Assignment #6 due Today 11:59 PM

Assignment #7 available in Canvas
Due Thu March 12 11:59 PM

Capacitor

\[ q = CV \]

Where \( C \) is the capacitance in F

\( C \) is independent of \( q \) vs \( V \)

\[ C = \frac{\varepsilon A}{d} \]

\( \varepsilon \) = permittivity of the dielectric

The current flow is referred to as displacement current

\[ i = \frac{dQ}{dt} = \frac{d}{dt}(CV) = C \frac{dv}{dt} \]

Observation 1

If a dc (or constant) voltage is applied to a capacitor,

\[ i = C \frac{dv}{dt} = 0 \]

i.e., the capacitor acts (behaves) like an open circuit for dc

Observation 2

The voltage across a capacitor cannot change instantaneously (abruptly)

\[ \frac{dv}{dt} \rightarrow \infty \Rightarrow i = \infty \]

not practically possible

However, the current can change abruptly
Ex: \( v(t) = 7t + 3 \) \( V \) \( \forall \) \( C = 2 \text{ F} \)

\[ i = \frac{dv}{dt} = 2 \times 7 = 14 \text{ A} \]

Calculating voltage given the current
\[
\frac{dv}{dt} = \frac{1}{C} v \Rightarrow v(t) = \frac{1}{C} \int i \, dt
\]
\[
= \frac{1}{C} \int_{t_0}^{t} i \, dt + \frac{1}{C} \int_{-\infty}^{t_0} i \, dt
\]
\[
= v(t_0) + \frac{1}{C} \int_{t_0}^{t} i \, dt
\]

**Remark**

Since a capacitor stores charge, it is an energy storage element. The capacitor stores charge when charging and releases charge when discharging.

charging \( + \frac{1}{i} \)

\( \uparrow \)

discharging \( - \frac{1}{i} \)

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An ideal capacitor does not dissipate energy

**Power and energy**

\[
P = v i = v \left( \frac{cdv}{dt} \right) = c \frac{d}{dt} v^2
\]

Energy stored on the capacitor is

\[
w = \int_{-\infty}^{t} p \, dt = c \int_{-\infty}^{t} \frac{d}{dt} \left( \frac{1}{2} v^2 \right) \, dt = \frac{1}{2} C v^2
\]

Example

\[ v_c = \frac{cdv_c}{dt} \]

\[ P_c = v_c i_c \]

\[ W = \int_{-\infty}^{t} P_c \, dt \]

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\[ v_c(t) \]

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\[ i_c(t) \]

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\[ P_c(t) \]

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\[ w(t) \]

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\[ \text{charging} \]

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\[ \text{absorbing} \]

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\[ \text{supplying} \]

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\[ \text{energy delivered} \]

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\[ \text{energy stored} \]
Parallel Combination

\[ i \quad + \quad \frac{1}{C_1} \quad - \quad \frac{1}{C_2} \quad - \quad \frac{1}{C_3} \quad - \quad \cdots \quad - \quad \frac{1}{C_n} = i \quad + \quad \frac{1}{C_{eq}} \quad - \quad \frac{1}{C_1} \quad - \quad \frac{1}{C_2} \quad - \quad \frac{1}{C_3} \quad - \quad \cdots \quad - \quad \frac{1}{C_n} \]

\[ C_{eq} = C_1 + C_2 + \cdots + C_n \]

Series combination

\[ \frac{1}{c_1} \quad - \quad \frac{1}{c_2} \quad - \quad \frac{1}{c_3} \quad - \quad \cdots \quad - \quad \frac{1}{c_n} = \frac{1}{C_{eq}} \quad - \quad \frac{1}{c_1} \quad - \quad \frac{1}{c_2} \quad - \quad \frac{1}{c_3} \quad - \quad \cdots \quad - \quad \frac{1}{c_n} \]

\[ \frac{1}{C_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \cdots + \frac{1}{c_n} \]

For two capacitors in series:

\[ \frac{1}{C_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} \]

\[ \Rightarrow C_{eq} = \frac{c_1 c_2}{c_1 + c_2} \]

Ex:

\[ \begin{align*} a \quad 40 \mu F \quad b \\ \text{120} \mu F \quad a \quad 60 \mu F \quad b \\ \frac{120 \times 60}{120 + 60} = 40 \mu F \end{align*} \]

Inductor

An element that stores energy in its magnetic field

\[ v = L \frac{di}{dt} \]

Where \( L \) is the inductance in Henry (H)

\[ L \propto N, \mu, A \]
• For a dc (constant) current \( \frac{di}{dt} = 0 \Rightarrow v = 0 \)
  i.e. the inductor behaves like a short circuit for dc

• The current through an inductor cannot change instantaneously (abruptly)

\[
\text{not possible}
\]

An ideal inductor does not dissipate energy. The inductor absorbs power from the circuit when storing energy and delivers power to the circuit when releasing previously stored energy.