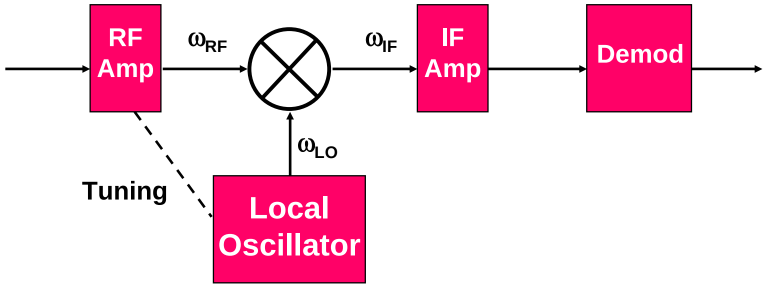
Down Conversion Mixer

- Super-heterodyne receiver

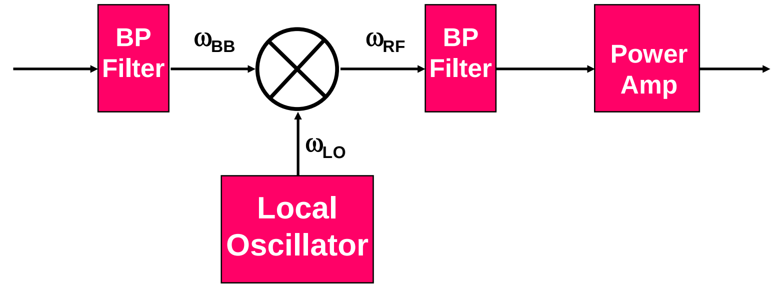


– Mixer translates high frequency RF signal to a lower frequency IF

$$\omega_{IF} = \omega_{LO} - \omega_{RF}$$

Up Conversion Mixer

- Super-heterodyne transmitter



– Mixer translates a low frequency baseband signal to a higher frequency RF

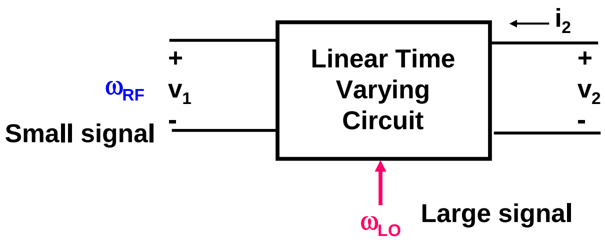
$$\omega_{RF} = \omega_{LO} + \omega_{BB}$$

Down Conversion Mixer

$\omega_{IF} = \omega_{LO} - \omega_{RF}$

- **Linear time invariant networks do not produce spectral components not in the input**

  $\Rightarrow$ Use time varying or nonlinear networks



i_2, v_2 represent components only due to v_1

Analysis of LTV Network

$$i_2 = g(t)v_1$$

g(t) is the time varying transconductance due to the LO at ω_{LO}

$$g(t) = g_0 + g_1 \text{Cos}\,\omega_{LO}t + g_2 \text{Cos}\,2\omega_{LO}t + \ldots$$

For $v_1(t) = V_1 \text{Cos}\,\omega_{RF}t$

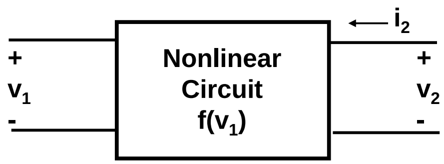
$$i_2(t) = (g_0 + g_1 \text{Cos}\,\omega_{LO}t + g_2 \text{Cos}\,2\omega_{LO}t + \ldots)V_1 \text{Cos}\,\omega_{RF}t$$

$$= g_0 V_1 \text{Cos}\,\omega_{RF}t + \frac{g_1 V_1}{2}\text{Cos}(\omega_{LO} \pm \omega_{RF})t + \frac{g_2 V_1}{2}\text{Cos}(2\omega_{LO} \pm \omega_{RF})t + \ldots$$

The component of $i_2(t)$ at the IF is $\frac{g_1 V_1}{2}\text{Cos}(\omega_{LO} - \omega_{RF})t$

$$G_{mconv} = \frac{|\text{IF current output}|}{|\text{RF voltage input}|} = \frac{g_1 V_1 / 2}{V_1} = \frac{g_1}{2}$$

Nonlinear Network



$$i_2 = f(v_1) = \sum_i a_i v_1^i$$

Consider a quadratic nonlinearity

$$i_2 = a_1 v_1 + a_2 v_1^2$$

$$v_1(t) = V_{RF}\text{Cos}\omega_{RF}t + V_{LO}\text{Cos}\omega_{LO}t$$

$$i_2(t) = a_1\left(V_{RF}\text{Cos}\omega_{RF}t + V_{LO}\text{Cos}\omega_{LO}t\right) + a_2\left(V_{RF}\text{Cos}\omega_{RF}t + V_{LO}\text{Cos}\omega_{LO}t\right)^2$$

$$a_2 V_{RF} V_{LO}\left[\text{Cos}(\underbrace{\omega_{LO} - \omega_{RF}}_{\omega_{IF}})t + \text{Cos}(\omega_{LO} + \omega_{RF})t\right]$$

$$G_{mconv} = \frac{|\text{IF current output}|}{|\text{RF voltage input}|} = \frac{a_2 V_{RF} V_{LO}}{V_{RF}} = a_2 V_{LO}$$

Transistor G_{mconv}

For a BJT under small signal conditions

$$a_2 = \frac{1}{2}\frac{I_C}{V_{th}^2} \Rightarrow G_{mconv} = \frac{1}{2}\frac{I_C}{V_{th}^2}V_{LO}$$

depends on bias, LO amplitude and temperature

For a MOSFET

$$a_2 = \frac{1}{2}k'\frac{W}{L} \Rightarrow G_{mconv} = \frac{1}{2}k'\frac{W}{L}V_{LO} = \frac{1}{2}\mu C_{ox}\frac{W}{L}V_{LO}$$

Independent of bias

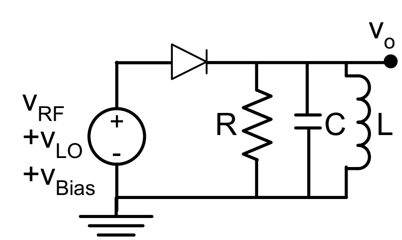
Depends on LO amplitude and temperature

Mixer Design Considerations

- **Linearity**
  - Third-order intermodulation (IP3) is an important consideration
- **Isolation**
  - Interaction between RF, IF, and LO ports has to be minimized
- **Conversion gain**
- **Noise performance**
- **Power**

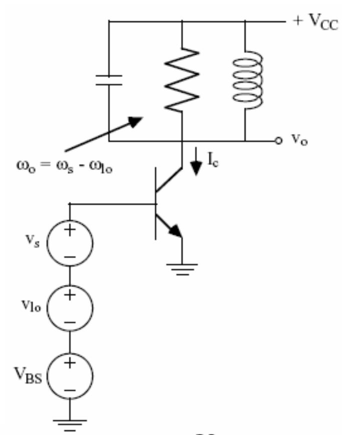
Single-Diode Mixer

- Simplest and oldest mixer
- Diode nonlinearity generates current at harmonic and intermod frequencies
- Tank filters except IF
- No isolation
- No conversion gain
- Suitable for high frequencies
- Used in early UHF TV tuners



Single-Device BJT Mixer

- High gain
- Collector tuned to IF



$$I_C = I_S \exp\left(\frac{V_{BS} + v_s + v_{lo}}{V_t}\right)$$

$$= I_{CA} \exp\left(\frac{v_s}{V_t}\right) \exp\left(\frac{v_{lo}}{V_t}\right)$$

$$I_{CA} = I_S \exp\left(\frac{V_{BS}}{V_t}\right)$$

$$\exp\left(\frac{V_{sA}\cos\omega_s t}{V_t}\right) = I_0(d) + 2I_1(d)\cos\omega_s t + \cdots \qquad d = \frac{V_{sA}}{V_t}$$

$$\exp\left(\frac{V_{loA}\cos\omega_{lo} t}{V_t}\right) = I_0(y) + 2I_1(y)\cos\omega_{lo} t + \cdots \qquad y = \frac{V_{loA}}{V_t}$$

Single-Device BJT Mixer Analysis

$$I_C = I_{CA} I_0(d) I_0(y) + \cdots 2I_{CA} I_1(d) I_1(y)\cos(\omega_s - \omega_{lo})t + \cdots$$

$$= I_{DC}\left[1 + \cdots + 2\frac{I_1(d)}{I_0(d)}\frac{I_1(y)}{I_0(y)}\cos(\omega_s - \omega_{lo})t + \cdots\right]$$

$$I_{DC} = I_{CA} I_0(d) I_0(y)$$

Since the RF is a small signal $V_{sA} << V_t \Rightarrow d << 1$

$$\Rightarrow \frac{I_1(d)}{I_0(d)} = \frac{1}{2}$$

$$I_C(\omega_{IF}) = I_{dc} d \frac{I_1(y)}{I_0(y)} \cos\omega_{IF} t = I_{dc} \frac{V_{sA}}{V_t} \frac{I_1(y)}{I_0(y)} \cos\omega_{IF} t$$

$$G_{mconv} = \frac{I_{dc}}{V_t}\frac{I_1(y)}{I_0(y)} \cong 0.95 \frac{I_{dc}}{V_t} \text{ for large b}$$

LO Feed Through

The collector current at the fundamental of the LO

$$I_C(\omega_{LO}) = I_{dc} 2 \frac{I_1(y)}{I_0(y)} \cos\omega_{LO} t = 1.9 I_{dc} \cos\omega_{LO} t \quad \text{for large b}$$

The collector current at IF

$$I_C(\omega_{IF}) = I_{dc} \frac{V_{sA}}{V_t} \frac{I_1(y)}{I_0(y)} \cos\omega_{IF} t = 0.95 I_{dc} \frac{V_{sA}}{V_t} \cos\omega_{IF} t \quad \text{for large b}$$

Therefore, $\left|\frac{I_C(\omega_{IF})}{I_C(\omega_{LO})}\right| = 2\frac{V_{sA}}{V_t}$

Example: VsA = 1mV $\quad \left|\frac{I_C(\omega_{IF})}{I_C(\omega_{LO})}\right| = 2\frac{V_{sA}}{V_t} = 2 \times 25.85 = 51.7$

$\Rightarrow$ output tuned circuit must provide adequate rejection at LO frequency

Avoiding LO Feed Through

Consider an ideal multiplier

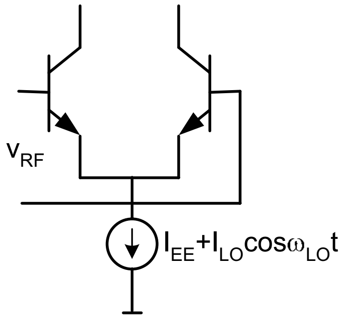
$$V_{RF}\text{Cos}\omega_{RF} t \times V_{LO}\text{Cos}\omega_{LO} t$$

$$= \frac{V_{RF}V_{LO}}{2}\left[\text{Cos}(\omega_{LO} - \omega_{RF})t + \text{Cos}(\omega_{LO} + \omega_{RF})t\right]$$

- There is perfect isolation from RF and LO to IF
  - $\Rightarrow$ Double-balanced mixer
- No third-order IM term!

Simplified Bipolar Multiplier (Mixer)

Assume v_{RF} is small



$$i_{out} = g_m v_{RF} = \frac{I_{EE}+I_{LO}\cos\omega_{LO}t}{2V_{th}} v_{RF}$$

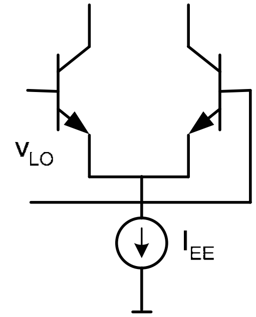
$$= \left(\frac{I_{EE}+I_{LO}\cos\omega_{LO}t}{2V_{th}}\right) V_{RF}\cos\omega_{RF}t$$

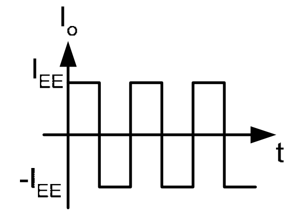
$$= \frac{I_{EE}}{2V_{th}} V_{RF}\cos\omega_{RF}t + \frac{I_{LO}}{4V_{th}} V_{RF}\left[\cos(\omega_{LO}-\omega_{RF})t + \cos(\omega_{LO}+\omega_{RF})t\right]$$

Output contains

- RF term and IF term (multiplicative term)
- No LO $\Rightarrow$ single-balanced mixer

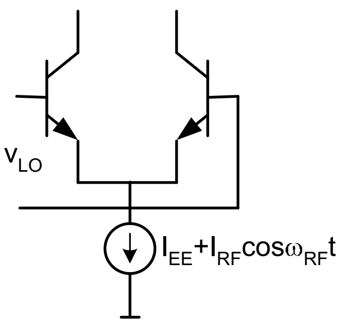
Switching Emitter-Coupled Pair





$$I_o = I_{c1} - I_{c2} = I_{EE}\left(\frac{4}{\pi}\cos\omega_{LO}t - \frac{4}{3\pi}\cos 3\omega_{LO}t + \ldots\right)$$

Single-Balanced Mixer



$$I_o = (I_{EE} + I_{RF}\cos\omega_{RF}t)\left(\frac{4}{\pi}\cos\omega_{LO}t - \frac{4}{3\pi}\cos 3\omega_{LO}t + \ldots\right)$$

$$= \underbrace{\frac{4}{\pi} I_{EE}\cos\omega_{LO}t}_{\text{LO feed through}} + \underbrace{\frac{2}{\pi} I_{RF}\cos(\omega_{LO}-\omega_{RF})t}_{\text{IF component}} + \frac{2}{\pi} I_{RF}\cos(\omega_{LO}+\omega_{RF})t + \ldots$$

LO feed through

IF component

V-I Conversion

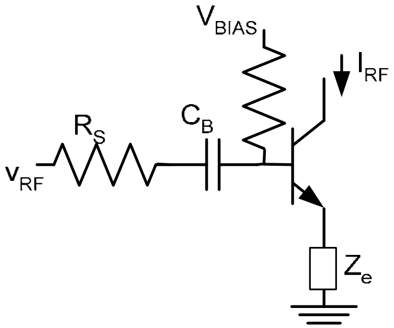
Assuming $I_{RF} = g_m v_{RF}$, where g_m is the transconductance of the V-I converter, we have

$$I_{IF} = \frac{2}{\pi} g_m v_{RF}$$

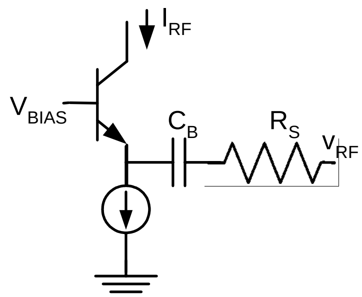
$\Rightarrow$ the conversion transconductance is

$$G_{mconv} = \frac{2}{\pi} g_m$$

Examples of Transconductors



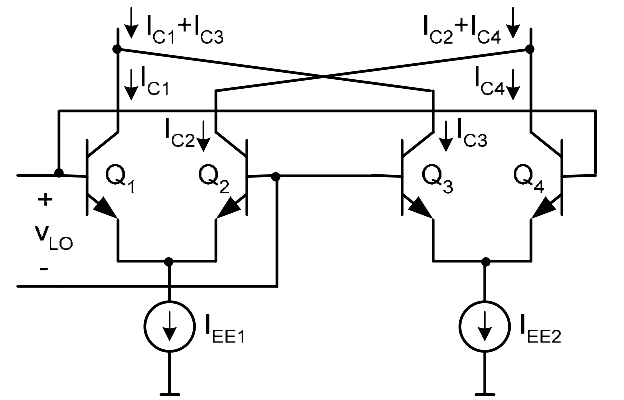
CE (CS) with degeneration



CB (CG) with linearization due to R_S

- **Inductive degeneration preferred**
  - **No thermal noise**
  - **No dc voltage drop**
  - **Attenuation of high frequency harmonics, intermod**

Eliminating LO Feedthrough



$$I_o = (I_{C1} + I_{C3}) - (I_{C2} + I_{C4}) = (I_{C1} - I_{C2}) - (I_{C4} - I_{C3})$$

$$= (I_{EE1} - I_{EE2})\left(\frac{4}{\pi}\cos\omega_{LO}t - \frac{4}{3\pi}\cos 3\omega_{LO}t + \ldots\right)$$

RF Currents

Now if we have

$$I_{EE1} = I_{EE} + I_{RF}\cos\omega_{RF}t$$

$$I_{EE2} = I_{EE} - I_{RF}\cos\omega_{RF}t$$

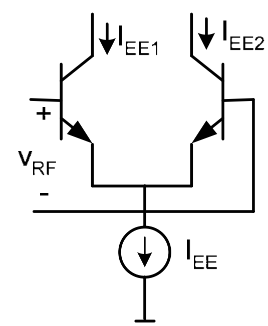
Then,

$$I_o = 2I_{RF}\cos\omega_{RF}t\left(\frac{4}{\pi}\cos\omega_{LO}t - \frac{4}{3\pi}\cos 3\omega_{LO}t + \ldots\right)$$

$$= \frac{4}{\pi}I_{RF}\cos(\omega_{LO} - \omega_{RF})t + \frac{4}{\pi}I_{RF}\cos(\omega_{LO} + \omega_{RF})t + \ldots$$

i.e., no LO or RF feedthrough!

Generation of RF Currents



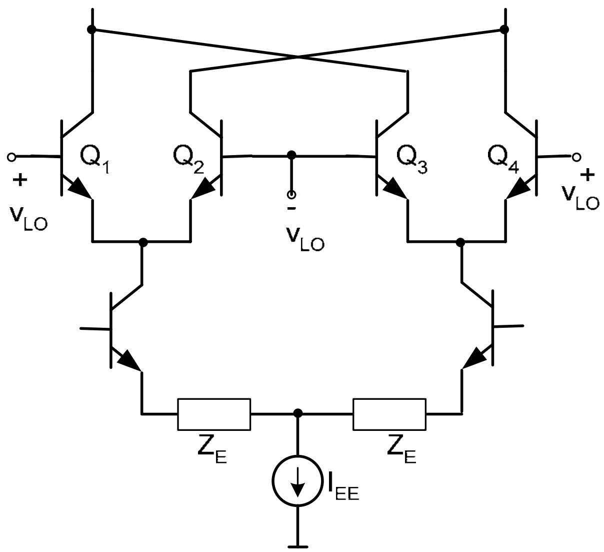
$$I_{EE1} - I_{EE2} = g_m v_{RF} = \frac{I_{EE}}{2V_{th}} v_{RF}$$

$$I_o = g_m V_{RF}\cos\omega_{RF}t\left(\frac{4}{\pi}\cos\omega_{LO}t - \frac{4}{3\pi}\cos 3\omega_{LO}t + \ldots\right)$$

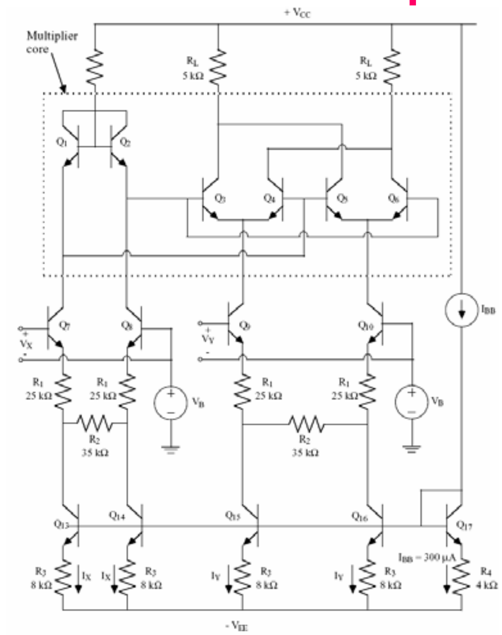
$$= \frac{2}{\pi}g_m V_{RF}\cos(\omega_{LO} - \omega_{RF})t + \frac{2}{\pi}g_m V_{RF}\cos(\omega_{LO} + \omega_{RF})t + \ldots$$

whereby, $G_{mconv} = \frac{2}{\pi}g_m$

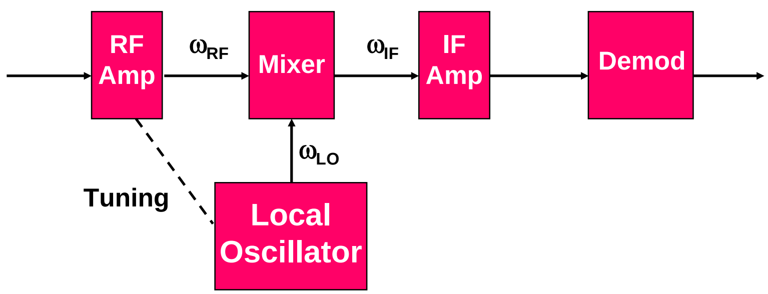
Double-Balanced (Gilbert) Mixer



Gilbert Cell Multiplier



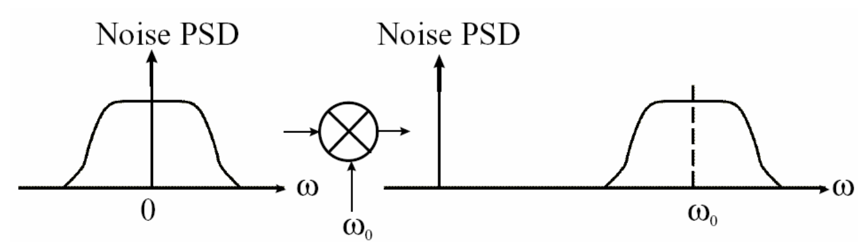
Down Conversion Mixer



– Mixer also translates noise to IF
– Noise figure is an important specification

Mixing Noise

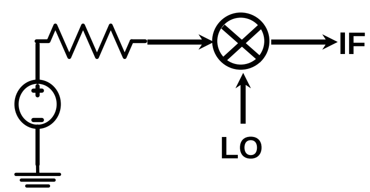
- Up/down conversion of noise due to mixing

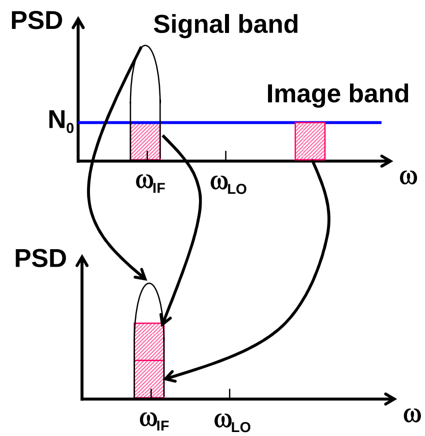


- SPICE noise analysis does not work
- Cyclostationarity/frequency correlation important
- Monte Carlo or stochastic methods

Noise in a Mixer

- Consider white noise at the input and a noiseless mixer





Assume that the mixer has a conversion gain of 1 for all frequencies

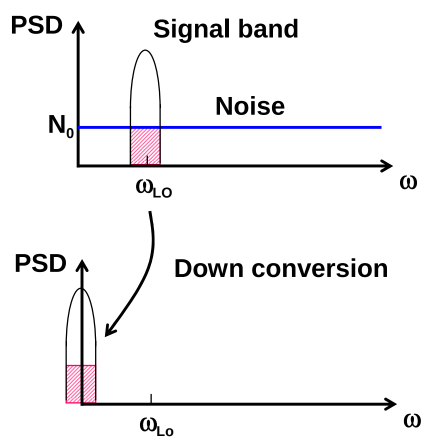
⇒ Output SNR is half of input SNR

Down Conversion of Noise

- Since noise in the image band is also downconverted to IF
  - ⇒ NF for noiseless mixer is 3dB
- This NF is called the single-sideband (SSB) NF, i.e., desired signal spectrum is on one side of the LO frequency as in heterodyne receivers

Homodyne Receiver

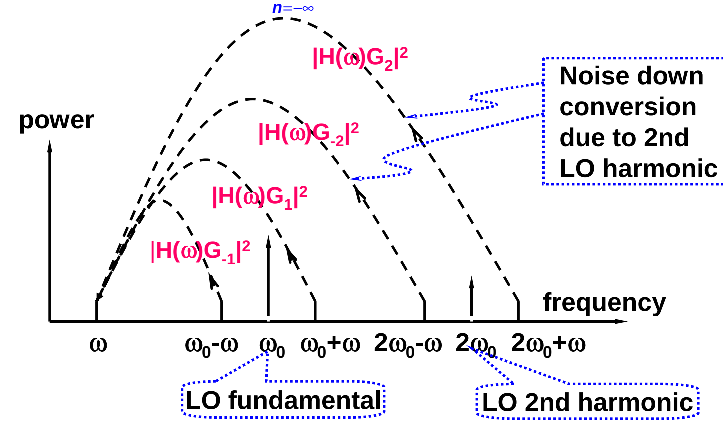
- For a noiseless mixer the input and output SNR are equal
  - ⇒ NF = 0dB
- This is the double-sideband (DSB) NF
  - ⇒ Signal spectrum exists on both sides of the LO



Noise Down Conversion in a Real Mixer

- For a stationary noise source

$$S_{out}(\omega) = \sum_{n=-\infty}^{+\infty} \left|H(\omega)\cdot G_n\right|^2 \cdot S_{in}(\omega + n\omega_0)$$

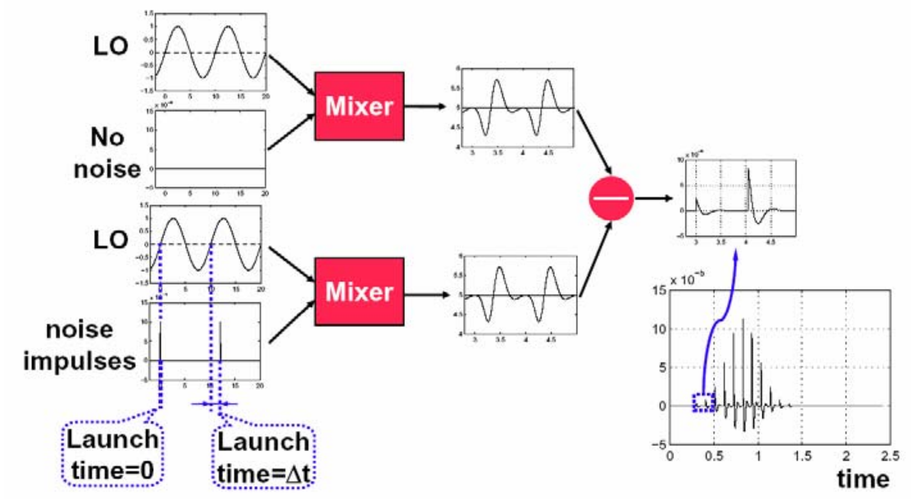


Mixer Noise Analysis

- **Simulation approach**
  - Available in SpectreRF, Agilent/EEsof ADS
  - Linear time-varying analysis used
    - **Circuit linearized around the LO**
    - **Noise applied as a small-signal**
  - Can be used for any circuit in periodic steady state
- **Indirect approach**
  - Applicable to mixers (Hull & Meyer, 1993)
  - SPICE simulations and 2D FFT used for calculating noise

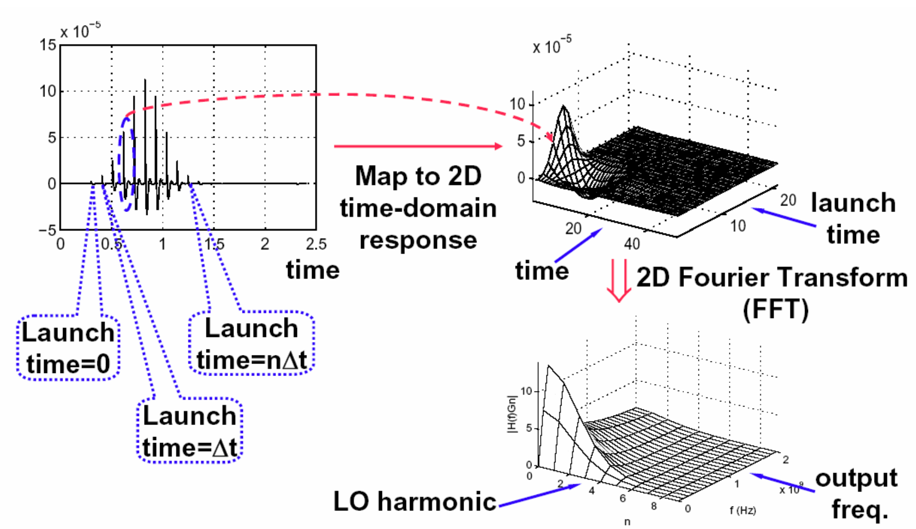
Hull & Meyer's Approach

- **Impulse response from noise source to output (SPICE transient analyses)**



Hull & Meyer's Approach

- **Frequency response (2D FFT)**



Noise in a Switching Mixer