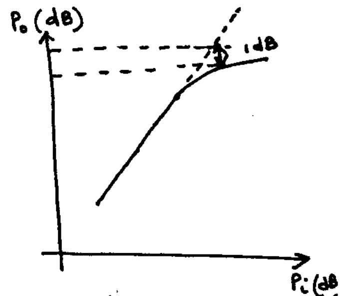
Compression and Intercept Points

Used for characterizing distortion in RF circuits

Compression Point

The 1-dB compression point is defined as the input power for which the fundamental power output is 1-dB below the extrapolated small-signal value



Consider the fundamental term obtained from the power series representation (p. 25)

$$b_1 = a_1 V_{iA} + \frac{3}{4} a_3 V_{iA}^3$$

Gain compression occurs for $a_3 < 0$

To calculate the 1-dB compression point

$$20 \log \left(a_1 V_{iA} - \frac{3}{4} |a_3| V_{iA}^3 \right) = 20 \log a_1 V_{iA} - 1\text{dB}$$

$$= 20 \log (a_1 V_{iA} / 1.12)$$

$$\therefore \quad a_1 V_{iA} - \frac{3}{4} |a_3| V_{iA}^3 = \frac{a_1 V_{iA}}{1.12}$$

$$\therefore \quad V_{iA} = \sqrt{\frac{4}{3} \frac{a_1}{|a_3|}} \sqrt{0.11} = \sqrt{0.145 \left| \frac{a_1}{a_3} \right|}$$

Note

This is the input voltage corresponding to the 1-dB compression point

$$\therefore \quad \text{Power} = \frac{V_{iA}^2}{2R_s}$$

Example

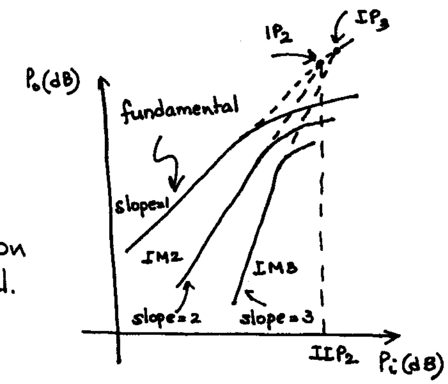
For a 1-dB compression point of -20dBm in a 50 Ω system

$$V_{iA} = \sqrt{2 R_s \text{(power)}} = \sqrt{2 \times 50 \times 10^{-5}} = 31.6 \text{ mV}$$

Note This is the 0-peak value

Intercept points (IP)

These are defined as the intercept of the extrapolated small-signal powers of the intermodulation terms with the fundamental.



The intercept points can be defined in terms of input or output powers.

To calculate the intercept points, e.g. IP_3 use the equations for IM terms

Require $\quad |a_1| V_{iA} = \frac{3}{4} |a_3| V_{iA}^3$

$$\text{or} \quad V_{iA} = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|}$$

$$\text{So} \quad IIP_3 = \frac{V_{iA}^2}{2R_s} = \frac{2}{3} \left| \frac{a_1}{a_3} \right| \frac{1}{R_s}$$

Relation between 1-dB compression and IIP_3

1-dB compression $\quad V_{iA} = \sqrt{0.145 \left| \frac{a_1}{a_3} \right|}$

IIP_3 $\quad V_{iA} = \sqrt{4/3 \left| \frac{a_1}{a_3} \right|}$

$$\therefore \frac{V_{iA}(1dB)}{V_{iA}(IP_3)} = \sqrt{\frac{0.145}{4/3}} = \sqrt{0.11} = -9.6dB$$

i.e. the 1dB compression occurs before the 3-rd order intercept point.

Cross modulation

This occurs when the modulation from an unwanted AM signal is transferred to the signal of interest due to circuit nonlinearities

Consider $v(t) = V_{iA}\cos\omega_1 t + V_{2A}[1 + m\cos\omega_m t]\cos\omega_2 t$
where $m < 1$ is the modulation index.

Due to the cubic term of the nonlinearity the modulation from the second signal is transferred to the first

$$a_1 V_{iA}\left(1 + 3\frac{a_3}{a_1}V_{2A}^2 m\cos\omega_m t\right)\cos\omega_1 t$$

Cross-modulation (CM) is defined as the ratio of the transferred modulation index to the original modulation

i.e. $CM = 3\left|\frac{a_3 V_{2A}^2}{a_1}\right|$

Blocking

Consider a weak desired signal in the presence of a strong interferer

Consider $v(t) = V_1\cos\omega_1 t + V_2\cos\omega_2 t$

Due to the cubic nonlinearity at ω_1

$$\left(a_1 V_1 + \frac{3}{4}a_3 V_1^3 + \frac{3}{2}a_3 V_1 V_2^2\right)\cos\omega_1 t$$

For $V_1 \ll V_2 \Rightarrow \left(a_1 + \frac{3}{2}a_3 V_2^2\right)V_1\cos\omega_1 t$

For $a_3 < 0$, the gain reduces as V_2 increases and the signal is "blocked". Need to withstand blockers 60-70dB larger.

Differential Error Approach

A technique useful for estimating small distortions
- Find small-signal gain at
  - quiescent operating point
  - positive peak
  - negative peak
- Define new parameters
- Develop expressions for HD_2 & HD_3

Recall the power series expansion

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

Small-signal gain $= \frac{dv_o}{dv_i} = a_1 + 2a_2 v_i + 3a_3 v_i^2 + \ldots$

at the operating point

$v_i = 0 \Rightarrow$ small-signal gain $= a_1$

For an input $v_i = V_{iA}\cos\omega_1 t$ we have

peak positive value $= V_{iA}$

" negative " $= -V_{iA}$

Define the two gains as a^+ and a^-

$$a^+ = a_1 + 2a_2 V_{iA} + 3a_3 V_{iA}^2 + \ldots$$
$$a^- = a_1 - 2a_2 V_{iA} + 3a_3 V_{iA}^2 - \ldots$$

Next define $E^+ = \frac{a^+ - a_1}{a_1} = \frac{2a_2 V_{iA} + 3a_3 V_{iA}^2 + \ldots}{a_1}$

$$E^- = \frac{a^- - a_1}{a_1} = \frac{-2a_2 V_{iA} + 3a_3 V_{iA}^2 + \ldots}{a_1}$$

From which we have

$$E^+ - E^- \simeq \frac{4a_2 V_{iA}}{a_1} \Rightarrow \frac{a_2}{a_1} V_{iA} = \frac{E^+ - E^-}{4} \quad (1)$$

$$E^+ + E^- \simeq \frac{6a_3 V_{iA}^2}{a_1} \qquad \frac{a_3}{a_1} V_{iA}^2 = \frac{E^+ + E^-}{6} \quad (2)$$

Recall

$$HD_2 = \frac{1}{2}\frac{a_2}{a_1} V_{iA}$$

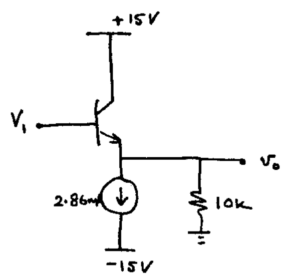
$$HD_3 = \frac{1}{4}\frac{a_3}{a_1} V_{iA}^2$$

which can be expressed in terms of E^+, E^- from (1) & (2)

$$\therefore \quad HD_2 = \frac{E^+ - E^-}{8}$$

$$HD_3 = \frac{E^+ + E^-}{24}$$

Example (EF stage)



$$\text{Small-signal gain} = \frac{(\beta+1)R_L}{r_\pi + (\beta+1)R_L}$$

$$= \frac{1}{1 + \frac{r_\pi}{(\beta+1)R_L}}$$

Since $g_m r_\pi = \beta \Rightarrow r_\pi = \frac{\beta}{g_m}$

$$\therefore \text{gain} = \frac{1}{1 + \frac{\beta}{(\beta+1)}\frac{1}{g_m R_L}} \qquad g_m = \frac{I_c}{V_{th}}$$

$$\Rightarrow \text{gain} = \frac{1}{1 + \frac{V_{th}}{|I_E| R_L}}$$

Operating point: $I_E = I_{EE} = 2.86\text{ mA}$, $V_{BE} = 0.8\text{ V}$

$V_{iA} = 10\text{ V} \Rightarrow I_E^+ = 2.86 + \frac{10\text{ V}}{10\text{K}} = 3.86\text{ mA}$

$V_{iA} = -10\text{ V} \Rightarrow I_E^- = 1.86\text{ mA}$

$$\therefore a_1 = \frac{1}{1 + \frac{V_{th}}{(2.86\text{mA})(10\text{K})}} = 0.9991$$

$$a^+ = \frac{1}{1 + \frac{V_{th}}{(3.86\text{mA})(10\text{K})}} = 0.9993$$

$$a^- = \frac{1}{1 + \frac{V_{th}}{(1.86\text{mA})(10\text{K})}} = 0.9986$$

From which $$E^+ = \frac{0.9993 - 0.9991}{0.9991} = 2\times10^{-4}$$

$$E^- = \frac{0.9986 - 0.9991}{0.9991} = -5\times10^{-4}$$

$$\Rightarrow HD_2 = \frac{E^+ - E^-}{8} = 8.75\times10^{-3}\% \quad (\text{SPICE: } 6.1\times10^{-3}\%)$$

$$HD_3 = \frac{E^+ + E^-}{24} = 1.25\times10^{-3}\% \quad (\text{SPICE: } 5\times10^{-4}\%)$$

Remarks

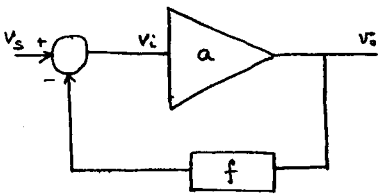
- The calculations need to be carried out in enough precision
- We have ignored the change in V_{BE}. For more accuracy V_{BE} should be calculated exactly using iteration.

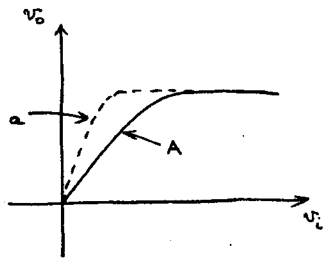
Distortion in Feedback Amplifiers

Negative feedback benefits:

- stabilization of gain
- modification of I/O impedances to suit needs
- reduced distortion
- increased bandwidth

We will focus on the reduction in distortion





open-loop gain $= a$

closed-loop gain $A = \frac{a}{1+af}$ i.e. gain is reduced

The reduction in gain results in a linearization of the characteristics and hence reduced distortion

Distortion reduction

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots$$

With feedback $v_i = v_s - f v_o$

$$\therefore \; v_o\Big|_{\text{with FB}} = a_1(v_s - fv_o) + a_2(v_s - fv_o)^2 + a_3(v_s - fv_o)^3 + \dots$$

$$= \alpha_1 v_s + \alpha_2 v_s^2 + \alpha_3 v_s^3 + \dots$$

Noting that the latter is a Taylor's series in v_s we have

$$\alpha_1 = \frac{\partial v_o}{\partial v_s}\Big|_{v_s=0}$$

$$2\alpha_2 = \frac{\partial^2 v_o}{\partial v_s^2}\Big|_{v_s=0}$$

$$6\alpha_3 = \frac{\partial^3 v_o}{\partial v_s^3}\Big|_{v_s=0}$$

Also note that $v_o = 0$ when $v_s = 0$

$$\therefore \; \alpha_1 = a_1\left(1 - f\frac{\partial v_o}{\partial v_s}\right) + 2a_2(v_s - fv_o)\left(1 - f\frac{\partial v_o}{\partial v_s}\right) + \dots\Big|_{v_s=0}$$

$$= a_1(1 - f\alpha_1) + 0$$

$$\therefore \; \alpha_1 = \frac{a_1}{1+a_1 f}$$ i.e. the usual small-signal case with $T = a_1 f$

Can show that

$$\alpha_2 = \frac{a_2}{(1+a_1 f)^3}$$

$$\alpha_3 = \frac{a_3(1+a_1 f) - 2a_2^2 f}{(1+a_1 f)^5}$$

<u>Open Loop</u> $HD_2 = \frac{1}{2}\frac{a_2}{a_1} V_{iA}$, $HD_3 = \frac{1}{4}\frac{a_3}{a_1} V_{iA}^2$

<u>Closed Loop</u> $HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1} V_{iA} = \frac{1}{2}\frac{a_2}{a_1} V_{iA} \frac{1}{(1+a_1 f)^2}$

$$= \frac{1}{(1+a_1 f)^2} \, HD_2\Big|_{\text{w/o feedback}}$$

$$HD_3 = \frac{1}{4}\frac{\alpha_3}{\alpha_1}V_{iA}^2 = \frac{1}{4}\frac{a_3}{a_1}V_{iA}^2 \frac{\left[1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right]}{(1+a_1 f)^3}$$

$$= \frac{\left[1 - \frac{2a_2^2 f}{a_3(1+a_1 f)}\right]}{(1+a_1 f)^3} \; HD_3 \Big|_{w/o\ feedback}$$

For a <u>constant output signal</u> V_{oA}

<u>Open Loop</u> $\quad HD_2 = \frac{1}{2}\frac{a_2}{a_1^2}V_{oA}\;, \quad HD_3 = \frac{1}{4}\frac{a_3}{a_1^3}V_{oA}^2$

<u>Closed Loop</u>

$$HD_2 = \frac{1}{2}\frac{\alpha_2}{\alpha_1^2}V_{oA} = \frac{1}{4}\frac{a_2}{a_1^2}V_{oA}\frac{1}{(1+a_1 f)}$$

i.e. HD_2 is reduced by $(1+T)$ where T is the loop gain.

$$HD_3 = \frac{1}{4}\frac{\alpha_3}{\alpha_1^3}V_{oA}^2 = \frac{1}{4}\frac{a_3}{a_1^3}V_{oA}^2\left[\frac{1 - 2a_2^2 f/((1+a_1 f)a_3)}{(1+a_1 f)}\right]$$

<u>Remarks</u>

- If $a_2 = 0$ i.e. no 2nd order distortion in the amplifier then HD_3 is also reduced by $(1+T)$.
- Cancellation of third-order distortion occurs when

$$\frac{2a_2^2 f}{a_3(1+a_1 f)} = 1$$

This is important when IM_3 needs to be reduced

<u>Noise in ICs</u>

The sensitivity of communication systems is limited by noise.

Noise generally means everything except the desired signal. We are not interested in man-made noise or artificial noise sources (e.g. 60 Hz power line hum). We will examine noise phenomena caused by small current fluctuations in devices (BJT, diode MOSFET) themselves.

These fluctuations give rise to an uncertainty of current at the various nodes of a circuit which is noise.

Noise is present basically due to the fact that charge is not continuous but is carried in discrete amounts $= q = 1.6\times10^{-19}$ C (quantization of charge).

<u>Why is noise a problem?</u>

- it represents a lower limit to the amplitude of the signal that can be amplified (or the circuit can process) without seriously affecting the signal quality

- it also places an upper limit on the useful gain of an amplifier. If the gain is increased without limit the output stages of the circuit eventually begin to limit - on the amplified noise from the input stages.

Nyquist, Johnson & Schottky were the pioneers who explained the origins of noise sources.