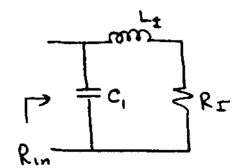
$$\therefore \frac{R_P}{R_I} = 1 + Q_2^2 \Rightarrow Q_2 = \sqrt{\frac{R_P}{R_I} - 1}$$

Also $\quad Q_2 = \omega_0 \frac{L_2}{R_I} \Rightarrow \frac{\omega_0 L_2}{R_I} = \sqrt{\frac{R_P}{R_I} - 1}$

2) <u>Upward transformer</u>



$$R_{in} = (1 + Q_1^2) R_I$$

$$\therefore Q_1 = \sqrt{\frac{R_{in}}{R_I} - 1} = \frac{\omega_0 L_1}{R_I}$$

The overall Q of the network is

$$Q = \frac{\omega_0 (L_1 + L_2)}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_P}{R_I} - 1} \quad \text{—} ①$$

Given Q & R_{in}, R_p $\quad R_I$ can be computed

$$\Rightarrow L_1 + L_2 = \frac{Q R_I}{\omega_0} \quad \text{i.e. } L \text{ can be determined}$$

Then capacitors can be calculated as

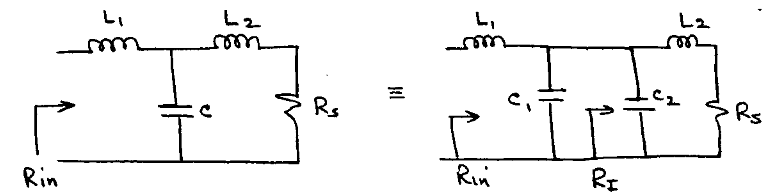
$$C_1 = \frac{Q_1}{\omega_0 R_{in}}, \quad C_2 = \frac{Q_2}{\omega_0 R_p}$$

<u>Remarks</u>

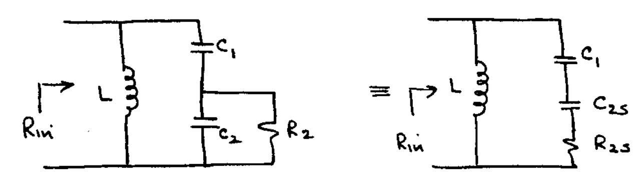
- Solution of ① can be determined iteratively for R_I
- If Q is large $\quad R_I \cong \left(\frac{\sqrt{R_{in}} + \sqrt{R_P}}{Q^2} \right)^2$

- Parasitic capacitances can be absorbed in network design. This is important because capacitance parasitics can dominate in many cases

<u>The T-match</u>



<u>Tapped Capacitor Resonator</u>



$$C_{2s} = C_2 \frac{(Q_2^2 + 1)}{Q_2^2}, \quad R_{2s} = \frac{R_2}{Q_2^2 + 1}$$

where Q_2 is the Q of the parallel $R_2 C_2$ sectio

$$R_s = \frac{R_{in}}{Q^2 + 1} = R_{2s}$$

$$\Rightarrow \frac{R_{in}}{Q^2 + 1} = \frac{R_2}{Q_2^2 + 1}$$

whereby $\quad Q_2 = \sqrt{\frac{R_2}{R_{in}} (Q^2 + 1) - 1} = \omega_0 R_2 C_2$

$$\therefore\ C_2 = \frac{Q_2}{\omega_0 R_2} = \frac{1}{\omega_0 R_2}\sqrt{\frac{R_2}{R_{in}}(Q^2+1) - 1}$$

To obtain C_1 consider $C_{eq} = \dfrac{C_1 C_{2s}}{C_1 + C_{2s}}$

The network Q is also $Q = \dfrac{1}{\omega_0 R_{2s} C_{eq}}$

$$Q = \frac{1}{\omega_0 R_{2s}} \frac{C_1 + C_{2s}}{C_1 C_{2s}}$$

$$= \frac{1}{\omega_0 \frac{R_2}{\cancel{(Q_2^2+1)}} \cdot C_1 C_2 \frac{\cancel{(Q_2^2+1)}}{Q_2^2}} \cdot \left[C_1 + C_2(Q_2^2+1)/Q_2^2\right]$$

$$= \frac{1}{\omega_0 R_2 C_1 C_2}\left(Q_2^2 C_1 + C_2(Q_2^2+1)\right)$$

$$= \frac{Q_2^2 C_1 + Q_2^2 C_2 + C_2}{C_1 Q_2}$$

$$\therefore\ Q Q_2 C_1 = Q_2^2 C_1 + C_2(Q_2^2+1)$$

$$C_1(Q Q_2 - Q_2^2) = C_2(Q_2^2+1)$$

$$\Rightarrow C_1 = \frac{C_2(Q_2^2+1)}{Q Q_2 - Q_2^2}.$$

<u>To calculate L</u>

$$Q = \frac{R_{in}}{\omega_0 L} \Rightarrow L = \frac{R_{in}}{\omega_0 Q}.$$

The network Q is determined from $Q = \frac{\omega_0}{\omega_b}$, where ω_b is the BW over which the transformation ratio is approx. constant.

<u>Transformation Ratio</u>

We have $\dfrac{R_{in}}{Q^2+1} = \dfrac{R_2}{Q_2^2+1}$

$$\Rightarrow \frac{R_{in}}{R_2} \cong \frac{Q^2}{Q_2^2}$$

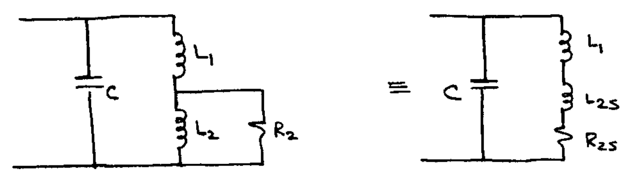
Also $$Q = \frac{Q_2^2 C_1 + C_2(Q_2^2+1)}{C_1 Q_2} \cong \frac{Q_2^2 C_1 + C_2 Q_2^2}{C_1 Q_2}$$

$$= \left(\frac{C_1 + C_2}{C_1}\right) Q_2 .$$

$$\therefore\ \frac{Q}{Q_2} = \frac{C_1 + C_2}{C_1}$$

$$\Rightarrow \frac{R_{in}}{R_2} \cong \left(\frac{C_1 + C_2}{C_1}\right)^2 \qquad \Rightarrow R_{in} > R_2 .$$

<u>Tapped Inductor Match</u>



<u>Remark</u>

- impractical values may be needed in the various transformation networks described so far. A double tapped resonator can be used to bring L & C to realizable values

Relationship between Q, ω_0 & poles

Denominator of transfer function

$$D(s) = s^2 + \frac{G}{C}s + \frac{1}{LC} = s^2 + \omega_b s + \omega_0^2$$

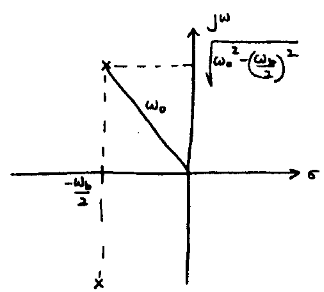
where $\omega_b = \frac{G}{C}$ & $\omega_0^2 = \frac{1}{LC}$

The poles are roots of $D(s)$, i.e., $s^2 + \omega_b s + \omega_0^2 = 0$

$$\Rightarrow s_{1,2} = -\frac{\omega_b}{2} \pm j\sqrt{\omega_0^2 - \left(\frac{\omega_b}{2}\right)^2}$$

Distance of poles from origin

$$= \sqrt{\cancel{\left(\frac{\omega_b}{2}\right)^2} + \omega_0^2 - \cancel{\left(\frac{\omega_b}{2}\right)^2}} = \omega_0$$



$$Q = \frac{\omega_0}{\omega_b} = \frac{\sqrt{Re^2 + Im^2}}{2\,Re}$$

where Re = real (s_1, s_2)
Im = imag (s_1, s_2)

Remark

- A Q can be defined for any complex conjugate pole pair.

$$|D(\omega)| = \left| -\omega^2 + \omega_b j\omega + \omega_0^2 \right| = \sqrt{(\omega_0^2 - \omega^2)^2 + \omega_b^2\omega^2}$$

$$= \frac{\omega\,\omega_0}{Q}\sqrt{1 + Q^2\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2} = \omega\,\omega_b\sqrt{1 + Q^2\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)^2}$$

A Typical Planar Spiral Inductor

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