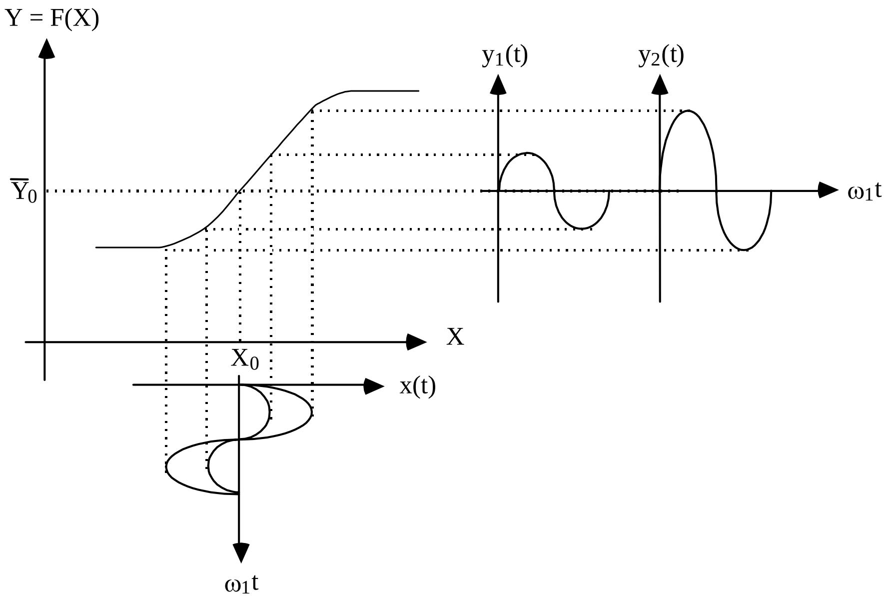
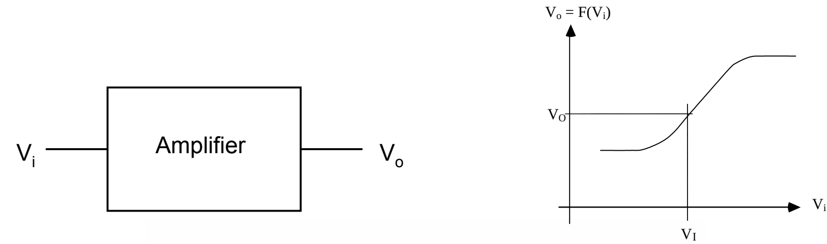
Distortion Generation



Power Series and Distortion

- Assume low frequency analysis ⇒ ignore capacitors
- Output expressed as a power series



$$V_O + v_o = F(V_I + v_i) = a_o + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots$$

$a_o = F(V_I) = V_O(V_I)$ is the quiescent output voltage

Power Series Analysis

The incremental output voltage is

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots$$

Assume a pure sinusoidal tone

$$v_i = V_{iA} \cos \omega_1 t$$

Using trigonometric identities can show that

$$v_o = b_0 + b_1 \cos \omega_1 t + b_2 \cos 2\omega_1 t + \dots$$

The Coefficients

$$\begin{aligned} b_0 &= \frac{a_2}{2} V_{iA}^2 + \dots \\ b_1 &= a_1 V_{iA} + \frac{3}{4} a_3 V_{iA}^3 + \dots \approx a_1 V_{iA} + \dots \\ b_2 &= \frac{a_2}{2} V_{iA}^2 + \dots \\ b_3 &= \frac{a_3}{4} V_{iA}^3 + \dots \\ &\dots \end{aligned}$$

- **A dc term is present**
  - ⇒ A dynamic shift of dc operating point due to distortion generation
- **Compression or expansion of fundamental and harmonics**
  - ⇒ Contribution of higher order terms

Harmonic Distortion Factors

$$HD_2 = \frac{|b_2|}{|b_1|} \approx \frac{1}{2}\left|\frac{a_2}{a_1}\right| V_{iA}$$

$$HD_3 = \frac{|b_3|}{|b_1|} \approx \frac{1}{4}\left|\frac{a_3}{a_1}\right| V_{iA}^2$$

$$\cdots$$

$$THD = \frac{\sqrt{b_2^2 + b_3^2 + \ldots}}{|b_1|}$$

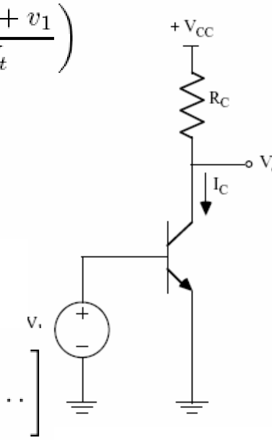
Example (Common-Emitter Stage)

$$V_1 = V_{BB} + v_1$$

$$I_c = I_C + i_c = I_S \exp\left(\frac{V_{be}}{V_t}\right) = I_S \exp\left(\frac{V_{BB} + v_1}{V_t}\right)$$

$$= \left[I_S \exp\left(\frac{V_{BB}}{V_t}\right)\right]\left[\exp\left(\frac{v_1}{V_t}\right)\right]$$

$$= I_{CA} \exp\left(\frac{v_1}{V_t}\right)$$



I_{CA} = quiescent value of collector current

$$\exp(x) = 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 + \ldots.$$

$$I_c = I_{CA}\left[1 + \left(\frac{v_1}{V_t}\right) + \frac{1}{2}\left(\frac{v_1}{V_t}\right)^2 + \frac{1}{6}\left(\frac{v_1}{V_t}\right)^3 + \ldots\right]$$

$$I_c = I_{CA}\left[1 + a_1' v_1 + a_2' v_1^2 + \ldots\right]$$

CE Stage Power Series

$$a_0 = I_c(V_{BB}) = I_{CA}$$

$$a_1 = a_1' I_{CA} = I_{CA}\left(\frac{1}{V_t}\right)$$

$$a_2 = a_2' I_{CA} = \frac{1}{2} I_{CA}\left(\frac{1}{V_t}\right)^2$$

$$a_3 = a_3' I_{CA} = \frac{1}{6} I_{CA}\left(\frac{1}{V_t}\right)^3$$

$$\Rightarrow$$

$$b_0 \approx \frac{I_{CA}}{4}\left(\frac{V_{1A}}{V_t}\right)^2$$

$$b_1 \approx I_{CA}\left(\frac{V_{1A}}{V_t}\right)$$

$$b_2 \approx \frac{I_{CA}}{4}\left(\frac{V_{1A}}{V_t}\right)^2$$

$$b_3 \approx \frac{I_{CA}}{24}\left(\frac{V_{1A}}{V_t}\right)^3$$

CE Stage Harmonic Distortion Factors

$$HD_2 \approx \frac{1}{4}\left(\frac{V_{1A}}{V_t}\right)$$

$$HD_3 \approx \frac{1}{24}\left(\frac{V_{1A}}{V_t}\right)^2$$

Example $V_{BB} = 0.774$ V $\quad I_S = 1 \times 10^{-16}$ A

$V_t = 25.85$ mV and $I_{CA} = 1.0$ mA $\quad V_{1A} = 10$ mV

$$b_0 = 0.19 \text{ V}, \quad b_1 = 1.93 \text{ V}, \quad b_2 = 0.19 \text{ V}, \quad b_3 = 0.012 \text{ V}$$

$$HD_2 = 9.8\%$$

$$HD_3 = 0.62\%$$

CE Stage with a Large Sinusoidal Input

$$I_c = I_{CA}\exp\left(\frac{v_1}{V_t}\right) = I_{CA}\exp\left(\frac{V_{1A}\cos\omega_1 t}{V_t}\right) = I_{CA}\exp\left(d\cos\omega_1 t\right) \qquad d = \frac{V_{1A}}{V_t}$$

$$\exp(d\cos\omega_1 t) = I_0(d) + 2I_1(d)\cos\omega_1 t + 2I_2(d)\cos 2\omega_1 t + \ldots + 2I_n(d)\cos n\omega_1 t + \ldots$$

where $I_n(d)$ are modified Bessel functions of order n

$$I_c = I_{CA}[I_0(d) + 2I_1(d)\cos\omega_1 t + \ldots + 2I_n(d)\cos n\omega_1 t + \ldots]$$
$$= I_{CA}I_0(d)\left[1 + \frac{2I_1(d)}{I_0(d)}\cos\omega_1 t + \ldots + \frac{2I_n(d)}{I_0(d)}\cos n\omega_1 t + \ldots\right]$$
$$= I_{dc}\left[1 + \frac{2I_1(d)}{I_0(d)}\cos\omega_1 t + \ldots + \frac{2I_n(d)}{I_0(d)}\cos n\omega_1 t + \ldots\right]$$

Harmonic Distortion Factors

$$I_{dc} = I_{CA}I_0(d)$$

$$b_1 = I_{dc}\left[\frac{2I_1(d)}{I_0(d)}\right]$$
$$b_2 = I_{dc}\left[\frac{2I_2(d)}{I_0(d)}\right]$$
$$b_n = I_{dc}\left[\frac{2I_n(d)}{I_0(d)}\right]$$

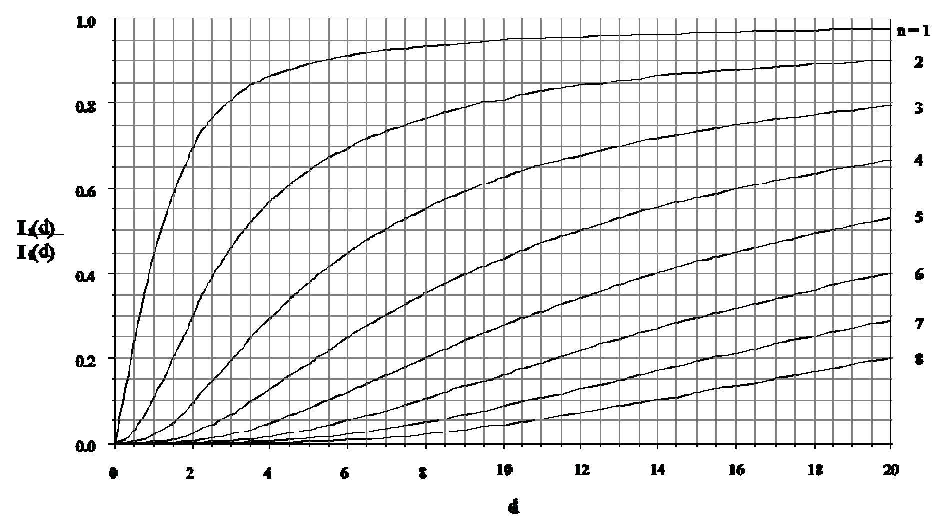
$$HD_2 = \frac{|b_2|}{|b_1|} = \frac{I_2(d)}{I_1(d)}$$
$$HD_3 = \frac{|b_3|}{|b_1|} = \frac{I_3(d)}{I_1(d)}$$

Modified Bessel Functions

x	$I_0(x)$	$I_1(x)$	$I_2(x)$	$I_3(x)$	$\frac{I_1(x)}{I_0(x)}$	$\frac{I_2(x)}{I_0(x)}$	$\frac{I_3(x)}{I_0(x)}$
0.0	1.0000	0.0000	0.0000	0.0000	0.000	0.000	0.000
0.1	1.0025	0.0501	0.0013	0.0000	0.050	0.001	0.000
0.2	1.0100	0.1005	0.0050	0.0002	0.100	0.005	0.000
0.4	1.0404	0.2040	0.0203	0.0013	0.196	0.020	0.001
0.6	1.0920	0.3137	0.0464	0.0046	0.287	0.043	0.004
0.8	1.1665	0.4329	0.0844	0.0111	0.371	0.072	0.010
1.0	1.2661	0.5652	0.1357	0.0222	0.446	0.107	0.018
1.2	1.3937	0.7147	0.2026	0.0394	0.513	0.145	0.028
1.4	1.5534	0.8861	0.2875	0.0645	0.570	0.185	0.041
1.6	1.7500	1.0848	0.3940	0.0999	0.620	0.225	0.057
1.8	1.9896	1.3172	0.5260	0.1482	0.662	0.264	0.074
2.0	2.2796	1.5906	0.6889	0.2127	0.698	0.302	0.093
2.2	2.6291	1.9141	0.8891	0.2976	0.728	0.338	0.113
2.4	3.0493	2.2981	1.1342	0.4079	0.754	0.372	0.134
2.6	3.5533	2.7554	1.4337	0.5496	0.776	0.404	0.155
2.8	4.1573	3.3011	1.7994	0.7305	0.794	0.433	0.176
3.0	4.8808	3.9534	2.2452	0.9598	0.810	0.460	0.197
3.2	5.7472	4.7343	2.7883	1.2489	0.824	0.485	0.217
3.4	6.7848	5.6701	3.4495	1.6119	0.836	0.508	0.238
3.6	8.0277	6.7927	4.2540	2.0661	0.846	0.530	0.257
3.8	9.5169	8.1404	5.2325	2.6326	0.855	0.550	0.277
4.0	11.302	9.7595	6.4222	3.3373	0.864	0.568	0.295
4.2	13.442	11.706	7.8684	4.2120	0.871	0.585	0.313
4.4	16.010	14.046	9.6258	5.2955	0.877	0.601	0.331
4.6	19.093	16.863	11.761	6.6355	0.883	0.616	0.347
4.8	22.794	20.253	14.355	8.2903	0.889	0.630	0.364
5.0	27.240	24.336	17.506	10.331	0.893	0.643	0.379
5.2	32.584	29.254	21.332	12.845	0.898	0.655	0.394
5.4	39.009	35.182	25.978	15.939	0.902	0.666	0.409
5.6	46.738	42.328	31.620	19.742	0.906	0.677	0.422
5.8	56.038	50.946	38.470	24.415	0.909	0.687	0.436
6.0	67.234	61.342	46.787	30.151	0.912	0.696	0.448
6.2	80.718	73.886	56.884	37.187	0.915	0.705	0.461
6.4	96.962	89.026	69.141	45.813	0.918	0.713	0.472
6.6	116.54	107.30	84.021	56.383	0.921	0.721	0.484
6.8	140.14	129.38	102.08	69.328	0.923	0.729	0.495
7.0	168.59	156.04	124.01	85.175	0.926	0.736	0.505
7.2	202.92	188.25	150.63	104.57	0.928	0.742	0.515
7.4	244.34	227.17	182.94	128.29	0.930	0.749	0.525
7.6	294.33	274.22	222.17	157.29	0.932	0.755	0.534
7.8	354.68	331.10	269.79	192.75	0.933	0.761	0.543
8.0	427.56	399.87	327.60	236.08	0.935	0.766	0.552

Equation for higher order terms: $I_{n+1}(x) = I_{n-1}(x) - \frac{2n}{x} I_n(x)$

Normalized Modified Bessel Functions



Distortion Calculation From Waveforms

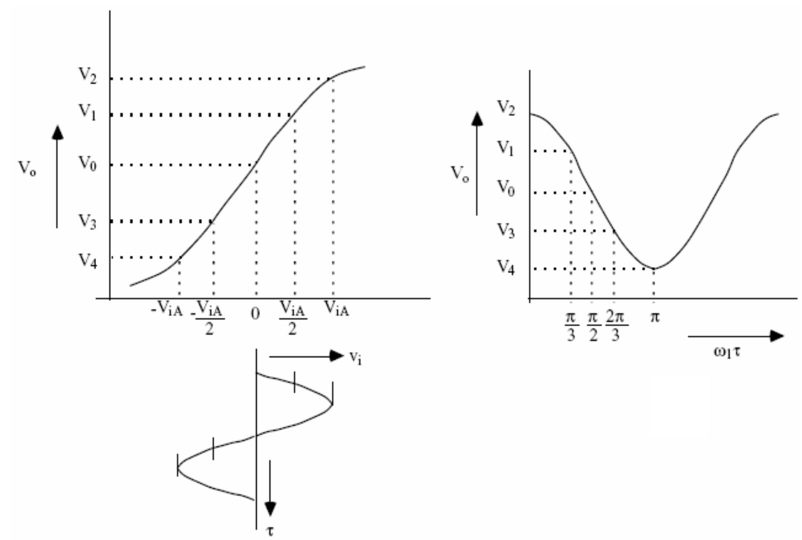
Assume the waveform can be described as

$$V_O + v_o = (V_O + b_0) + b_1 \cos \omega_1 t + b_2 \cos 2\omega_1 t + b_3 \cos 3\omega_1 t + b_4 \cos 4\omega_1 t$$

Select a set of input voltages

- maximum value, V_{iA} (for a cosine: at $\omega_1 t = 0$)
- the half maximum, $V_{iA}/2$ ($\omega_1 t = 60^o$)
- the zero value, 0 ($\omega_1 t = 90^o$)
- the half minimum, $-V_{iA}/2$ ($\omega_1 t = 120^o$)
- and the input minimum, $-V_{iA}$ ($\omega_1 t = 180^o$)

Selection of Waveform Samples



Calculation of b0', b1, b2, b3, b4

$$V_O + b_0 = \frac{1}{6}(V_2 + 2V_1 + 2V_3 + V_4)$$

$$b_1 = \frac{1}{3}(V_2 + V_1 - V_3 - V_4)$$

$$b_2 = \frac{1}{4}(V_2 - 2V_0 + V_4)$$

$$b_3 = \frac{1}{6}(V_2 - 2V_1 + 2V_3 - V_4)$$

$$b_4 = \frac{1}{12}(V_2 - 4V_1 + 6V_0 - 4V_3 + V_4)$$

This is referred to as the 5-point method

3-Point Method

Only 3 input points are used ($\omega_1 t = 0$, $\omega_1 t = 90^o$, $\omega_1 t = 180^o$)

$$V_O + b_0 = \frac{1}{4}(V_2 + 2V_0 + V_4)$$

$$b_1 = \frac{1}{2}(V_2 - V_4)$$

$$b_2 = \frac{1}{4}(V_2 - 2V_0 + V_4)$$

$$HD_2 = \frac{b_2}{b_1} = \frac{1}{2}\left(\frac{V_2 - 2V_0 + V_4}{V_2 - V_4}\right)$$

- **Provides a rough estimate for HD2**
- **For higher-order harmonics an FFT program has to be used**

Common-Source Amplifier

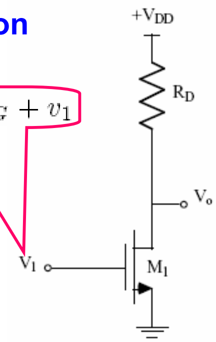
Assume operation in MOS saturation region

$$I_d = \frac{k'}{2}\frac{W}{L}\left(V_{GS} - V_T\right)^2$$

$$V_1 = V_{GS} = V_{GG} + v_1$$

$$I_d = \left[\frac{k'}{2}\frac{W}{L}(V_{GG} - V_T)^2\right]\left[1 + \frac{v_1}{V_{GG} - V_T}\right]^2$$

$$I_d = I_{DA}\left[1 + 2\frac{v_1}{V_{GG} - V_T} + \left(\frac{v_1}{V_{GG} - V_T}\right)^2\right]$$



$$a_0 = I_{DA}$$

$$a_1 = I_{DA}\frac{2}{V_{GG} - V_T}$$

$$a_2 = \frac{I_{DA}}{(V_{GG} - V_T)^2}$$

$$b_0 = \frac{1}{2}\left[\frac{I_{DA}}{(V_{GG} - V_T)^2}\right]V_{1A}^2$$

$$b_1 = 2\left[\frac{I_{DA}}{(V_{GG} - V_T)}\right]V_{1A}$$

$$b_2 = \frac{1}{2}\left[\frac{I_{DA}}{(V_{GG} - V_T)^2}\right]V_{1A}^2$$

Harmonic Distortion Factors

$$HD_2 = \frac{|b_2|}{|b_1|} = \frac{1}{4}\frac{V_{1A}}{V_{GG} - V_T}$$

There are no higher order distortion components since the IV relationship is a quadratic

Example: V_{GG} = 1.5V, V_{1A} = 0.5V, V_T = 1V

$V_o = V_{DD} - I_D R_D$ = 5 – 37.5μA (66.7kΩ) = 2.5V

$$b_0 = 0.0125\ \text{V}$$

$$b_1 = 0.50\ \text{V}$$

$$b_2 = 0.0125\ \text{V}$$

$$HD_2 = \frac{|b_2|}{|b_1|} = 2.5\%$$

SPICE: 2.46%

Intermodulation Distortion (IMD)

IM occurs when 2 or more sinusoidal inputs are applied to a nonlinear circuit

Recall power series representation:

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

Let the input be

$$v_i = V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t$$

Then

$$\begin{aligned} v_o = {} & a_1\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right) + \\ & a_2\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^2 + \\ & a_3\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^3 + \\ & \ldots \end{aligned}$$

Second-Order Term

$$a_2 v_i^2 = a_2\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^2$$

$$\begin{aligned} &= a_2 V_{1A}^2\cos^2\omega_1 t + a_2 V_{2A}^2\cos^2\omega_2 t \\ &\quad + 2a_2 V_{1A}V_{2A}\cos\omega_1 t\cos\omega_2 t \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}a_2\left(V_{1A}^2 + V_{2A}^2\right) + \frac{1}{2}a_2 V_{1A}^2\cos 2\omega_1 t + \frac{1}{2}a_2 V_{2A}^2\cos 2\omega_2 t \\ &\quad + a_2 V_{1A}V_{2A}\cos(\omega_1 + \omega_2)t + a_2 V_{1A}V_{2A}\cos(\omega_1 - \omega_2)t \end{aligned}$$

Intermodulation (Quadratic Term)

Frequency	0	$2f_1$	$2f_2$	$f_1 \pm f_2$
Amplitude	$\frac{a_2}{2}\left[V_{1A}^2 + V_{2A}^2\right]$	$\frac{a_2}{2}V_{1A}^2$	$\frac{a_2}{2}V_{2A}^2$	$a_2 V_{1A} V_{2A}$

- Second-order term produces a dc shift
- Output has 2nd harmonics of input signals
- Terms at sum/difference frequencies $\rightarrow$ beat terms
- IM_2 refers to 2nd-order terms
- IM_2 produced by even-order terms of power series
- Low input levels IM_2 due to quadratic term

IM_2 Distortion

IM_2 is the ratio of the amplitude of one of the beat terms to the amplitude of the fundamental

$$IM_2 = \frac{a_2 V_{1A} V_{2A}}{a_1 V_{1A}} = \frac{a_2}{a_1} V_{2A}$$

For simplicity, IM_2 is defined for equal amplitudes of the two input signals ($V_{1A} = V_{2A}$)

$$IM_2 = \frac{a_2}{a_1} V_{1A} = 2\left(\frac{1}{2}\frac{a_2}{a_1} V_{1A}\right) = 2HD_2$$

IM_2 in terms of the fundamental of the output ($V_{oA} \approx a_1 V_{1A}$)

$$IM_2 = \frac{a_2}{a_1^2} V_{oA}$$