Intermodulation Distortion (IMD)

IM occurs when 2 or more sinusoidal inputs are applied to a nonlinear circuit

Recall power series representation:

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots$$

Let the input be

$$v_i = V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t$$

Then

$$\begin{aligned} v_o = {} & a_1\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right) + \\ & a_2\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^2 + \\ & a_3\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^3 + \\ & \dots \end{aligned}$$

Second-Order Term

$$a_2 v_i^2 = a_2\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^2$$

$$\begin{aligned} &= a_2 V_{1A}^2\cos^2\omega_1 t + a_2 V_{2A}^2\cos^2\omega_2 t \\ &\quad + 2a_2 V_{1A}V_{2A}\cos\omega_1 t\cos\omega_2 t \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2}a_2\left(V_{1A}^2 + V_{2A}^2\right) + \frac{1}{2}a_2 V_{1A}^2\cos 2\omega_1 t + \frac{1}{2}a_2 V_{2A}^2\cos 2\omega_2 t \\ &\quad + a_2 V_{1A}V_{2A}\cos(\omega_1+\omega_2)t + a_2 V_{1A}V_{2A}\cos(\omega_1-\omega_2)t \end{aligned}$$

Intermodulation (Quadratic Term)

Frequency	**0**	**$2f_1$**	**$2f_2$**	**$f_1 \pm f_2$**
Amplitude	$\frac{a_2}{2}\left[V_{1A}^2 + V_{2A}^2\right]$	$\frac{a_2}{2}V_{1A}^2$	$\frac{a_2}{2}V_{2A}^2$	$a_2 V_{1A}V_{2A}$

- **Second-order term produces a dc shift**
- **Output has 2nd harmonics of input signals**
- **Terms at sum/difference frequencies → beat terms**
- **IM_2 refers to 2nd-order terms**
- **IM_2 produced by even-order terms of power series**
- **Low input levels IM_2 due to quadratic term**

IM_2 Distortion

IM_2 is the ratio of the amplitude of one of the beat terms to the amplitude of the fundamental

$$IM_2 = \frac{a_2 V_{1A}V_{2A}}{a_1 V_{1A}} = \frac{a_2}{a_1}V_{2A}$$

For simplicity, IM_2 is defined for equal amplitudes of the two input signals ($V_{1A} = V_{2A}$)

$$IM_2 = \frac{a_2}{a_1}V_{1A} = 2\left(\frac{1}{2}\frac{a_2}{a_1}V_{1A}\right) = 2HD_2$$

IM_2 in terms of the fundamental of the output ($V_{oA} \approx a_1 V_{1A}$)

$$IM_2 = \frac{a_2}{a_1^2}V_{oA}$$

Intermodulation (Cubic Term) IM_3

$$a_3 v_i^3 = a_3\left(V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t\right)^3$$
$$= a_3 V_{1A}^3\cos^3\omega_1 t + 3a_3 V_{1A}V_{2A}^2\cos\omega_1 t\cos^2\omega_2 t$$
$$+3a_3 V_{1A}^2 V_{2A}\cos^2\omega_1 t\cos\omega_2 t + a_3 V_{2A}^3\cos^3\omega_2 t$$

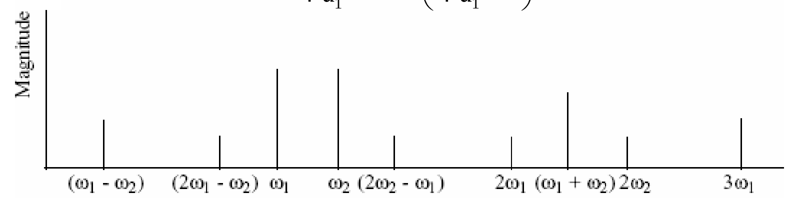
Frequency	f_1	f_2	$2f_1 \pm f_2$	$2f_2 \pm f_1$	$3f_1$	$3f_2$
Amplitude	$\frac{3a_3}{4}\left[V_{1A}^3 + V_{1A}V_{2A}^2\right]$	$\frac{3a_3}{4}\left[V_{2A}^3 + V_{1A}^2 V_{2A}\right]$	$\frac{3}{4}a_3 V_{1A}^2 V_{2A}$	$\frac{3}{4}a_3 V_{1A} V_{2A}^2$	$\frac{1}{4}a_3 V_{1A}^3$	$\frac{1}{4}a_3 V_{2A}^3$

Common test condition for IM_3 is to have equal amplitudes of the two input signals ($V_{1A} = V_{2A}$)

$$V_o(IM_3) = \frac{3}{4}a_3 V_{1A}^3 \qquad IM_3 = \frac{3}{4}\frac{a_3 V_{1A}^3}{a_1 V_{1A}} = \frac{3}{4}\frac{a_3}{a_1}V_{1A}^2$$

IM3 Distortion

$$IM_3 = \frac{3}{4}\frac{a_3}{a_1}V_{1A}^2 = 3\left(\frac{1}{4}\frac{a_3}{a_1}V_{1A}^2\right) = 3HD_3$$



- **Third-order IM components ($2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$) are close to the fundamentals if ω_1 is close to ω_2**
- **This commonly occurs with broadcast receiver applications**
- **IM3 terms may fall in the passband of the receiver and cannot be filtered out**

Triple Beat (TB) Distortion

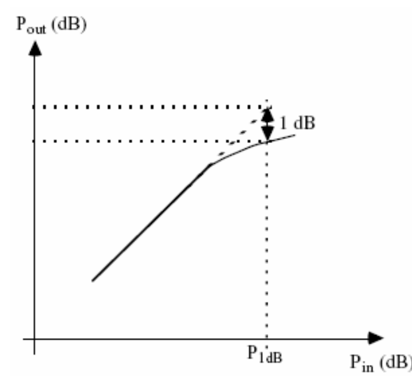
- **When three or more input signals are present at the input of an amplifier, more beat terms are obtained**
- **For the case of three input signals, a ‘triplebeat’ results**

$$V_O(TB) = \frac{3}{2}a_3 V_{1A}V_{2A}V_{3A}\cos\left(\omega_1 \pm \omega_2 \pm \omega_3\right)t$$

$$TB = \frac{3}{2}\frac{a_3}{a_1}V_{1A}^2 = 2IM_3 = 6HD_3$$

Compression and Intercept Points for Characterizing Distortion in RF Circuits

The 1 dB compression point is defined as the input power for which the fundamental output power is 1 dB below the extrapolated small-signal fundamental power



Calculation of 1-dB Compression Point

Fundamental term from power series analysis is

$$b_1 \approx a_1 V_{iA} + \frac{3}{4} a_3 V_{iA}^3$$

Fundamental power is

$$P_{fund} = 20 \log \left(a_1 V_{iA} - \frac{3}{4} |a_3| V_{iA}^3 \right)$$

Extrapolated small-signal power

$$P_{ss} = 20 \log (a_1 V_{iA})$$

1 dB compression point given by

$$P_{fund} = P_{ss} - 1 \text{ dB}$$

Input Voltage for 1-dB Compression Point

$$20 \log \left(a_1 V_{iA} - \frac{3}{4} |a_3| V_{iA}^3 \right) = 20 \log (a_1 V_{iA}) - 1 \text{ dB}$$

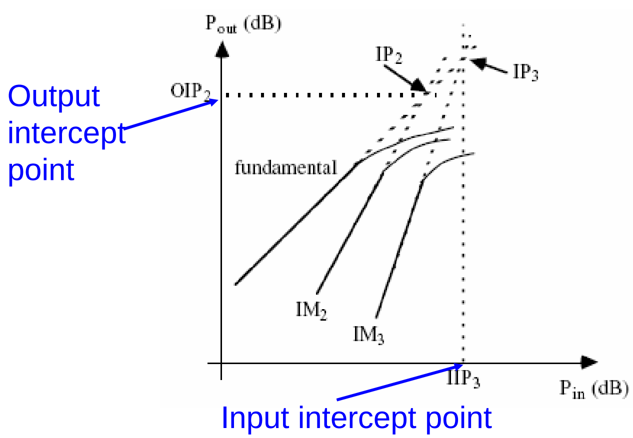
$$= 20 \log \left(\frac{a_1 V_{iA}}{1.12} \right)$$

Input voltage corresponding to 1 dB compression point given by

$$V_{iA} = \sqrt{0.145 \left| \frac{a_1}{a_3} \right|}$$

Intercept Points

The intercept points (IP) are defined as the intercept of the extrapolated small-signal power of the intermodulation terms with the fundamental



Calculation of IIP3

From definition and power series analysis

$$|a_1| V_{iA} = \frac{3}{4} |a_3| V_{iA}^3$$

or

$$V_{iA} = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|}$$

Relation between 1-dB compression and IIP3

1 dB compression $\left\{ V_{iA} = \sqrt{0.145 \left| \frac{a_1}{a_3} \right|} \right.$ IIP3 $\left\{ V_{iA} = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|} \right.$

$$\frac{V_{iA}(1dB)}{V_{iA}(IP3)} = \sqrt{\frac{0.145}{4/3}} = \sqrt{0.11} = -9.6 \text{ dB}$$

Cross Modulation (CM)

Cross modulation is due to the transfer of the modulation from an unwanted AM signal to a signal of interest due to circuit nonlinearities

$$v_i = v_1 + v_2 = V_{1A}\cos\omega_1 t + V_{2A}(1 + m\cos\omega_m t)\cos\omega_2 t$$

m < 1 is the modulation index
Due to the cubic term of the nonlinearity the ω_1 term is

$$a_1 V_{1A}\left(1 + \frac{3a_3 V_{2A}^2 m}{a_1}\cos\omega_m t\right)\cos\omega_1 t$$

CM is defined as

$$CM = \frac{3a_3 V_{2A}^2}{a_1}$$

Blocking

Consider a weak signal in the presence of a strong interferer

$$v_i = V_{1A}\cos\omega_1 t + V_{2A}\cos\omega_2 t$$

Due to the cubic term of the nonlinearity the ω_1 term is

$$\left(a_1 V_{1A} + \frac{3}{4}a_3 V_{1A}^3 + \frac{3}{2}a_3 V_{1A}V_{2A}^2\right)\cos\omega_1 t$$

For $V_{1A} << V_{2A}$ $\left(a_1 + \frac{3}{2}a_3 V_{2A}^2\right)V_{1A}\cos\omega_1 t$

- **For $a_3 < 0$, gain reduces as V_2 increases and the signal is "blocked"**
- **Need to withstand blockers 60-70dB larger**

Differential Error Approach

- **Useful for estimating small distortions**
  - Find small-signal gain at
    - Quiescent operating point
    - Positive peak of input
    - Negative peak of input
  - Define new parameters
  - Develop expressions for HD2 and HD3

Differential Error Distortion Analysis

Consider the power series expression

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

The small-signal gain is

$$a = \frac{dv_o}{dv_i} = a_1 + 2a_2 v_i + 3a_3 v_i^2 + \ldots$$

At the quiescent operating point $v_i = 0 \Rightarrow$ gain = a_1

For input $v_i = V_{iA}\cos\omega_1 t$

$$a^+ = a_1 + 2a_2 V_{iA} + 3a_3 V_{iA}^2 + \ldots$$
$$a^- = a_1 + 2a_2(-V_{iA}) + 3a_3(-V_{iA})^2 + \ldots$$

Distortion Factors

Define two new coefficients

$$E^+ = \frac{a^+ - a_1}{a_1} = \frac{2a_2V_{iA} + 3a_3V_{iA}^2 + \ldots}{a_1}$$

$$E^- = \frac{a^- - a_1}{a_1} = \frac{-2a_2V_{iA} + 3a_3V_{iA}^2 + \ldots}{a_1}$$

From this

$$E^+ - E^- \approx \frac{4a_2V_{iA}}{a_1} \qquad E^+ + E^- \approx \frac{6a_3V_{iA}^2}{a_1} \quad \Rightarrow \quad a_2 = \frac{a_1(E^+ - E^-)}{4V_{iA}} \qquad a_3 = \frac{a_1(E^+ + E^-)}{6V_{iA}^2}$$

$$HD_2 \approx \frac{1}{2}\frac{a_2}{a_1}V_{iA} \qquad HD_3 \approx \frac{1}{4}\frac{a_3}{a_1}V_{iA}^2 \quad \Rightarrow \quad HD_2 = \frac{E^+ - E^-}{8} \qquad HD_3 = \frac{E^+ + E^-}{24}$$

Negative Feedback - Advantages

- **Stabilize gain of an amplifier against parameter changes in devices due to supply voltage variation, temperature changes, or device aging**
- **Modify input and output impedances to suit needs**
  - e.g., voltage amplifier: large input impedance and small output impedance
- **Reduce distortion in amplifiers (the input-output transfer characteristics become more linear)**
  - All audio amplifiers
- **Increase bandwidth of circuits**
  - Widely used in broadband amplifiers

Negative Feedback - Disadvantages

- **Gain is reduced in almost direct proportion to other benefits (add extra gain stages)**
- **Feedback causes instability and there is a tendency for oscillation**

  $\Rightarrow$ An amplifier may become an oscillator!

Review of Feedback Terminology

$$\frac{S_o}{S_i} = A = \frac{a}{1+af}$$

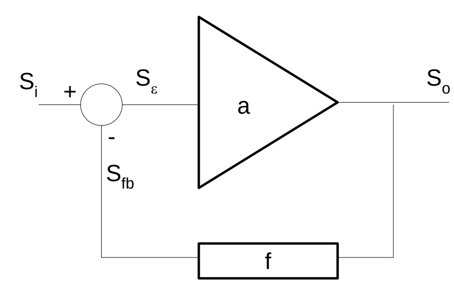
a = open-loop gain

$T = af$ = loop gain

A = closed-loop gain

$$A = \frac{a}{1+T}$$

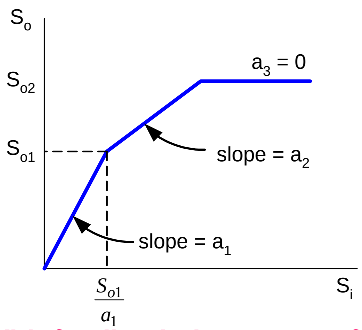
For $T >> 1$ $\quad A \cong \frac{1}{f}$

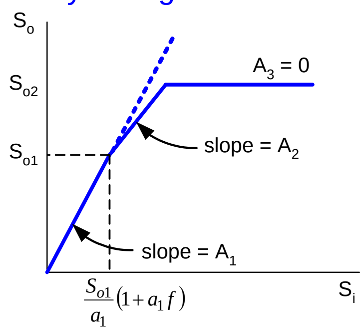


Effect on Distortion

Negative feedback keeps overall gain A approximately constant

⇒ it should be effective in reducing distortion, because distortion is caused by changes in 'a'

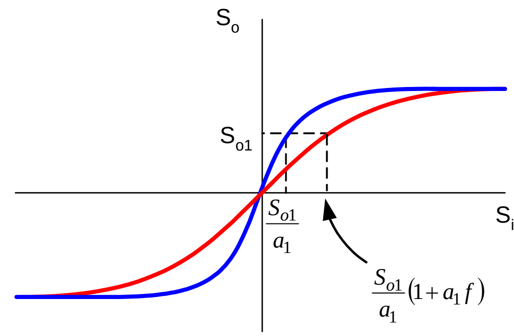




With feedback the slopes of the transfer function curves are

$$A_1 = \frac{a_1}{1+a_1 f} = \frac{1}{f+\frac{1}{a_1}} \cong \frac{1}{f} \qquad A_2 = \frac{a_2}{1+a_2 f} = \frac{1}{f+\frac{1}{a_2}} \cong \frac{1}{f}$$

Effect on Distortion



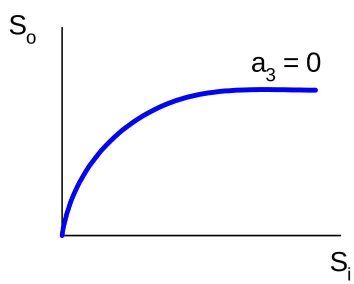
The transfer characteristic is much less nonlinear than that for the original amplifier

Negative feedback <u>reduces the gain</u> but <u>improves the linearity</u>

What About a Hard Nonlinearity?

When the output shows hard saturation then the gain is zero and negative feedback cannot improve the situation

$$A_3 = \frac{a_3}{1+a_3 f} = \frac{0}{1} = 0$$



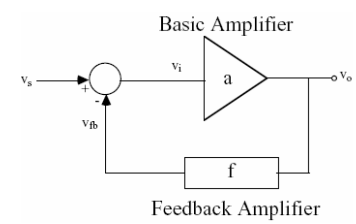
Distortion Analysis with Feedback

Consider the power series expression

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

With feedback

$$v_i = v_s - v_{fb} = v_s - f v_o$$



The output voltage with feedback is

$$v_o = a_1(v_s - f v_o) + a_2(v_s - f v_o)^2 + \ldots$$

Therefore, the output voltage is a power series in v_s

$$v_o = a_1' v_s + a_2' v_s^2 + \ldots$$

Power Series with Feedback

Since the power series is a Taylor's series

$$v_o = \left.\frac{dv_o}{dv_s}\right|_{0,0} v_s + \frac{1}{2}\left.\frac{d^2 v_o}{dv_s^2}\right|_{0,0} v_s^2 + \ldots$$

The coefficients of the power series are evaluated using the fact that $v_s = 0 \Rightarrow v_o = 0$

$$\left.\frac{dv_o}{dv_s}\right|_{0,0} = a_1'$$

$$\left.\frac{d^2 v_o}{dv_s^2}\right|_{0,0} = 2a_2'$$

$$\left.\frac{d^3 v_o}{dv_s^3}\right|_{0,0} = 6a_3'$$

Distortion Factors with Feedback

The coefficients are

$$a_1' = \frac{a_1}{(1+a_1 f)}$$ ← Usual small-signal case T = af

$$a_2' = \frac{a_2}{(1+a_1 f)^3}$$

$$a_3' = \frac{a_3(1+a_1 f) - 2a_2^2 f}{(1+a_1 f)^5}$$

Harmonic distortion factors are

$$HD_2 \approx \frac{1}{2}\frac{a_2'}{a_1'}V_{sA} = \frac{1}{2}\frac{a_2}{a_1}V_{sA}\frac{1}{(1+a_1 f)^2}$$

$$\approx \frac{1}{(1+a_1 f)^2}\, HD2|_{\mathrm{w/o\ fb}}$$

$$HD_3 \approx \frac{1}{4}\frac{a_3'}{a_1'}V_{sA}^2 = \frac{1}{4}\frac{a_3}{a_1}V_{sA}^2\frac{\left|1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right|}{(1+a_1 f)^3}$$

$$\approx \frac{\left|1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right|}{(1+a_1 f)^3}\, HD3|_{\mathrm{w/o\ fb}}$$

Distortion Factors for Constant V_{oA}

Open loop

$$HD_2 \approx \frac{1}{2}\frac{a_2}{a_1^2}V_{oA}$$

$$HD_3 \approx \frac{1}{4}\frac{a_3}{a_1^3}V_{oA}^2$$

Closed loop

$$HD_2 \approx \frac{1}{2}\frac{a_2'}{a_1'^2}V_{oA} = \frac{1}{2}\frac{a_2}{a_1^2}V_{oA}\frac{1}{1+a_1 f}$$

$$\approx \frac{1}{1+a_1 f}\, HD2|_{\mathrm{w/o\ fb}}$$

$$HD_3 \approx \frac{1}{4}\frac{a_3'}{a_1'^3}V_{oA}^2 = \frac{1}{4}\frac{a_3}{a_1^3}V_{oA}^2\frac{\left|1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right|}{1+a_1 f}$$

$$\approx \frac{\left|1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right|}{1+a_1 f}\, HD3|_{\mathrm{w/o\ fb}}$$

Observations on HD2 and HD3

- **HD2 is reduced by $(1 + a_1 f)^2$ when input voltage is held constant and is reduced by $(1+a_1 f)$ when output voltage is kept constant**
- **If $a_2 = 0$, i.e., no second-order distortion then HD3 is also reduced by $(1+a_1 f)$ for a constant output**
- **Cancellation of third-order distortion occurs for**

$$\frac{2a_2^2 f}{a_3(1+a_1 f)} = 1$$

  – Important when IM3 needs to be reduced