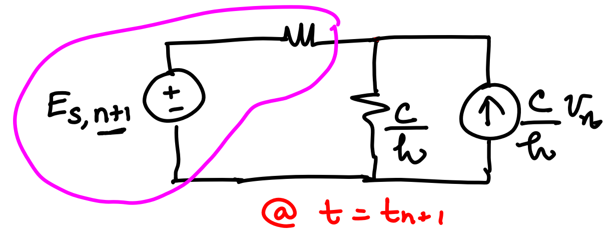
$\frac{dV_2}{dt} = \alpha V_2 + \beta$





Companion model of a capacitor for transient analysis

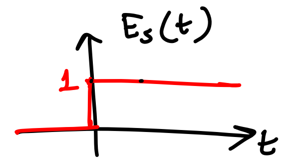


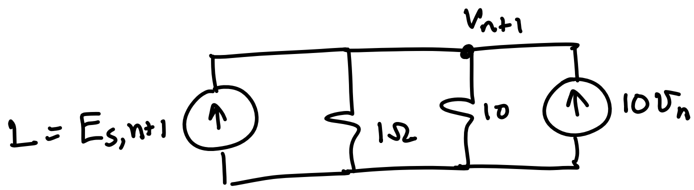


Given an initial condition across the capacitor v_0 we can solve $v_1, v_2, v_3, \ldots v_M$

Example $R = 1\,\Omega$, $C = 1\,F$, $E_s(t) = 1V$ step

$h = 0.1$ sec $\quad v_c(0) = 0$





$v(t) = 1 - e^{-t}$

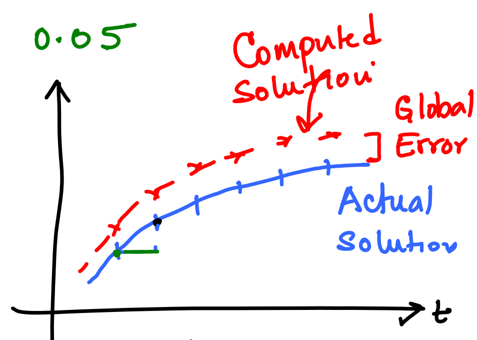
$$1.\, V_{n+1} + 10\, V_{n+1} = 10\, v_n + 1$$

$$v_{n+1} = \frac{1}{11}\left(1 + 10\, v_n\right)$$

	Numerical	Exact
$v(0.1)$	0.0909	0.095
$v(0.2)$	0.174	0.181
$v(0.3)$	0.249	0.259
$v(0.4)$	0.317	0.33

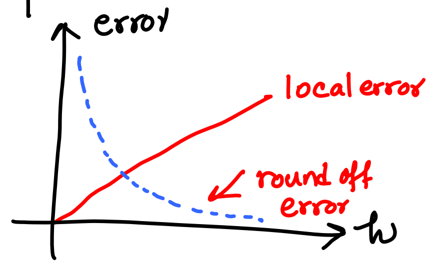
Change h from 0.1 to 0.05

Global error is the error accumulated over the whole time interval



What about the local error (one-step error)

Naturally, as $h \to 0$ this error should go to zero



$$\frac{dv}{dt} = \frac{v(t+h)-v(t)}{h}$$

Convergent method

Consistency

Stability

Consistency + Stability $\Longleftrightarrow$ Convergence