Current (I or i)

- The time rate of change of charge is known as electric current
  - Units of current are amperes (A) \( 1 \text{ A} = 1 \text{ C/s} \)

\[
i(t) = \frac{dq}{dt} \quad \text{(A)}
\]

- Conventional current flow is the movement of positive charge
- For all circuit elements \( i_{\text{in}} = i_{\text{out}} \)
- Charge transferred between time \( t_0 \) and \( t \)

\[
q(t) = \int_{t_0}^{t} i(t) dt \quad \text{(C)}
\]
Water Flow Analogy (Current)

- Consider the flow of water in a pipe
  - flow of water ⇔ flow of charge
  - flow rate ⇔ current

Water Flow Analogy (Voltage)

- The higher tank results in more water flow
Water Flow Analogy

- A pump forces water to a height $h$ above the reference level $\leftrightarrow$ potential energy

Voltage (Potential Difference)

- The voltage $V_{ab}$ between two points in an electric circuit is the energy (or work) needed to move a unit positive charge from $b$ to $a$
- The units of voltage are Volts (V)
  - $1 \text{ V} = 1 \text{ J}/1 \text{ C}$
  - $v_{ab} = \frac{dW}{dq}$
- Electrical system would do 12 J of work to move 1 C from $b$ to $a$
- $V_{ab}$ is the voltage (potential difference) between $a$ and $b$
  - $a$ is at a potential $V_{ab}$ higher than $b$
  - the potential at $a$ w.r.t. $b$ is $V_{ab}$
  - the potential at $b$ is $V_{ab}$ lower than $a$
Voltage

- A voltage drop from $a$ to $b$ is equivalent to a voltage rise from $b$ to $a$
  \[ V_{ab} = -V_{ba} \]

- $a$ is 12 V above $b$
- $b$ is 12 V below $a$
- 12 V voltage rise from $b$ to $a$
- 12 V voltage drop from $a$ to $b$
Electrical/Mechanical/Fluid/Thermal Analogs

<table>
<thead>
<tr>
<th>General Description</th>
<th>Electrical</th>
<th>Mechanical</th>
<th>Fluid</th>
<th>Thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motion (flow) “through” variable</td>
<td>Current, I</td>
<td>Velocity</td>
<td>Flow rate</td>
<td>Heat flow</td>
</tr>
<tr>
<td>Push (effort) “across” variable</td>
<td>Voltage, V</td>
<td>Force</td>
<td>Pressure</td>
<td>Temperature</td>
</tr>
</tbody>
</table>

Measuring Voltage and Current

**Measurement Instruments**

- **Voltmeter**
  - Measures voltage (potential difference) in Volts (V)
  - Connected in parallel (voltage is an “across” variable)
  - Ideally does not draw current

- **Ammeter**
  - Measures current in Amperes (A)
  - Connected in series (current is a “through” variable)
    - **Current must flow through the instrument**
  - Ideally does not drop voltage
### Independent Voltage and Current Sources

**Independent Sources**

<table>
<thead>
<tr>
<th>Source Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ideal Voltage Source</strong></td>
<td>Provides a specified voltage completely independent of the circuit elements connected to it</td>
</tr>
<tr>
<td>Battery</td>
<td>or</td>
</tr>
<tr>
<td>dc source</td>
<td>Any source*</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ideal Current Source</strong></td>
<td>Provides a specified current completely independent of the circuit elements connected to it</td>
</tr>
<tr>
<td>dc source</td>
<td>Any source</td>
</tr>
</tbody>
</table>

### Example

- **Circuit 1**
  - 2 V
  - 1 A

- **Circuit 2**
  - 2 V
  - 1 A
Reference direction for current

\[ \rightarrow 2A \]

Power and Energy

Useful for electric circuits
- Light bulbs: 60W, 100W power ratings
- Electric bills: kW-h based on energy consumed

Power
- Work done per unit time
- Rate of expending or absorbing energy
\[ P = \frac{dw}{dt} \]

\[ 1W = 1J/s \]

\[ P = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = \vec{v} \cdot \vec{i} \quad (W = V \cdot A) \]

Power is a time varying quantity (in general) and at a given instance of time you measure instantaneous power.

An element can either absorb (or dissipate) power or it can supply power.

**Passive element**
- Element that dissipates power, e.g., bulb.

**Active element**
- Element that supplies power, e.g., battery, generator.

**Passive sign convention**
- Current enters into the positive terminal
\[ P = VI \]
\[ p > 0 \implies \text{power absorbed} \]
\[ p < 0 \implies \text{power supplied} \]

\[ p = 4 \times 3 = 12 \text{W} \]

Absorbed

Power absorbed = - Power supplied

\[ E = \int_{t_0}^{t} p(t) \, dt \]

\[ W = \int_{t_0}^{t} p(t) \, dt \]

is the energy absorbed or supplied by an element from time \( t_0 \) to \( t \)

Electric bill measures energy in kWh (kilo watt hour)

\[ 1 \text{ kWh} = \left(10^3 \text{ W}\right) \left(1 \text{ h}\right) = 1000 \text{ Wh} \]

\[ 1 \text{ W} = 1 \text{ J/s} \]

\[ 1000 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^6 \text{ J} = 3.6 \text{ MJ} \]