ENGR 201  Lecture #16  MARCH 5, 2020

\[ i = C \frac{dv}{dt} \]
\[ v = \frac{1}{C} \int i \, dt + v(t_0) \]
\[ i(t) = \frac{1}{L} \int v \, dt + i(t_0) \]

At dc: \( i = 0 \Rightarrow \) open circuit
At dc: \( v = 0 \Rightarrow \) short circuit

Voltage across a capacitor cannot change abruptly
Current through an inductor cannot change abruptly

\[ L \]
\[ v = \frac{1}{L} \int i \, dt \]
\[ v = \frac{1}{L} \int i \, dt \]

\[ \text{Ex} \]
Find \( i_L \) and \( v_c \)
(\( dc \))

For \( dc \): Inductor \( \Rightarrow \) short circuit
Capacitor \( \Rightarrow \) open circuit

\[ i_L = \frac{3}{3+1} \times 4 = 3 \text{A} \]
Current division

\[ v_c = \text{voltage drop across } 1 \Omega \text{ resistor} \]
\[ = 3 \times 1 = 3 \text{V} \]

Power and Energy for Inductor

\[ P = \frac{1}{2} L \frac{dv}{dt} = \frac{1}{2} L i^2 \]

Energy: \( W = \int_{-\infty}^{t} P \, dt = \frac{1}{2} L i^2 \)
Series Combination of Inductors

\[ L_{eq} = L_1 + L_2 + \cdots + L_n \]

Inductors in Parallel

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n} \]

For 2 inductors in parallel

\[ L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \]

Ex:

\[ L_{eq} = \frac{30 \times 60}{30 + 60} = 20 \text{ H} \]

\[ \begin{align*}
\begin{array}{c}
\text{At } t = 0^- \\
5V \\
\text{open circuit for } C
\end{array}
\end{align*} \]

\[ \begin{align*}
\begin{array}{c}
\text{At } t = 0^+ \\
5V \\
\text{closed circuit for } C
\end{array}
\end{align*} \]

\[ L_{eq} \]

\[ \begin{align*}
\begin{array}{c}
\text{At } t = 0^- \\
5V \\
\text{short circuit for } L
\end{array}
\end{align*} \]

\[ \begin{align*}
\begin{array}{c}
\text{At } t = 0^+ \\
5V \\
\text{closed circuit for } L
\end{array}
\end{align*} \]

KCL:

\[ i_C + i_R = 0 \]

\[ \frac{di_C}{dt} \frac{V}{R} + i_C - \frac{V}{R} = 0 \]

\[ L \frac{di}{dt} + Ri = 0 \]

\[ \frac{di}{dt} + i = 0 \]

\[ L_{eq} \]

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\begin{array}{c}
\text{At } t = 0^- \\
5V \\
\text{short circuit for } L
\end{array}
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\begin{array}{c}
\text{At } t = 0^+ \\
5V \\
\text{closed circuit for } L
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KCL:

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\[ L \frac{di}{dt} + Ri = 0 \]

\[ \frac{di}{dt} + i = 0 \]

\[ L_{eq} \]
Generic equation: \( \frac{dx}{dt} + x = 0 \) where \( x = v \) or \( i \)

First-order differential equation (source free)

Solution of the differential equation is

\[ x(t) = X_0 e^{-\frac{t}{\tau}} \]

where \( X_0 = x(0) \) initial condition

For the RC circuit: \( v(t) = V_0 e^{-\frac{t}{\tau}} \)

where \( V_0 = 5V \); \( \tau = RC \)

For the RL circuit: \( i(t) = I_0 e^{-\frac{t}{\tau}} \)

where \( I_0 = 5A \); \( \tau = LR \)

For the RC circuit:

\[ v(t) = 5e^{-\frac{t}{\tau}} = 5e^{-\frac{t}{0.37}} \]

\( \tau = \text{time constant (seconds)} \)

it is the time required for the response to decay by a factor of \( \frac{1}{e} \) or 0.37 of its initial value

\[ 5\tau < 1\% \text{ of the initial value} \]

\( \tau = \frac{RC}{1} \)

Ex

At \( t = 0^- \)

\( v_c(0^-) = \text{voltage drop across 9Ω resistor} \)

voltage division: \( v_c(0) = \frac{9}{3+9} \times 20 = 15V = v_c(0) \)

For \( t > 0 \) \( \tau = 9Ω + 1Ω = 10Ω \)

\( \tau = \text{RL} C = 10 \times 20 \times 10^{-3} \text{ s} \)

\[ v_c(t) = 15e^{-\frac{t}{0.2}} = 15e^{-5t} \text{ V} \]
\[ V_1 \quad 0 \quad V_2 \quad R \quad C \]