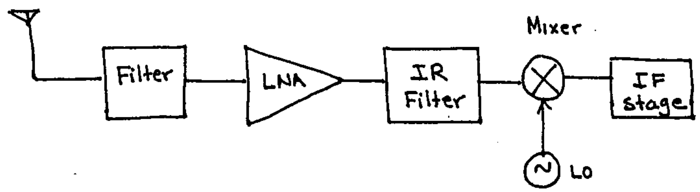
Low Noise Amplifier (LNA)

Consider the superheterodyne architecture



Note

- LNA needs to provide enough gain to overcome the noise of subsequent stages
- should add as little noise as possible
- present a specific impedance (50Ω) to the input source
- handle large signals without distortion

So the design objectives should be

- gain
- minimum noise figure
- input match
- linearity
- power consumption

Classical 2-port noise theory has several shortcomings

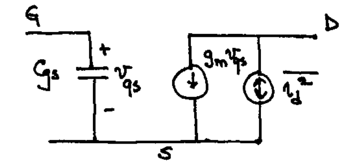
- source impedance for F_{min} differs from that which maximizes power
- power consumption is ignored

The assumption is that one is given a device with fixed characteristics. This is not true for the IC scenario!

MOSFET Two-port Noise Parameters

Ignoring 1/f noise since it is not important for high-frequency amplifiers we have

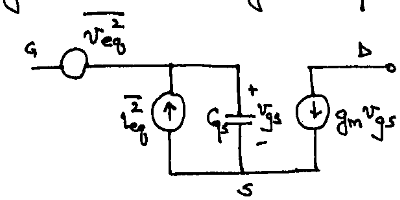
$$\overline{i_d^2} = 4kT\gamma g_{do}\Delta f$$



The equivalent input noise generators are given by

$$\overline{v_{eq}^2} = \frac{\overline{i_d^2}}{g_m^2} = \frac{4kT\gamma g_{do}\Delta f}{g_m^2}$$

$$\Rightarrow R_n = \frac{\gamma g_{do}}{g_m^2}$$



and $$\overline{i_{eq}^2} = \frac{\omega^2 C_{gs}^2}{g_m^2}\overline{i_d^2} = \omega^2 C_{gs}^2 \overline{v_{eq}^2}$$

whereby $\overline{i_{eq}^2}$ and $\overline{v_{eq}^2}$ are correlated.

Now let us introduce the induced gate noise

$$\overline{i_g^2} = 4kT\delta g_g \Delta f$$

where $$g_g = \frac{\omega^2 C_{gs}^2}{5 g_{do}}$$

The gate noise is correlated with the drain noise with a correlation coefficient

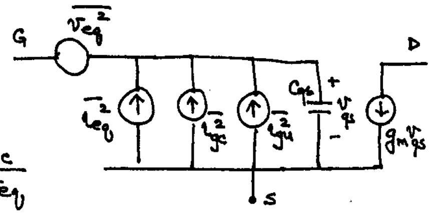
$$c = \frac{\overline{i_g i_d^*}}{\sqrt{\overline{i_g^2}\,\overline{i_d^2}}} = -0.395j$$ for long-channel devices

$$\overline{i_g^2} = \overline{(i_{gc} + i_{gu})^2} = \overline{i_{gc}^2} + \overline{i_{gu}^2}$$

$$= \underbrace{4kT\delta\, g_g\, |c|^2 \Delta f}_{\overline{i_{gc}^2}} + \underbrace{4kT\delta\, g_g\, (1-|c|^2)\, \Delta f}_{\overline{i_{gu}^2}}$$

Now we consider the complete MOSFET noise

The correlation admittance Y_c can be calculated as



$$Y_c = \frac{i_{eq} + i_{gc}}{v_{eq}} = j\omega C_{gs} + \frac{i_{gc}}{v_{eq}}$$

Since $v_{eq} = \frac{i_d}{g_m} \Rightarrow Y_c = j\omega C_{gs} + g_m \frac{i_{gc}}{i_d}$

Let us examine the term $\frac{i_{gc}}{i_d}$

$$\frac{i_{gc}}{i_d} = \frac{\overline{i_{gc}\, i_d^*}}{\overline{i_d\, i_d^*}} = \frac{\overline{i_{gc}\, i_d^*}}{\overline{i_d^2}} = \frac{\overline{i_{gc}\, i_d^*}}{\sqrt{\overline{i_d^2}}\sqrt{\overline{i_d^2}}}$$

$$= \frac{\overline{i_g\, i_d^*}}{\sqrt{\overline{i_g^2}}\sqrt{\overline{i_d^2}}}\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}} = c\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}} = -j|c|\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}}$$

assuming that c is a purely imaginary number

$$\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}} = \sqrt{\frac{\delta\omega^2 C_{gs}^2}{5\gamma\, g_{do}^2}} = \frac{\omega C_{gs}}{g_{do}}\sqrt{\frac{\delta}{5\gamma}}$$

$$\therefore\ Y_c = j\omega C_{gs} - j|c|\frac{\omega C_{gs}}{g_{do}}\, g_m\sqrt{\frac{\delta}{5\gamma}}$$

$$\Rightarrow Y_c = j\omega C_{gs}\left(1 - \alpha|c|\sqrt{\frac{\delta}{5\gamma}}\right)$$

where $\alpha = \frac{g_m}{g_{do}}$ $= 1$ for long-channel devices
< 1 " short-channel "

Observations

- Y_c is purely imaginary ; $G_c = 0$
- $Y_c \propto \omega C_{gs}$ ($= Y_{in}$)
  i.e. noise figure cannot be minimized when maximizing power transfer

The uncorrelated portion of the gate noise current $\overline{i_{gu}^2}$ gives G_u

$$G_u = \frac{\overline{i_u^2}}{4kT\Delta f} = \frac{\delta\omega^2 C_{gs}^2}{5 g_{do}}\left(1-|c|^2\right)$$

From the above analysis we have

Summary of two-port noise parameters

$$G_c \sim 0$$
$$B_c = \omega C_{gs}\left(1 - \alpha|c|\sqrt{\frac{\delta}{5\gamma}}\right)$$
$$R_n = \frac{\gamma g_{do}}{g_m^2} = \frac{\gamma}{\alpha}\frac{1}{g_m}$$
$$G_u = \frac{\delta\omega^2 C_{gs}^2}{5 g_{do}}\left(1-|c|^2\right)$$

From the two-port noise theory, to minimize the noise figure

$$B_{opt} = -B_c = -\omega C_{gs}\left(1 - \alpha|c|\sqrt{\frac{\delta}{5\gamma}}\right)$$

$$G_{opt} = \sqrt{G_c^2 + \frac{G_u}{R_n}} = \sqrt{\frac{\delta \omega^2 C_{gs}^2}{5 g_{do}} \left(1-|c|^2\right) \frac{\alpha g_m}{\gamma}}$$

$$= \alpha \omega C_{gs} \sqrt{\frac{\delta}{5\gamma}\left(1-|c|^2\right)}$$

and $F_{min} = 1 + 2R_n\left(G_{opt} + G_c\right)$

$$= 1 + 2\frac{\gamma}{\cancel{\alpha}} \frac{1}{g_m} \cdot \cancel{\alpha} \omega C_{gs} \sqrt{\frac{\delta}{5\gamma}\left(1-|c|^2\right)}$$

$$= 1 + \frac{2}{\sqrt{5}} \frac{\omega}{\omega_T} \sqrt{\delta\gamma\left(1-|c|^2\right)}$$

where $\omega_T = \dfrac{g_m}{C_{gs}}$

Remarks

- If there was no gate current noise, i.e. $\delta = 0$ then $F_{min} = 1$ An unrealistic value
  $\Rightarrow$ gate noise must exist
- increasing the correlation c would improve noise figure
- $F_{min} \propto \dfrac{\omega}{\omega_T}$ i.e. a technology with a higher ω_T will improve F for a given ω

Example $\gamma = 2$, $\delta = 4$, $|c| = 0.395$

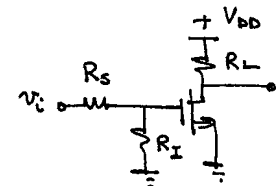
ω_T/ω	F_{min} (dB)
20	0.5
10	0.9
5	1.6

Summary

- For a MOSFET, the source impedance for F_{min} is inductive in nature
- The input impedance of the MOSFET is inherently capacitive
- Z_s for F_{min} unrelated to maximum power transfer

How do we provide a good match to a 50 Ω source?

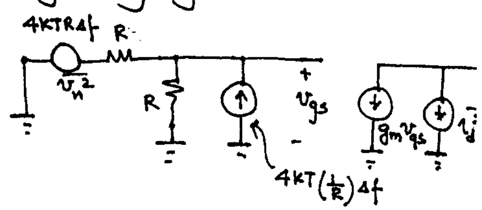
A straight forward approach
common-source amplifier with shunt input resistor



Note

R_I adds its own thermal noise and attenuates input signal by a factor of 2

For low frequencies and ignoring gate noise



$$\overline{i_o^2} = \overline{i_d^2} + \frac{g_m^2}{4}\overline{v_n^2} + g_m^2 R^2 \frac{\overline{i_n^2}}{4}$$

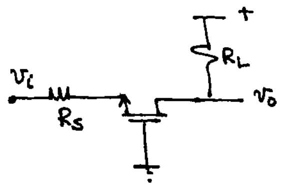
$$i_{o,R} = \frac{g_m^2}{4}\overline{v_n^2}$$

$$\therefore F = \frac{\overline{i_d^2}}{\frac{g_m^2}{4}\overline{v_n^2}} + 2 = 2 + \frac{4\cancel{KT}\gamma g_{do} \cancel{\Delta f}}{\frac{g_m^2}{4} \cdot 4\cancel{KT} R \cancel{\Delta f}}$$

$$= 2 + \frac{4\gamma}{\alpha} \frac{1}{g_m R}$$

Another choice

Common-gate configuration



For input match require

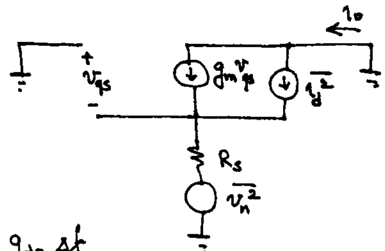
$$R_s = \frac{1}{g_m} = 50\,\Omega$$

By a proper choice of device size and bias current can provide the input match.

What about F ?

Assuming low frequencies and ignoring gate noise

$$R_s = 1/g_m$$



$$\overline{i_o^2} = g_m^2 \frac{\overline{v_n^2}}{4} + \frac{\overline{i_d^2}}{4}$$

since $\quad G_m = \frac{g_m}{1+g_m R_s} = \frac{g_m}{2}$

$$\therefore\ F = 1 + \frac{\overline{i_d^2}}{g_m^2 \overline{v_n^2}} = 1 + \frac{\cancel{4kT}\,\gamma\, g_{do}\, \cancel{\Delta f}}{g_m^2\, \cancel{4kT}\, R\, \cancel{\Delta f}}$$

$$= 1 + \gamma \frac{g_{do}}{g_m} = 1 + \frac{\gamma}{\alpha} \qquad \text{since } \alpha = \frac{g_m}{g_{do}}$$

Example

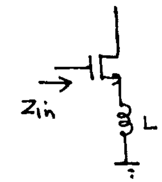
Long-channel device $\quad \gamma = 2/3, \quad \alpha = 1$

$$\therefore\ F = 1 + \frac{2}{3} \Rightarrow 2.2\,dB$$

How can we provide a resistive input impedance without using resistors (or the MOSFET channel resistance) ?

Consider a MOSFET with inductive source degeneration

$$Z_{in} = sL + \frac{1}{sC_{gs}} + \frac{g_m}{C_{gs}} L$$



At the frequency $\omega_o = \frac{1}{\sqrt{LC_{gs}}}$

$$Z_{in} = \frac{g_m}{C_{gs}} L \quad \text{which is real}$$

i.e. at resonance the input impedance is purely resistive.

Note

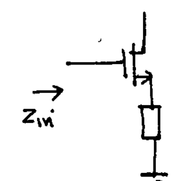
This circuit provides only a narrow band input match, which is desirable in several cases.

At resonance $\quad Z_{in} = \frac{g_m}{C_{gs}} L = \omega_T L$

which indicates that L is used to provide the desired input resistance. To have a desired resonance frequency an inductor is added in series with the gate.

For a general case:

$$Z_{in}(j\omega) = \frac{1}{j\omega C_{gs}} + Z + \frac{\omega_T}{j\omega} Z$$



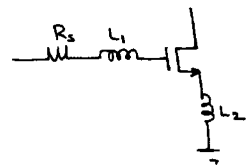
Therefore, capacitive degeneration contributes a negative resistance at the input

$\Rightarrow$ C_{sb} offsets the positive resistance from inductive degeneration

Example (cs amplifier)

RF LNA

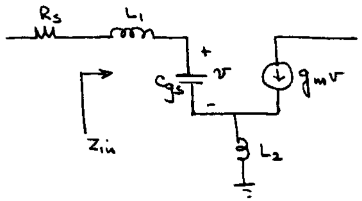
Find F assuming matched conditions



$$Z_{IN}(s) = sL_1 + \frac{1}{sC_{gs}} + \left(\frac{g_m}{sC_{gs}} + 1\right) sL_2$$

$$= sL_1 + \frac{1}{sC_{gs}} + sL_2 + \frac{g_m L_2}{C_{gs}}$$

$$= s(L_1 + L_2) + \frac{1}{sC_{gs}} + \frac{g_m L_2}{C_{gs}}$$



At the resonance frequency ω_0 $Z_{IN}(j\omega_0)$ is real because of matching considerations

i.e. $j\omega_0(L_1 + L_2) + \frac{1}{j\omega_0 C_{gs}} = 0$

$$\omega_0(L_1 + L_2) - \frac{1}{\omega_0 C_{gs}} = 0$$

$$\Rightarrow \omega_0 = \sqrt{\frac{1}{(L_1 + L_2)C_{gs}}}$$

Also $R_s = \frac{g_m L_2}{C_{gs}} = \omega_T L_2$ (ω_T = unity gain frequency)

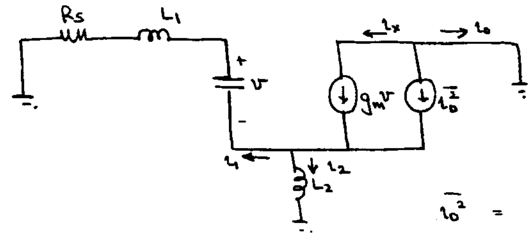
To determine noise factor we consider noise due to the amplifier only and then noise due to R_s

$$F = 1 + \frac{N_{OA}}{N_{ORS}}$$

and we consider the output short circuit current

At high frequencies $1/f$ noise is negligible

i_o



$$\overline{i_o^2} = 4kT\,\frac{2}{3}\,g_m \Delta f$$

$$g_m v + i_o = i_1 + i_2$$

$$i_1 = -j\omega_0 C_{gs} v$$

$$i_2 = \Big[(R_s + j\omega_0 L_1)\underbrace{(-j\omega_0 C_{gs} v)}_{=\,i_1} - v\Big]\frac{1}{j\omega_0 L_2}$$

$$\therefore\ i_1 + i_2 = \left[-j\omega_0 C_{gs} - \frac{(R_s + j\omega_0 L_1)(j\omega_0 C_{gs}) + 1}{j\omega_0 L_2}\right] v$$

$$= -\left[\frac{\cancel{-\omega_0^2 L_2 C_{gs}} + R_s \cancel{j\omega_0} C_{gs} \cancel{- \omega_0^2 L_1 C_{gs} + 1}}{\cancel{j\omega_0} L_2}\right] v$$

$$= -\frac{R_s C_{gs}}{L_2} v = -g_m v \quad \text{from matching}$$

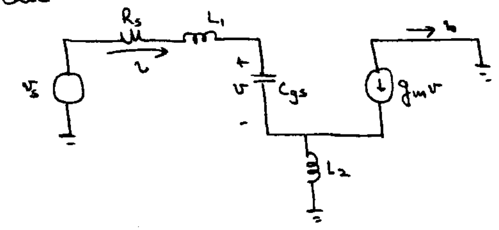
$$\therefore\ g_m v + i_o = -g_m v$$

$$\Rightarrow v = -\frac{i_o}{2g_m} \quad \& \quad i_x = g_m v = -\frac{i_o}{2}$$

$$\therefore\ i_o = i_o + i_x = i_o - \frac{i_o}{2} = \frac{1}{2} i_o$$

$$\Rightarrow \overline{i_o^2} = \frac{1}{4}\,\overline{i_o^2} = \frac{1}{\cancel{4}}\,\frac{\overset{2}{\cancel{8}}}{3}\,kT g_m \Delta f = \frac{2}{3} kT g_m \Delta f$$

output noise due to R_s



$i = \frac{v_s}{2R_s}$ due to the match condition

$\therefore v = \frac{1}{j\omega_0 C_{gs}} \frac{1}{2R_s} v_s$

and $i_o = -\frac{1}{j\omega_0 C_{gs}} \frac{g_m}{2R_s} v_s$

$$\Rightarrow \overline{i_o^2} = \frac{1}{\omega_0^2 C_{gs}^2}\frac{g_m^2}{4R_s^2}\overline{v_s^2} = \frac{1}{\omega_0^2 C_{gs}^2}\frac{g_m^2}{\cancel{4}R_s^{\cancel{2}}}\cdot \cancel{4}kT\cancel{R_s}\Delta f$$

$$\therefore F = 1 + \frac{2/3\, kT\, \cancel{g_m}\, \cancel{\Delta f}}{\frac{1}{\omega_0^2 C_{gs}^2}\frac{g_m^{\cancel{2}}}{R_s}\cdot \cancel{kT}\,\cancel{\Delta f}}$$

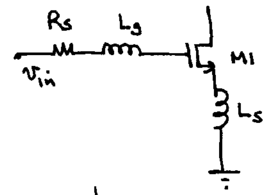
$$= 1 + \frac{2}{3}\frac{\omega_0^2 C_{gs}^2 R_s}{g_m} = 1 + \frac{2}{3}\overbrace{\frac{1}{g_m} C_{gs} R_s}^{= L_2}\frac{1}{L_1+L_2}$$

$$= 1 + \frac{2}{3}\frac{L_2}{L_1+L_2} = 1 + \frac{2}{3}\frac{1}{1+L_1/L_2}$$

Remark
- F decreases as L_1 increases

Overall stage transconductance

$R_s = \omega_T L_s$ at resonance



$G_m = g_m \frac{v_{gs}}{v_{in}} = g_m Q_{in}$

where $Q_{in} = \frac{1}{\omega_0 C_{gs}(R_s + \omega_T L_s)} = \frac{1}{2\omega_0 C_{gs} R_s}$

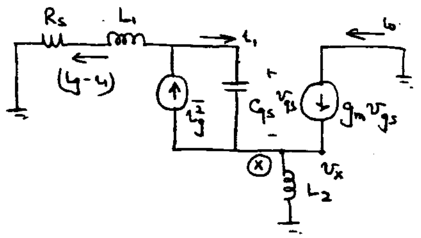
$$\therefore G_m = \frac{g_m}{2\omega_0 C_{gs} R_s} = \frac{g_m}{C_{gs}}\frac{1}{2\omega_0 R_s} = \frac{\omega_T}{2\omega_0 R_s}$$

i.e. the overall transconductance is independent of the device g_m

Analysis of Common-source LNA (including gate noise)

Contribution due to $\overline{i_g^2}$

First let us determine the output current i_o due to a gate current i_g



$v_{gs} = \frac{i_1}{j\omega_0 C_{gs}}$

$v_x = (R_s + j\omega_0 L_1)(i_g - i_1) - \frac{i_1}{j\omega_0 C_{gs}}$

Using KCL at ⊗ we have

$$i_g - i_1 - \frac{g_m i_1}{j\omega_0 C_{gs}} + \left[(R_s + j\omega_0 L_1)(i_g - i_1) - \frac{i_1}{j\omega_0 C_{gs}}\right]\frac{1}{j\omega_0 L_2} = 0$$

$$\therefore i_g\left[1 + \frac{R_s + j\omega_0 L_1}{j\omega_0 L_2}\right] = i_1\left[1 + \frac{g_m}{j\omega_0 C_{gs}} + \frac{R_s + j\omega_0 L_1}{j\omega_0 L_2} - \frac{1}{\omega_0^2 L_2 C_{gs}}\right]$$

Note that $\frac{1}{\omega_0^2 L_2 C_{gs}} = \frac{1}{\frac{1}{(L_1+L_2)C_{gs}} L_2 C_{gs}} = 1 + \frac{L_1}{L_2}$

and $\frac{R_s C_{gs}}{L_2} = g_m \Rightarrow \frac{R_s}{L_2} = \frac{g_m}{C_{gs}}$

$$\therefore i_g\left[1 + \underbrace{\frac{R_s}{j\omega_0 L_2}}_{\frac{g_m}{j\omega_0 C_{gs}}} + \frac{L_1}{L_2}\right] = i_1\left[\cancel{1} + \frac{g_m}{j\omega_0 C_{gs}} + \underbrace{\frac{R_s}{j\omega_0 L_2}}_{\frac{g_m}{j\omega_0 C_{gs}}} + \cancel{\frac{L_1}{L_2}} - \left(\cancel{1+\frac{L_1}{L_2}}\right)\right]$$

$$i_g\left[\frac{1}{\omega_0^2 L_2 C_{gs}} - j\frac{g_m}{\omega_0 C_{gs}}\right] = \frac{2g_m}{j\omega_0 C_{gs}}\, i_1$$

$$i_o = g_m v = \frac{g_m i_1}{j\omega_0 C_{gs}} = \frac{i_g}{2}\cdot\frac{g_m}{\omega_0 C_{gs}}\left[\frac{1}{\omega_0 g_m L_2} - j\right]$$

But $\frac{1}{\omega_0 g_m L_2} = \frac{1}{\omega_0 R_s C_{gs}} = Q_L$

$$\therefore i_o = \frac{i_g}{2}\,\frac{g_m}{\omega_0 C_{gs}}\left[Q_L - j\right]$$

Now consider the correlation i.e. i_d & i_{gc} are correlated

<u>Recall</u> $i_o = \frac{1}{2} i_d$.

Correlation $\Rightarrow$ $$\overline{i_o^2} = \overline{\left|\frac{i_d}{2} + \frac{i_{gc}}{2}\,\frac{g_m}{\omega_0 C_{gs}}(Q_L - j)\right|^2}$$

$$= \left|1 + \frac{g_m}{\omega_0 C_{gs}}(Q_L - j)\,\frac{i_{gc}}{i_d}\right|^2 \frac{\overline{i_d^2}}{4}$$

Also recall that $\frac{i_{gc}}{i_d} = -j|c|\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}}$

when c is purely imaginary

and $\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}} = \frac{\omega C_{gs}}{g_m}\sqrt{\frac{\delta\alpha^2}{5\gamma}}$

$$\therefore \overline{i_o^2} = \left|1 - \frac{\cancel{g_m}}{\cancel{\omega_0 C_{gs}}}(Q_L - j)j|c|\frac{\cancel{\omega_0 C_{gs}}}{\cancel{g_m}}\sqrt{\frac{\delta\alpha^2}{5\gamma}}\right|^2 \frac{\overline{i_d^2}}{4}$$

$$= \underbrace{\left[\left(1 - |c|\sqrt{\frac{\delta\alpha^2}{5\gamma}}\right)^2 + |c|^2 Q_L^2 \frac{\delta\alpha^2}{5\gamma}\right]}_{\kappa} \frac{\overline{i_d^2}}{4}$$

So we have the contribution due to the drain and correlat part of the gate noise at the output

$$\overline{i_o^2} = \kappa\,\frac{\overline{i_d^2}}{4} = \kappa\,\frac{4kT\gamma g_{do}}{4}$$

Now determine the contribution due to the uncorrelated p

$$\overline{i_o^2} = \frac{\overline{i_{gu}^2}}{4}\,\frac{g_m^2}{\omega_0^2 C_{gs}^2}(Q_L^2+1)$$

$$= \frac{1}{4}\,4kT\delta\,\frac{\cancel{\omega_0^2 C_{gs}^2}}{5 g_{do}}(1-|c|^2)\frac{g_m^2}{\cancel{\omega_0^2 C_{gs}^2}}(Q_L^2+1)$$

$$= \frac{1}{4}\,4kT\frac{\delta}{5\gamma}\frac{g_m^2}{g_{do}^2}(1-|c|^2)(Q_L^2+1)\,\gamma g_{do}$$

$$= \underbrace{\frac{\delta\alpha^2}{5\gamma}(1-|c|^2)(Q_L^2+1)}_{\xi}\,\frac{\overline{i_d^2}}{4}$$

The output noise due to i_d & i_g is

$$\overline{i_{o\,d,g}^2} = 4kT\gamma\chi\frac{g_{do}}{4}$$

where $\chi = \kappa + \xi = 1 - 2|c|\sqrt{\frac{\delta\alpha^2}{5\gamma}} + \frac{\delta\alpha^2}{5\gamma}\left(1+Q_L^2\right)$

$$\therefore\; F = 1 + \frac{\cancel{4}kT\gamma\chi g_{do}/\cancel{4}}{\frac{1}{\omega_o^2 C_{gs}^2}\,\frac{g_m^2}{\cancel{4}R_s^{\cancel{2}}}\cdot\cancel{4}kT\cancel{R_s}}$$

$$= 1 + \omega_o^2\frac{C_{gs}^2}{g_m^2}R_s\gamma\chi g_{do}$$

$$= 1 + \left(\frac{\omega_o}{\omega_T}\right)^2 R_s\gamma\chi g_{do}$$

Note that $g_{do}Q_L = \frac{g_m}{\alpha}\frac{1}{\omega_o R_s C_{gs}} = \frac{\omega_T}{\alpha\omega_o R_s}$

whereby

$$F = 1 + \frac{\gamma}{\alpha}\frac{\chi}{Q_L}\left(\frac{\omega_o}{\omega_T}\right)$$

<u>Remarks</u>

- χ contains a constant term and a term $\propto Q_L^2$

  $\Rightarrow$ F contains a term in Q_L, $\frac{1}{Q_L}$

  $\Rightarrow$ minimum of F exists for a particular Q_L

We will now consider the power consumption in the amplifier. To do this revisit the short-channel MOSFET I-V characteristics.

In saturation: $I_d = W C_{ox} v_{sat}\frac{(V_{gs}-V_T)^2}{V_{gs}-V_T+LE_{sat}}$

$$\therefore\; I_d = \frac{W}{L}C_{ox}\frac{v_{sat}}{E_{sat}}\frac{(V_{gs}-V_T)^2}{1+\frac{V_{gs}-V_T}{LE_{sat}}}$$

$$= \frac{\mu_{eff}}{2}\frac{W}{L}C_{ox}\frac{(V_{gs}-V_T)^2}{1+\rho}$$

where $v_{sat} = \frac{\mu_{eff}}{2}E_{sat}$ and $\rho = \frac{V_{gs}-V_T}{LE_{sat}}$

<u>Calculate g_m</u> $= \frac{\partial I_d}{\partial V_{gs}} = \frac{\mu_{eff}}{2}\frac{W}{L}C_{ox}\left[\frac{2(V_{gs}-V_T)}{1+\rho} - \frac{(V_{gs}-V_T)^2}{(1+\rho)^2}\frac{1}{LE_s}\right.$

$$\Rightarrow g_m = \frac{\mu_{eff}}{2}\frac{W}{L}C_{ox}(V_{gs}-V_T)\left[\frac{2}{1+\rho} - \frac{\rho}{(1+\rho)^2}\right]$$

$$= \mu_{eff}\frac{W}{L}C_{ox}(V_{gs}-V_T)\frac{1+\rho/2}{(1+\rho)^2}$$

<u>Calculate power consumption</u>

$$P_d = V_{DD}I_d = V_{DD}\,W C_{ox} v_{sat}\frac{(V_{gs}-V_T)^2}{(V_{gs}-V_T)+LE_{sat}}$$

Since $C_{gs} \simeq \frac{2}{3}WLC_{ox} \Rightarrow Q_L = \frac{1}{\omega_o R_s C_{gs}} = \frac{1}{\omega_o R_s\cdot\frac{2}{3}WLC_{ox}}$

$$= \frac{3}{2}\frac{1}{\omega_o R_s WLC_{ox}}$$

Since $P_d = V_{DD}\,WLC_{ox}E_{sat}v_{sat}\frac{\rho^2}{1+\rho}$

$$\Rightarrow \frac{1}{WLC_{ox}} = \frac{V_{DD}E_{sat}v_{sat}}{P_D}\frac{\rho^2}{1+\rho}$$

which gives $Q_L = \frac{3}{2}\frac{1}{P_d}\frac{V_{DD}E_{sat}v_{sat}}{\omega_o R_s}\frac{\rho^2}{1+\rho}$

$$= \frac{P_0}{P_d}\frac{\rho^2}{1+\rho}$$

Where $\quad P_o = \dfrac{3}{2}\,\dfrac{V_{dd}\,E_{sat}\,v_{sat}}{\omega_o R_s}$

Observations

- v_{sat} & E_{sat} are physical (technological) parameters
- V_{DD}, ω_o, R_s are design specifications

$\Rightarrow$ P_o is a constant i.e. independent of bias

Now express ω_T in terms of device parameters

$$\omega_T = \frac{g_m}{C_{gs}} = \frac{g_m}{\frac{2}{3}WLC_{ox}} = \frac{3}{2}\,\mu_{eff}\,\frac{V_{gs}-V_T}{L^2}\,\frac{1+\rho/2}{(1+\rho)^2}$$

$$= \frac{3}{2}\,\mu_{eff}\,\frac{V_{gs}-V_T}{L^2}\,\alpha$$

Since $\quad \alpha = \dfrac{g_m}{g_{do}} = \dfrac{1+\rho/2}{(1+\rho)^2}$

$$\therefore\ \omega_T = \frac{3}{\chi}\,\underbrace{\frac{\chi v_{sat}}{E_{sat}}\,\frac{V_{gs}-V_T}{L}}_{=\rho}\,\frac{1}{L}\,\alpha = \frac{3\alpha\,\rho\,v_{sat}}{L}$$

Now we can rewrite the expression for F

$$F = 1 + \frac{\gamma}{\alpha}\,\frac{\chi}{Q_L}\left(\frac{\omega_o}{\omega_T}\right)$$

$$= 1 + \frac{\gamma}{\alpha}\,\frac{\chi}{\frac{P_o}{P_d}\,\frac{\rho^2}{1+\rho}}\cdot\frac{\omega_o}{\frac{3\alpha\,\rho\,v_{sat}}{L}}$$

$$= 1 + \frac{\gamma\,\omega_o L}{3 v_{sat}}\,\frac{\chi}{\frac{P_o}{P_d}\,\frac{\rho^2}{1+\rho}\,\alpha^2\rho}$$

$$= 1 + \frac{\gamma\,\omega_o L}{3 v_{sat}}\,P(\rho, P_d)$$

where $\quad P(\rho, P_d) = \dfrac{\chi}{\frac{P_o}{P_d}\,\frac{\rho^2}{1+\rho}\,\alpha^2\rho}$

$$= \frac{1 - 2|c|\,\alpha\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta\alpha^2}{5\gamma}\left[1 + \left(\frac{P_o}{P_d}\,\frac{\rho^2}{1+\rho}\right)^2\right]}{\frac{P_o}{P_d}\,\frac{\rho^2}{1+\rho}\,\rho\,\underbrace{\frac{(1+\rho/2)^2}{(1+\rho)^4}}_{\alpha^2}}$$

$$\therefore\ P(\rho, P_d) = \frac{\frac{P_d}{P_o}\left[1 - 2|c|\,\frac{(1+\rho/2)}{(1+\rho)^2}\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta}{5\gamma}\,\frac{(1+\rho/2)^2}{(1+\rho)^4}\right] + \frac{\delta}{5\gamma}\,\frac{P_o}{P_d}\,\frac{\rho^4}{(1+\rho)^2}\cdot\frac{(1+\rho/2)^2}{(1+\rho)^4}}{\frac{\rho^3(1+\rho/2)^2(1+\rho)}{(1+\rho)^6}}$$

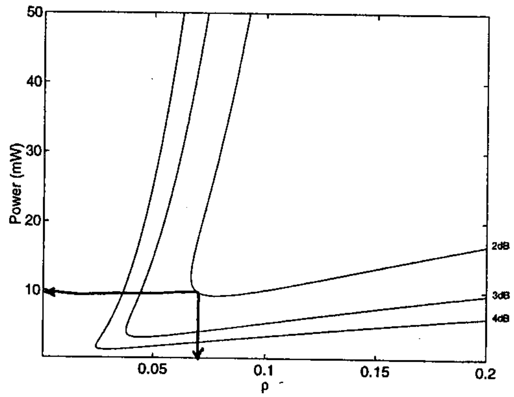
$$= \frac{\frac{P_d}{P_o}\left[(1+\rho)^6 - 2|c|\sqrt{\frac{\delta}{5\gamma}}\,(1+\rho)^4(1+\rho/2) + \frac{\delta}{5\gamma}\,(1+\rho/2)^2(1+\rho)^2\right] + \frac{P_o}{P_d}\,\frac{\delta}{5\gamma}\,\rho^4(1+\rho/2)^2}{\rho^3\,(1+\rho/2)^2\,(1+\rho)}$$

Note $\quad$ F is a function of two variables ρ & P_d.

Therefore, we can fix P_d and find ρ for the minimum F.

OR

Given F we can find the minimum P_d and the corresponding ρ



Constant NF contours can be plotted for different values of P_D and ρ as shown above.

Example For NF = 2dB $P_d(\min) \cong 10\,mW$, $\rho = 0.07$
For NF = 4dB $P_d(\min) \cong 1.5\,mW$, $\rho = 0.025$

For a special case $\rho \ll 1$, we can derive closed-form expressions that allow us to minimize F for a given P_d.

Assume $\rho \ll 1$ then

$$P(\rho, P_d) \doteq \frac{\frac{P_d}{P_o}\left[1 - 2|c|\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta}{5\gamma}\right] + \frac{P_o}{P_d}\frac{\delta}{5\gamma}\rho^4}{\rho^3}$$

Recall $F = 1 + \frac{\gamma \omega_o L}{3 v_{sat}} P(\rho, P_d)$

So minimize $P(\rho, P_d)$ for a given $P_d \Rightarrow \frac{\partial P(\rho, P_d)}{\partial \rho} = 0$

$$\Rightarrow \frac{P_d}{P_o}\left[1 - 2|c|\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta}{5\gamma}\right]\left(-\frac{3}{\rho^4}\right) + \frac{P_o}{P_d}\frac{\delta}{5\gamma} = 0$$

$$\therefore \frac{P_o}{P_d}\frac{\delta}{5\gamma}\rho^4 = \frac{P_d}{P_o}\, 3\left[1 - 2|c|\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta}{5\gamma}\right]$$

$$\Rightarrow \rho_{opt}^2 = \frac{P_d}{P_o}\sqrt{\frac{15\gamma}{\delta}}\sqrt{1 - 2|c|\sqrt{\frac{\delta}{5\gamma}} + \frac{\delta}{5\gamma}}$$

and $Q_{L\,opt} = \frac{P_o}{P_d}\frac{\rho_{opt}^2}{1+\rho_{opt}} \cong \frac{P_o}{P_d}\rho_{opt}^2$

$$= \sqrt{\frac{15\gamma}{\delta}}\sqrt{\left(1 + \frac{\delta}{5\gamma}\right) - 2|c|\sqrt{\frac{\delta}{5\gamma}}}$$

Substitute $\gamma = 2$, $\delta = 4$ & $|c| = 0.395$ then
$Q_{L,opt} \cong 2.6$

From Q_{Lopt} we have $\chi_{opt} = 1 - 2|c|\sqrt{\frac{\delta\alpha^2}{5\gamma}} + \frac{\delta\alpha^2}{5\gamma}\left(1 + Q_{Lopt}^2\right)$

where $\alpha = \frac{1 + \rho/2}{(1+\rho)^2} \cong 1$ i.e. a long-channel devic

$\therefore \frac{\chi_{opt}}{Q_{Lopt}} \cong 1.4 \Rightarrow F_{min} = 1 + 1.4\frac{\gamma}{\alpha}\left(\frac{\omega_o}{\omega_T}\right)$

For a long-channel device $\gamma = 2/3$, $\delta = 4/3$, $|c| = 0.395$
$Q_{L\,opt} \cong 3.8$ since $\frac{\delta}{\gamma} = 2$.

$\therefore F_{min} = 1 + 1.4\left(\frac{2}{3}\right)\left(\frac{\omega_o}{\omega_T}\right) = 1 + 0.93\left(\frac{\omega_o}{\omega_T}\right)$

For short-channel devices we have
$F_{min} > 1 + 0.93\left(\frac{\omega_o}{\omega_T}\right)$

Once $Q_{L,opt}$ is known then W for the optimum device can be calculated

$$Q_L = \frac{1}{\omega_0 R_s C_{gs}} = \frac{1}{\omega_0 R_s \frac{2}{3} W L C_{ox}}$$

$$\therefore \quad W_{L,opt} = \frac{3}{2}\,\frac{1}{L C_{ox} R_s Q_{L,opt}} \cong \frac{1}{3\omega_0 L C_{ox} R_s}$$

assuming $Q_{L,opt} \cong 4.5$

Remarks

- absolute minimum F is the transistor F_{min}

$$F_{min} = 1 + \frac{2}{\sqrt{5}}\,\frac{\omega}{\omega_T}\sqrt{\gamma\delta\left(1-|c|^2\right)} = 1 + 2.3\left(\frac{\omega}{\omega_T}\right)$$

- For the common-source LNA

$$F_{min} = 1 + 1.4\,\frac{\gamma}{\alpha}\left(\frac{\omega}{\omega_T}\right)$$

Example $\quad \omega/\omega_T = \frac{1}{10}$, $\gamma = 2$, $\alpha \cong 1$

F_{min} (device) $= 1 + 2.3\left(\frac{1}{10}\right) = 1.23$ (0.9dB)

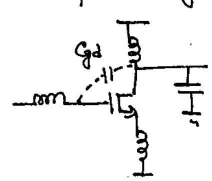
F_{min} (CS amp) $= 1 + 1.4\,\frac{2}{1}\left(\frac{1}{10}\right) = 1.28$ (1.1dB)

i.e. excellent noise figures are possible.

Remark

For narrowband applications it is advantageous to tune out the output capacitance for improved gain.

C_{gd} can cause problems due to its effect on the input impedance

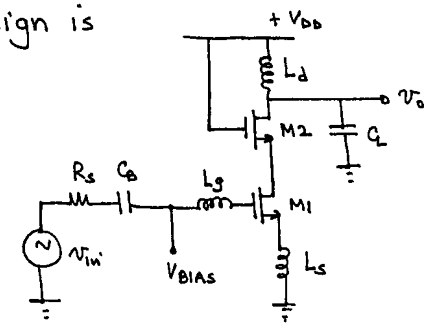


C_{gd} causes several problems by coupling the input and output tuned circuits

- loading of the output tank
- detuning of the tank
- potential for instability

To avoid this problem decouple the input and output by a cascode device

Hence, the overall LNA design is



M2 reduces the effect of M1's C_{gd} and the interaction of the input and output tuned circuits

L_d resonates with capacitance at drain of M2

$\Rightarrow$ increased gain & bandpass filtering

C_B is a blocking capacitor that allows M1 to be biased properly. It should have a negligible effect at the signal frequency.

Note $\quad C_B$ is in series with C_{gs}, so the effective series capacitance is

$$\frac{1}{C_{eff}} = \frac{1}{C_B} + \frac{1}{C_{gs}} \cong \frac{1}{C_{gs}} \quad \text{for } C_B \gg C_{gs}$$

If M2 and M1 are sized the same then the Miller capacitance at the input is $2C_{gd}$ in parallel with C_{gs}