## Noise in Integrated Circuits

- **Sensitivity of communication systems limited by noise**
- **We are interested in noise phenomena caused by small current and voltage fluctuations generated within the devices**
  - These fluctuations give rise to uncertainty in current/voltage at nodes of a circuit
- **Noise is present due to the fact that charge is not continuous but is carried in discrete amounts $q=1.6\times10^{-19}$ C (quantization of charge)**

**Nyquist, Johnson, Schottky were pioneers who explained the origins of noise**

## Why is Noise a Problem?

- **Noise represents a lower limit to the amplitude of the signal that can be amplified (or the circuit can process) without seriously affecting the signal quality**
- **Noise places an upper limit on the useful gain of an amplifier**
  - If gain is increased without limit, output stages of the circuit eventually begin to limit – on the amplified noise from the input stages

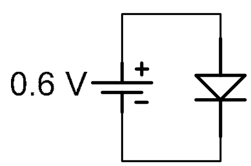
## Shot Noise

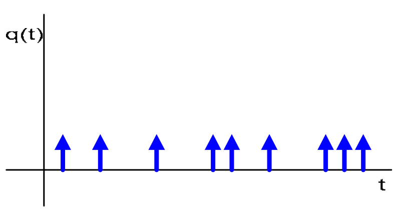
- **Described and explained by Schottky in 1918**
- **Associated with flow of carriers across a barrier (pn junction)**

**Consider a diode:**

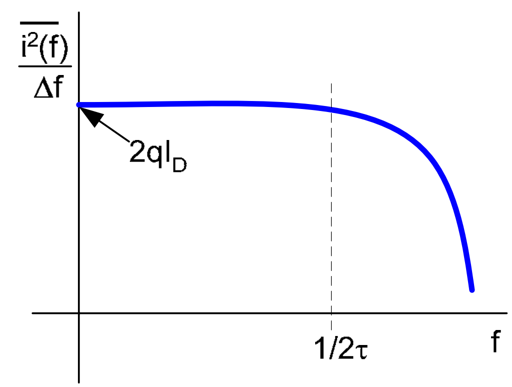
**Some carriers get enough energy to cross over the barrier**

**Since they cross the barrier randomly arrival of charge is random**



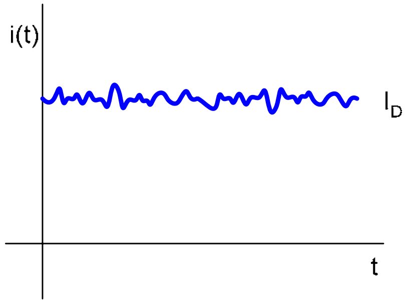

## Shot Noise (Frequency Spectrum)




$$\frac{\overline{i^2}}{\Delta f} = 2qI_d$$

**is constant over the frequency range of interest ⇒ white noise**

# Physical Interpretation



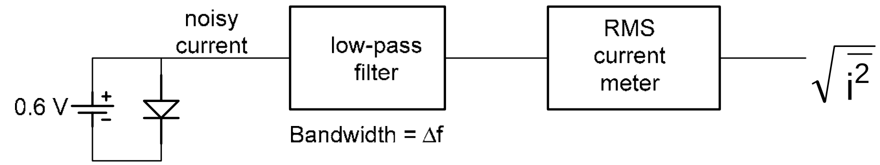

**i(t) is the sum of a DC term $I_D$ and a time varying term that is random**

**$I = I_D + i$ (random, zero mean)**

**$\overline{i^2}$ = mean square variation = $\overline{(I - I_D)^2}$** $= \lim_{T\to\infty} \frac{1}{T}\int_0^T (I - I_D)^2 dt$

$= 2qI_D\Delta f$

# Measurement of Noise



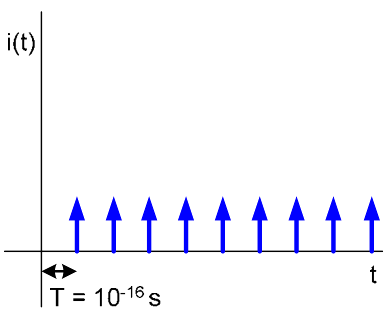

**$\overline{i^2}$ = mean squared value ($A^2$)**
**power delivered by the noise current over a bandwidth of $\Delta f$ to a 1Ω resistor**

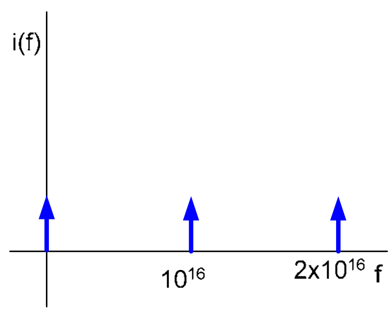
**$\sqrt{\overline{i^2}}$ = root mean squared value (A) (RMS)**

**$\frac{\overline{i^2}}{\Delta f}$ = power per unit Hz (spectral density)**

# Why is This a Random Phenomena

**$I_D$ = 1.6mA ⇒ current pulses every $(1/1.6 \times 10^{-3})1.6\times10^{-19}$ s = $10^{-16}$ s**




**The spectrum is periodic with a fundamental of $10^{16}$ Hz, i.e., no noise would be produced in the normal frequency range**

**⇒ Shot noise would not even be a problem!**

# Properties of Shot Noise

- **Related to a dc current**
- **Independent of temperature**
- **Spectral density independent of frequency**

$$\frac{\overline{i^2}}{\Delta f} = 2qI_D \quad \Rightarrow \text{white noise}$$

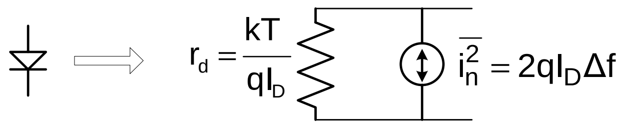
- **Noise power $\propto \Delta f, I_D$**

## Diode: Example + Small-Signal Model

$$I_D = 1\text{mA}, \qquad \Delta f = 1\text{MHz}$$

$$\overline{i_D^2} = 2q(1\text{mA})\Delta f = 2 \times 1.6 \times 10^{-19} \times 10^{-3} \times 10^{6}$$
$$= 3.2 \times 10^{-16}\ \text{A}^2$$

$$\therefore\ \text{rms value} = \sqrt{\overline{i_D^2}} = 1.7 \times 10^{-8}\ \text{A} = 17\text{nA}$$

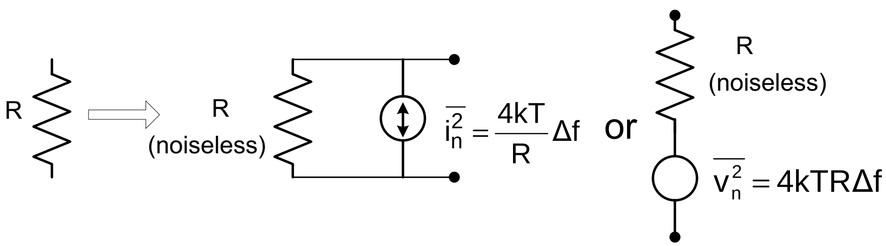



**Since the noise signal has random phase, and is defined solely in terms of its mean square value, it has no polarity**

## Thermal Noise

- **Also called Johnson or Nyquist noise**
- **Associated with random thermal motion of electrons within the physical body of a resistor**
- **Thermally agitated charge carriers in a conductor result in a randomly varying current, hence, a random voltage**
- **Thermal origin $\Rightarrow$ directly related to temperature. As temperature approaches 0 absolute, thermal noise also approaches 0**

## Resistor Thermal Noise




## Properties of Thermal Noise

- **White noise, uniformly distributed in frequency domain**
- **Proportional to absolute temperature**
- **Noise source not associated with a dc current or barrier**
- **Present in any physical resistor (not for resistors used in small signal equivalent circuits such as $r_o$, $r_\pi$)**

## Thermal Noise Example

For a 1k resistor at 300°K

$$\frac{\overline{v_n^2}}{\Delta f} = 16\times10^{-18} \quad v^2/Hz$$

$$\therefore \quad \sqrt{\overline{v_n^2}} = (4nV/\sqrt{Hz})\sqrt{\Delta f}$$

**Thermal noise current of a 1kΩ resistor at 300°K is the same as 50 μA of direct current exhibiting shot noise**

**The only way to reduce thermal noise of a given resistance is to keep the temperature as low as possible and the limit the bandwidth to the minimum useful value**

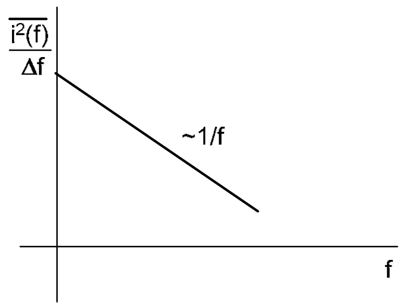
## Flicker Noise or 1/f Noise (Pink Noise)

- **Origins of flicker noise are varied**
- **Present in diodes, BJTs, and MOSFETs**
- **Tends to be associated with surface states**
  - Surface states capture and release carriers in a random manner
- **The noise signal has energy concentrated at low frequencies**

$$\overline{i^2} = k_1 \frac{I^a}{f^b}\Delta f$$

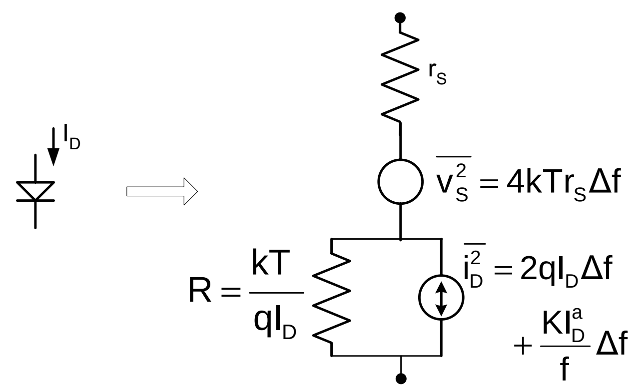
**I = direct current**
**a = a constant 0.5 < a < 2**
**b = a constant ~ 1 ⇒ "1/f noise"**
**$k_1$ varies with device**

## Flicker Noise Spectrum



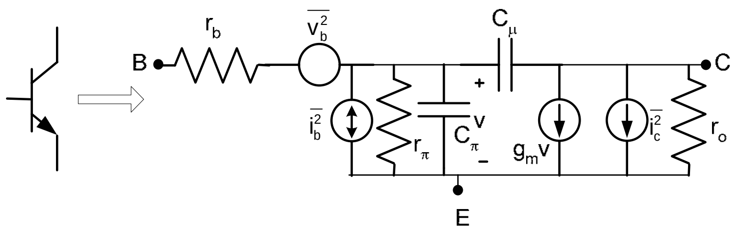

- **1/f noise important for low frequency operation**
- **$K_1$ can vary orders of magnitude from one device type to another**
- **$K_1$ can also vary widely for different transistors on the same wafer due to dependence on defects**

## Small-Signal Diode Noise Model




**$r_s$ = series ohmic resistance (a physical resistor)**

## Small-Signal BJT Noise Model



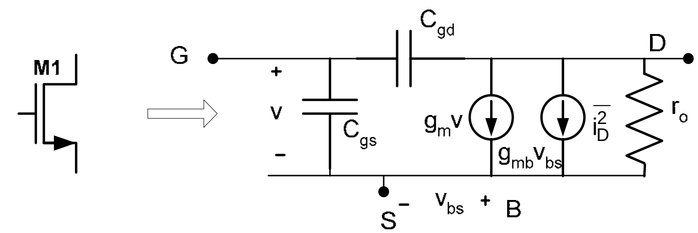

$$\overline{v_b^2} = 4kTr_b\Delta f$$

$$\overline{i_b^2} = 2qI_B\Delta f + K_1\frac{I_B^a}{f}\Delta f$$

$$\overline{i_c^2} = 2qI_C\Delta f$$

**$r_b$ = series base resistance (a physical resistor)**

## Small-Signal MOSFET Noise Model




$$\overline{i_D^2} = 4kT\gamma g_{d0}\Delta f + \frac{K}{f}\frac{g_m^2}{WLC_{ox}^2}\Delta f$$

where $g_{d0} = g_{ds}\big|_{V_{ds}=0}$

$\gamma = 1$ at zero $V_{DS}$ and 2/3 in saturation for long channel devices

In short-channel devices $\gamma$ is typically 2-3 and could be larger (??)

$$g_{ds}\big|_{V_{ds}=0} = g_{m,sat}$$

$$4kT\gamma g_{d0}\Delta f = 4kT\left(\frac{2}{3}\right)g_m$$

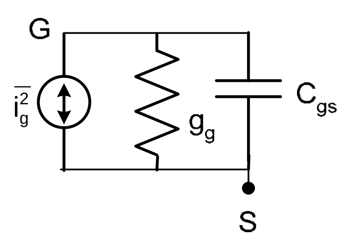
## MOSFET Induced Gate Noise

- Current flow through resistive channel ⇒ thermal noise (drain current noise)
- Because of capacitive coupling between channel and gate ⇒ thermal noise current at gate
- Important at high frequencies

$$\overline{i_g^2} = 4kT\delta g_g\Delta f$$

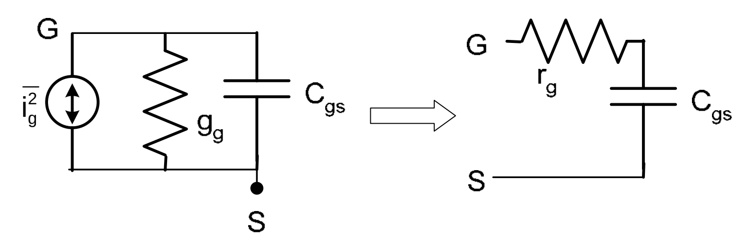
where $g_g = \dfrac{\omega^2 C_{gs}}{5g_{do}}$

$\delta = 4/3$ for long channel devices




## More on Induced Gate Noise

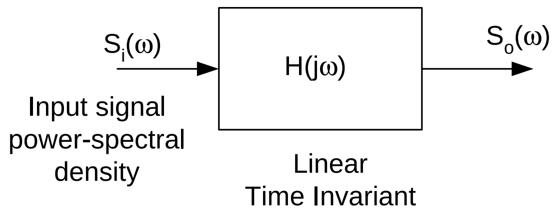
- The spectral density of gate current increases with frequency ⇒ important at high frequencies
- Noise source can be recast into equivalent form




$$\overline{v_g^2} = 4kT\delta r_g\Delta f$$

where $r_g = \dfrac{1}{5g_{do}}$

# Circuit Noise Calculations




$$S_o(\omega) = S_i(\omega)\left[H(j\omega)H^*(j\omega)\right]$$
$$= S_i(\omega)\left|H(j\omega)\right|^2$$
$$\therefore \sqrt{S_o(\omega)} = \sqrt{S_i(\omega)}\left|H(j\omega)\right|$$

# Analysis Procedure

**Assume noise sources are uncorrelated**
**(separate, independent physical mechanisms)**

**For each source $\overline{i_n^2}$ $(\overline{v_n^2})$ :**

**1)Replace with a sinusoidal, deterministic source of value $i_n$ $(v_n)$**

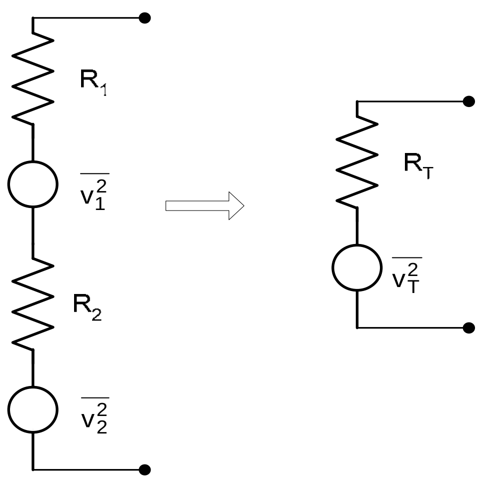
$$\sqrt{\overline{i_n^2}} \quad \left(\sqrt{\overline{v_n^2}}\right)$$

**2)Calculate** $v_{on}(\omega) = i_n(\omega)\left|H(j\omega)\right|$

$$\frac{\overline{v_{on}^2}}{\Delta f} = \text{output voltage spectral density due to } \overline{i_n^2}$$

$$\overline{v_{o_{total}}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \ldots$$

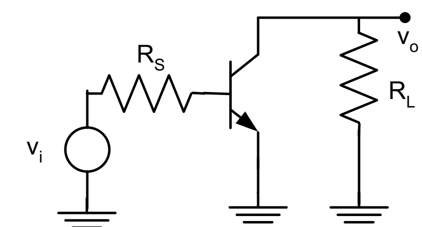
# Resistor Example

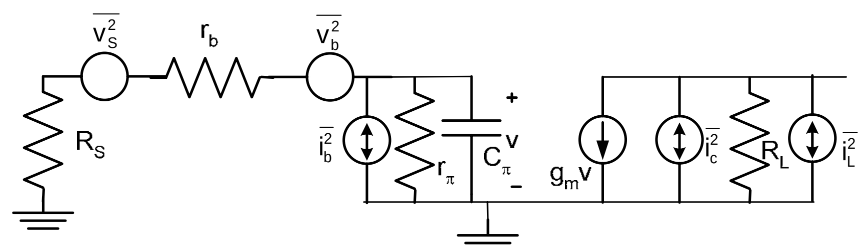



$$\overline{v_T^2} = \overline{v_1^2} + \overline{v_2^2} = 4kT(R_1 + R_2)\Delta f$$

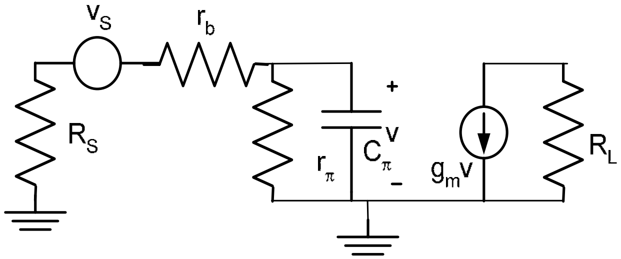
# BJT Example

$$\overline{v_S^2} = 4kTR_S\Delta f$$
$$\overline{v_b^2} = 4kTr_b\Delta f$$
$$\overline{i_b^2} = 2qI_B\Delta f \text{ (ignore 1/f noise)}$$
$$\overline{i_c^2} = 2qI_C\Delta f$$
$$\overline{i_L^2} = 4kT\frac{1}{R_L}\Delta f$$



## Output Noise Due to $\overline{v_s^2}$




$$v=\frac{r_\pi \| 1/j\omega C_\pi}{R_S+r_b+r_\pi \| 1/j\omega C_\pi}v_s=\frac{r_\pi/(1+j\omega C_\pi r_\pi)}{R_S+r_b+r_\pi/(1+j\omega C_\pi r_\pi)}v_s$$

$$v_o=-g_m R_L v$$

$$=-g_m R_L r_\pi \frac{1}{r_\pi+(R_S+r_b)(1+j\omega C_\pi r_\pi)}v_s$$

$$v_o=-\frac{\beta R_L}{r_\pi+R_S+r_b}\left(\frac{1}{1+j\omega\tau}\right)v_s$$

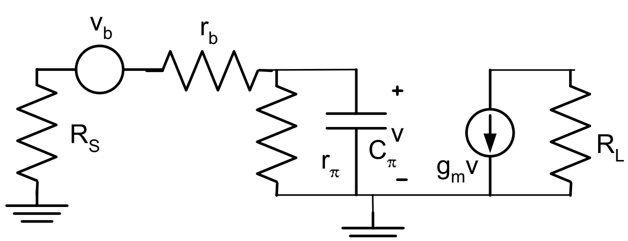
where $\tau=C_\pi\left(r_\pi \| (R_S+r_b)\right)$

## Output Noise Due to $\overline{v_s^2}$

$$\overline{v_{o1}^2}=\left(\frac{\beta R_L}{r_\pi+R_S+r_b}\right)^2\left(\frac{1}{1+(\omega\tau)^2}\right)\overline{v_s^2}$$

$$=\left(\frac{\beta R_L}{r_\pi+R_S+r_b}\right)^2\frac{1}{1+\left(\frac{f}{f_1}\right)^2}\overline{v_s^2} \qquad \text{where } f_1=\frac{1}{2\pi\tau}$$

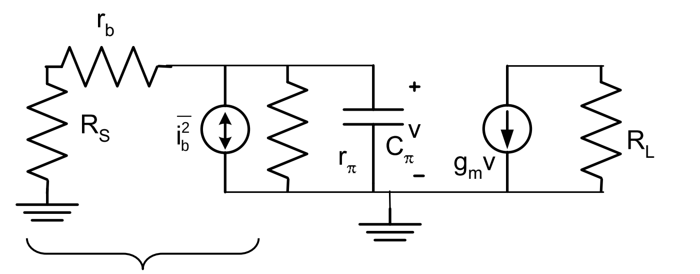
## Output Noise Due to $\overline{v_b^2}$



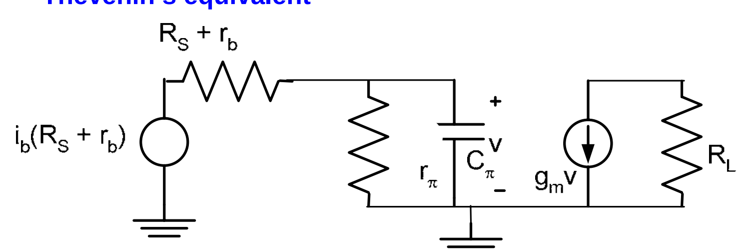

$$\overline{v_{o2}^2}=\left(\frac{\beta R_L}{r_\pi+R_S+r_b}\right)^2\left(\frac{1}{1+(\omega\tau)^2}\right)\overline{v_b^2}$$

$$=\left(\frac{\beta R_L}{r_\pi+R_S+r_b}\right)^2\frac{1}{1+\left(\frac{f}{f_1}\right)^2}\overline{v_b^2} \qquad \text{where } f_1=\frac{1}{2\pi\tau}$$

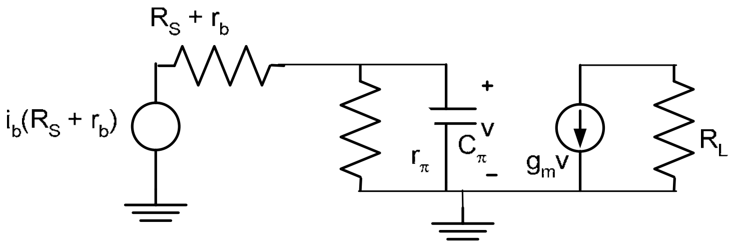
## Output Noise Due to $\overline{i_b^2}$




Thevenin's equivalent

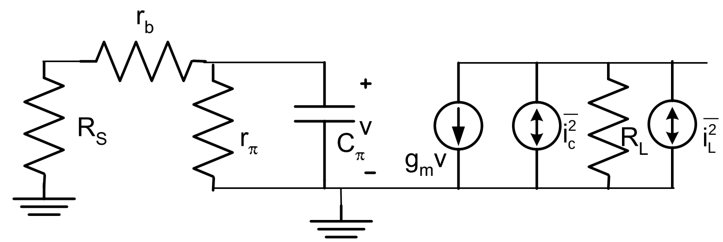


## Output Noise Due to $\overline{i_b^2}$




$$\overline{v_{o3}^2} = \left(\frac{\beta R_L}{r_\pi + R_S + r_b}\right)^2 \frac{1}{1+\left(\frac{f}{f_1}\right)^2}(R_S + r_b)^2\overline{i_b^2} \quad \text{where } f_1 = \frac{1}{2\pi\tau}$$

## Output Noise Due to $\overline{i_c^2}$ and $\overline{i_L^2}$




$$\overline{v_{o4}^2} = R_L^2\overline{i_c^2}$$
$$\overline{v_{o5}^2} = R_L^2\overline{i_L^2}$$

## Total Output Noise

$$\overline{v_{oT}^2} = \sum_{i=1}^{5}\overline{v_{oi}^2}$$

$$\overline{v_{oT}^2} = |A_V|^2\left(\overline{v_s^2} + \overline{v_b^2} + (r_b + R_S)^2\overline{i_b^2}\right) + R_L^2\overline{i_c^2} + R_L^2\overline{i_L^2}$$

$$\text{where } A_V = \frac{\beta R_L}{r_\pi + R_S + r_b}\frac{1}{1 + j\frac{f}{f_1}}$$

$$\overline{v_{oT}^2} = |A_V|^2\left(4KT(r_b + R_S) + 2qI_B(r_b + R_S)^2\right)\Delta f + 2qI_C R_L^2\Delta f + 4KTR_L\Delta f$$